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Effect of Trapped Particles on the Drift-Cyclotron Instability in a Multipole*

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The effect of trapped particles on the drift-cyclotron instability in a multipole machine has been considered. Dispersion equations have been obtained for this instability in a nonuniform collisionless plasma using a simplified approach which takes into consideration the magnetic field strength and curvature variation along field lines. The results show the existence of a branch of oscillations with frequency $|\omega| \sim \Omega_0 - k\bar{v}_g - \bar{\omega}_b$, where Ω_0, $\bar{v}_g$, and $\bar{\omega}_b$ are the average gyrofrequency, the drift velocity due to magnetic field gradient, and the ion "bounce" frequency, respectively, with k being the wave vector transverse to the magnetic field. This may explain the discrepancy between the experimentally observed frequency and the theoretical value obtained previously. This branch of the oscillation also shows a larger growth rate.

INTRODUCTION

Low-frequency fluctuations[1] observed in the Gulf General Atomic octopole device have characteristics consistent with the theoretical prediction of the drift-cyclotron instability.[2] The theoretical prediction of the dependence of the frequency on the magnetic field and plasma density appears to be verified by the experimental results except that the absolute value of the calculated frequency is larger than the observed value by about a factor of 2 to 3. The theoretical calculations were made after neglecting the inhomogeneity of the magnetic field along the lines of force, and thus the effect of magnetically trapped particles was neglected.

In the octopole device, the magnetic field strength varies by a factor of 4 or more along the line of force. The particle orbits used in the calculation would thus be affected and, hence, the use of proper orbit in calculating perturbed distributions may be important enough to make up for the difference of a factor 2 or 3 between the observed and calculated frequencies. It has been suggested[3] that the "bounce" frequency of trapped particles (ions executing "bouncing" motion) may introduce a large enough Doppler shift (over and above that due to curvature) so that the shifted cyclotron frequency would be low enough to agree with the experimental value.

A fairly comprehensive theory of low-frequency drift waves in an average well has been given by Rutherford *et al.*,[4] where they have considered the low-frequency stability of low-β plasmas in axisymmetric tori of field strength along the field lines. They have obtained various modes and derived general stability criteria. But in order to obtain more quantitative information about any of these modes for the sake of comparison with experimental results, it is necessary to carry out more elaborate calculations using realistic particle orbits. As is obvious, such a calculation is prohibitively difficult for particles that follow complicated orbits in a realistic field geometry, e.g., a multipole configuration. Hence, some suitably simplified model for the particle orbits needs to be used. Coppi *et al.*[5] have considered collisionless microinstabilities in configurations with periodic magnetic curvature using a two-dimensional model simulating the effects of magnetic curvature variation, magnetic shear, and particle trapping. In their model, they have considered a fixed "gravity" (in the sense that all the particles have the same drift velocity) but allowed for the existence of "good" and "bad" curvature regions. The model used in this paper is somewhat similar to that of Coppi *et al.*, though it is comparatively simple minded and straightforward. We also allow for the dispersion in the curvature drift velocities of the particles.

In our calculations, we use the local approximation, which may not be justifiable in view of the fact that, in the octopole device, the relative change in the density and magnetic field in one gyroradius is not negligible. In spite of this, the idea is to fit the experimental results quantitatively as far as possible with the theoretically calculated drift-cyclotron mode of instability, now that the qualitative agreement between the two results looks encouraging.

In the following, we describe a simplified model which is later used to obtain the dispersion relation of the drift-cyclotron mode of oscillation. Our objective is to determine the effect of trapped particles on such a mode and possibly explain the quantitative discrepancy between the experimental and theoretical results. A complete numerical solution of the dispersion relation has been obtained, which indicates the existence of two branches for the mode of oscillation, one with $|\omega| \sim \Omega_0 - k\bar{v}_g$ and the other with $|\omega| \sim \Omega_0 - k\bar{v}_g - \bar{\omega}_b$ with different growth rates (Ω_0, $\bar{v}_g$, and $\bar{\omega}_b$ are the average values for gyrofrequency, drift velocity, and bounce frequency, respectively).

THEORY

We consider a system having low β and we limit our consideration to electrostatic perturbations from equilibrium and then linearizing the Vlasov equation and integrating along particle trajectories, we have

$$\hat{f}_j = (e_j/m_j) \int_{-\infty}^{t} dt' \nabla\hat{\varphi} \circ \nabla_v f_{0j} \ ,$$

$$\nabla^2 \hat{\varphi} = \sum_j e_j \int \hat{f}_j \, d^3 v,$$

where the equilibrium distribution function for the species j is given by

$$f_{0j} = n_{0j} (\alpha_j/\pi)^{3/2} e^{-\alpha_j v^2} [1 - \epsilon_j (x + v_y/\Omega_j)], \quad (1)$$

$$j = i, e \ ,$$

together with

$$\Omega_j = (e_j B)/(m_j c) \ , \qquad \epsilon_j = \langle\langle d(\ln n)/dx \rangle\rangle_j \ ,$$
$$\alpha_j = m_j/(2kT_j) \ , \quad (2)$$

remembering that $\langle\langle \partial(\ln n)/\partial x \rangle\rangle_j = 1/r$ is to be taken small, so that $a_i/r \ll 1$, a_i being the ion Larmor radius. In order that charge neutrality holds, we have

$$\langle\langle \partial(\ln n)/\partial x \rangle\rangle_i = \langle\langle \partial(\ln n)/\partial x \rangle\rangle_e \ .$$

In our model, we have to take into consideration two important facts. Remembering that a pure multipole field is two-dimensional with closed field lines, our model must include the facts that the magnetic field varies in strength along the line of force and that the magnetic field curvature has "good" ("cusplike") and "bad" ("mirrorlike") regions. Thus, in the system there are some particles which have bad drift (in the sense that their velocity is such that they spend a long time in the bad curvature region) although, on the average, the particles have good drift. First, in order to allow for the bad particles, we consider a model which has been described elsewhere.[6,7] A brief description is given herewith for completeness.

It may be shown that in a magnetic field produced by an axially symmetric electron beam, a charged particle drifts in the z direction with a velocity given by

$$v_g = -[mc/(eBr)][(v_\perp{}^2/2)(\partial(\ln B/\partial(\ln r)) - v_\parallel{}^2],$$

where, after averaging over θ (the angle between the resultant velocity vector and the z direction in the spherical coordinate system), we have

$$\langle v_g \rangle = -\tfrac{1}{3}[mcv^2/(eBr)]\,[(2(\ln\beta)/\partial(\ln r)) - 1] \ .$$

Thus, we may have $\langle v_g \rangle \gtrless 0$ if $\partial(\ln B)/\partial(\ln r) \gtrless 1$, whereas there always are particles with $v_g > 0$.

For the case of a magnetic well produced by a multipole field geometry, the actual drift velocity would be a very complicated function of v and θ. Figure 1(a) shows the typical behavior of magnetic field curvature as a function of θ. A particle of type (1) with $\theta = \frac{1}{2}\pi$ experiences a good curvature in the region G, whereas a particle of type (2), which is moving along the field line ($\theta = 0$), goes through regions of good and bad curvature and, on the average (in the sense of $\oint dl/B$), may experience a good curvature. The most affected particles would be those with some intermediate value of θ such that they spend a lot of time in the bad curvature region C. Figure 1(b) shows schematically the regions of good and bad gravity as experienced by particles with angular orientation lying between $\frac{1}{2}\pi$ and 0. Around $\theta = \frac{1}{2}\pi$, the particle drift is always in the good direction. Near $\theta = 0$, i.e., for the circulating particle, the drift is good or bad depending on whether the field line (along which the particle is moving) lies within the

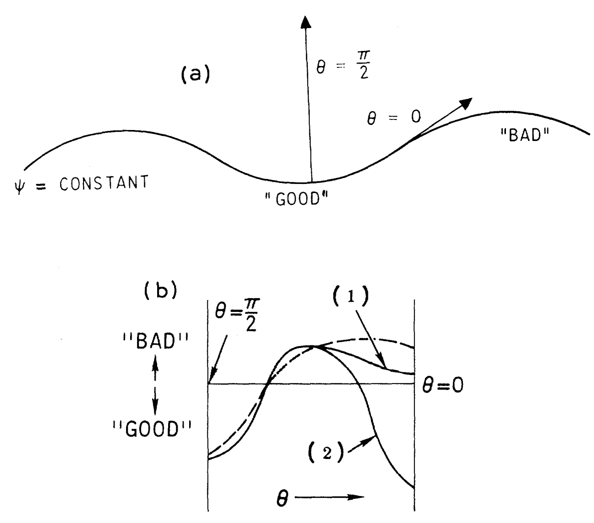

FIG. 1. (a) Typical flux line; (b) schematic diagram for the dependence of drift velocity on θ.

minimum of $\oint dl$ or not.[8] The curves (1) and (2) in Fig. 1(b) schematically represent the situation outside the minimum of $\oint dl$ and inside of the minimum, respectively. Experimental measurements on fluctuations generally have been made outside of the minimum of $\oint dl$ but inside of $\oint dl/B$. For the octopole, the two minima are located close. A curve that approximately simulates this behavior of magnetic field curvature is shown as dotted lines in Fig. 1(b). From this, an approximate expression for drift velocity may be written

$$v_g(v) \equiv v^2(\cos^2\theta/R_1 + \sin^2\theta/R_2)$$
$$= v_{\parallel}{}^2/R_1 + v_{\perp}{}^2/R_2 \,, \tag{2'}$$

where the magnitude and polarity of the quantities R_1 and R_2 have to be suitably chosen to simulate realistic conditions as close as possible.

In order to take into consideration the fact that the magnetic field along a field line is changing, we consider the following simplified lowest-order expressions for the particle orbit:

$$\begin{aligned} x(t') - x &= \frac{v_\perp}{\Omega_0}\{\sin[\int^{t'} \Omega(s(t))\,dt - \alpha] - \sin\alpha\}, \\ y(t') - y &= \frac{v_\perp}{\Omega_0}\{\cos[\int^{t'} \Omega(s(t))\,dt - \alpha] - \cos\alpha\} \\ &\quad + \int^{t'} v_g(s(t))\,dt, \\ z(t') - z &= \int^{t'} v_g^*(s(t))\,dt, \end{aligned} \tag{3}$$

where we write

$$\Omega(s(t)) = \Omega_0(1 + \delta\cos s/L),$$

$$v_g(s(t)) = v_g(v) + \tilde{v}_g(s(t)),$$

$$\tilde{v}_g(s(t)) = v_{g_1}\cos s/L,$$

$$v_g^*(s(t)) = g^*\cos s/L, \text{ for trapped particles}$$
$$= g^{**}\cos s/L_s,$$

for circulating particles, ;

s is the coordinate along a line of force; v_{g_1}, g^*, g^{**}, and δ are certain parameters; L is a characteristic length connection between good and bad regions (for the *GGA* multipoles $L \sim R_C$, radius of curvature of the line of force); and L_s is the line length. The quantity $v_g(v)$ has been defined before [see v_g in Eq. (2')]. For simplicity, we have assumed the pertinent physical quantities to have simple periodic dependence on s.

For particles with $\theta \approx 0$, i.e., $v_\parallel > v_\perp$, the motion could be considered to be a free circulation along the line of force. Hence, for these particles $s \approx v_\parallel t$. For particles whose $v_\perp > v_\parallel$, the motion along the line of force may be considered to be determined by a magnetic field of the form

$$B(s) \approx B_{\min}(0) + \tfrac{1}{2}B''s^2,$$

where we have expanded the magnetic field near its minimum $B_{\min}(0)$ at $s=0$, B'' being the second derivative of B with respect to s. The equation of motion then may be shown to be given by

$$m(d^2s/dt^2) = -\mu B''s\,, \tag{4}$$

where $\mu = mv_\perp{}^2/B$, which is obtained from the equation $m(d^2s/dt^2) = -(\partial/\partial s)(\mu B(s))$. Equation (4) has a solution of the form

$$s = s_0\sin(qv_\perp t), \tag{5}$$

where $q = (B''/B)^{1/2} \sim h/L$,

h being the modulation factor ($h \sim R_C/L \sim 1$). Assuming $s_0 \sim L$ and noting that the range of t that we are interested in is such that $vt/L \ll 1$, we have

$$s \sim L\frac{h|v_\perp|t}{L} \sim |v_\perp|t, \quad |v_\perp| > |v_\parallel|\,.$$

This is equivalent to the assumption that the mode is characterized by poor communication over the distance L (this applies to the ions).

We consider modes of the form $\hat{\phi}(x,z)e^{i(ky+\omega t)}$ assuming that they are localized in the x direction and $k \gg \partial/\partial x$. This leads us to consider $\hat{\phi}(x,z) \approx \hat{\phi}(z)$ and n and dn/dx as constants in the lowest order. Thus, we consider $\hat{\phi}(z) = \phi_0$ a constant, reminding us that we have taken into consideration the effect of oscillation along the line of force in an approximate way through the g-drift term in the particle orbits. Thus, the Poisson equation now becomes (at $x \approx 0$)

$$k^2\phi = 4\pi\sum_j(e_j{}^2/m_j)\int d^3v(\alpha_j/\pi)^{3/2}(-2\alpha_j v_\perp)e^{-\alpha_j v^2} - 4\pi\sum_j(e_j{}^2/m_j)\int d^3v\int_{-\infty}^{\infty}dt'$$

$$\times e^{ik(y'-y)+i\omega t'}[2i\omega(\alpha_j/\pi)^{3/2}(-\alpha_j)e^{-\alpha_j v^2}-\epsilon_j'(ik/\Omega_j)e^{-\alpha_j v^2}(\alpha_j/\pi)^{3/2}] .$$

To do the above integrals, we use the following relations:

$$e^{iz\sin\theta}=\sum_l J_l(z)e^{il\theta} ,$$

$$e^{-iz\cos\theta}=\sum_l J_l(z)e^{-il(\pi/2+\theta)} ,$$

where $J_l(z)$ is the Bessel function of order l with argument z. Using these relations wherever necessary, one can show that

$$e^{ik(y'-y)+i\omega t'}=\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\sum_{l=-\infty}^{+\infty}J_m(kv_\perp/\Omega_j)J_n(kv_\perp/\Omega_j)e^{i(n-m)\theta+i(n-m)(\pi/2)}$$

$$\times e^{it'[kv_g(v)-n\Omega_0+\omega]}J_l[(kv_{g_1}-n\delta\Omega_{0_j})/\omega_b(v)]e^{-il\omega_b(v)t'} ,$$

where we have defined the following:

$$\omega_b(v)\sim v_{\parallel}/L,\quad v_{\parallel}>v_\perp ,\qquad \sim v_\perp/L,\quad v_\perp>v_{\parallel} .$$

After doing the time integral, we do the velocity integral after transforming to spherical coordinates in velocity space, i.e.,

$$d^3v=v^2dv\,\sin\varphi d\varphi d\theta ,\quad 0\leq\theta\leq 2\pi ,$$

$$0\leq\varphi\leq\pi ,\quad 0\leq v\leq\infty ;$$

note that ϕ here corresponds to θ in Fig. 1. The result, after doing the integral over θ, may be written

$$\int d^3v\int_{-\infty}^{0}dt'e^{ik(y'-y)+i\omega t'}=\int_0^\infty v^2dv\int_0^\pi \sin\phi d\phi\sum_n\sum_l\frac{J_n^2(kv_\perp/\Omega_j)J_l[(kv_{g_1}-n\delta\Omega_{0_j})/\omega_b(v)]}{i[\omega-kv_g(v)+n\Omega_{0_j}+l\omega_b(v)]} .$$

We now simplify the above expression in several ways so that it becomes suitable for discussion of the drift-cyclotron mode. Since, $T_i\gg T_e\simeq 0$, $\omega\ll\Omega_e$, and $ka_e<1$ (a_e being the electron Larmor radius), we consider only $n=0$ terms for the electron and neglect its bouncing motion; also we use the approximation

$$J_0^2(ka_e)\sim 1 .$$

For the ions, we keep the $n=1$ term and $l=-1$ term in order to look for possible resonances of the form $|\omega|\approx\Omega_0-k\bar{v}_g-\bar{\omega}_b$, with $\Omega_0=\Omega_{0i}$. We also include the $l=0$ term.

After doing all the above simplifications and other suitable arrangements, we have the dispersion relation

$$F=1+k^2\lambda_D^2+(kv_{c_i}-k\bar{v}_{g_i})/(\omega+kv_{g_e})-(\omega-kv_{c_i})\left(\frac{\alpha_i}{\pi}\right)^{3/2}\int e^{-\alpha_i v^2}v^2\sin\phi d\phi dv$$

$$\times\left\{\frac{J_1^2(kv_\perp/\Omega_0)J_0[(\delta\Omega_0-kv_{g_1})/\omega_b(v)]}{\omega-kv_{g_i}+\Omega_0}+\frac{J_1^2(kv_\perp/\Omega_0)J_1[(\delta\Omega_0-kv_{g_1})/\omega_b(v)]}{\omega-kv_{g_i}+\Omega_0+\omega_b(v)}\right\} , \tag{6}$$

where $\omega_b(v)$ is expressed in terms of the magnitude v and the angle ϕ in velocity space, and where $v_c=n'/(2n|\alpha\Omega|)$, v_{g_e} = a constant "drift" term for the electron. In order to bring Eq. (6) down to an amenable form, we make the further simplification of replacing the quantity v, magnitude of the total velocity of a particle, by its average value wherever it occurs in the quantity inside the $\{\}$ bracket in Eq. (6). Thus, in our model, we allow dispersion in the orientation of the velocity vector with respect to the magnetic field, not in the absolute magnitude. For our present purpose, this is a justifiable approximation.

$$\int_{-1}^{+1} dx J_1^{\,2}(s_i)\left(\frac{J_0(\sigma_i)}{\omega-(k_\perp v_i^2/\Omega_0)(x^2/R_1+(1-x^2)/R_2)}+\frac{J_1(\sigma_i)}{\omega-(k_\perp v_i^2/\Omega_0)(x^2/R_1+(1-x^2)/R_2)}\right), \tag{7}$$

where $x=\cos\phi$,

$$s_i=(k_\perp v_i/\Omega_0)(1-x^2)^{1/2},$$

$$\sigma_i=(\delta\Omega_0-kv_{g_1})/\omega_b(\phi),$$

and $\omega_b(\phi)$ was determined by means of the following considerations: Following Fig. 2, which shows a typical pattern along a line of force due to a multipole, we note that depending on the angle ϕ, particles will be either free-running or trapped between two successive "high" points in the magnetic field distribution. A plot of the particle oscillation frequency along a field line would look roughly like what is shown in Fig. 3. For particles of type (1) (i.e., particles moving totally along the field line), the frequency is that of a complete circulation along the field line; the frequency ω_0 may be roughly given by

$$\omega_0 \sim v_i/L_s,$$

L_S being the total line length. For particles of type (2) (i.e., just trapped particles, with $\phi=\phi^*$, ϕ^* being the corresponding mirror angle), the frequency of oscillation vanishes since these particles spend a long time near the mirror points. For particles of type (3) [i.e., particles with $(\pi-\phi^*)>\phi>\phi^*$], the bounce frequency is finite and is approximately given by $\omega_1\sim v_i/L$. Particles of type (4) (i.e., particles with $\phi=\pi$), again have free circulations. The situation represented in Fig. 4 may be mathematically written

$$\begin{aligned}\omega_b(\phi)&=\omega_0\cos[\pi\phi/(2\phi^*)]\ ,\\ &=\omega_1\cos[\pi(\phi-\tfrac{1}{2}\pi)/2(\phi^*-\tfrac{1}{2}\pi)]\,,\ (\pi-\phi^*)\geq\phi\geq\phi^*\,,\\ &=\omega_0\cos[\pi(\pi-\phi)/2\phi^*]\,,\ \pi\geq\phi\geq(\pi-\phi^*)\ .\end{aligned} \tag{8}$$

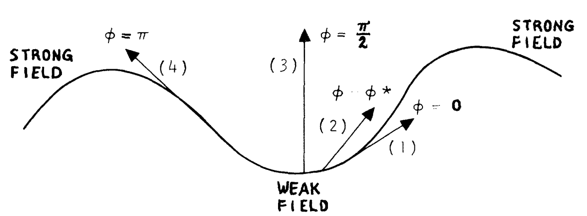

FIG. 2. Particle velocity vectors along a typical field line.

Expressions given in Eq. (8) necessitate the execution of the integral defined by Eq. (7) in three separate parts. Finally, the dispersion relation may be written as

$$F=1+k^2\lambda_D^{\,2}+\frac{kv_c-k\bar{v}_{g_i}}{\omega+kv_{g_e}}-\tfrac{1}{2}\,(\omega-kv_{c_i})D(\omega,k)=0, \tag{9}$$

where $D(\omega,k)$ is being defined by Eq. (7).

For large k such that $ka_i\gg1$ (a_i being the ion Larmor radius), one may expand the Bessel functions with arguments $k\sigma_i$, remembering that $J_1^2\times(k\sigma_i)\sim2/\pi k\sigma_i$. Then, if we assume that in the denominator the "gravitational" drift terms are constant [i.e., independent of ϕ and $\omega_b(\phi)\approx\bar{\omega}_b$, then keeping only the second term inside the parentheses in Eq. (6)], one obtains (cf. Ref. 3)

$$F\approx1+k^2\lambda_D^{\,2}+\frac{kv_{c_i}-k\bar{v}_{g_i}}{\omega+kv_{g_e}}+\frac{1}{\sqrt{\pi}\,|ka_i|}\,\frac{\omega-kv_{c_i}}{\omega+\Omega_0-k\bar{v}_{g_i}-\bar{\omega}_b}= \tag{10}$$

The resonance between the drift wave and the cyclotron motion takes place if

$$|\omega|\approx\Omega_0-k\bar{v}_{g_i}\approx kv_{c_i}/(1+k^2\lambda_D^{\,2}),$$

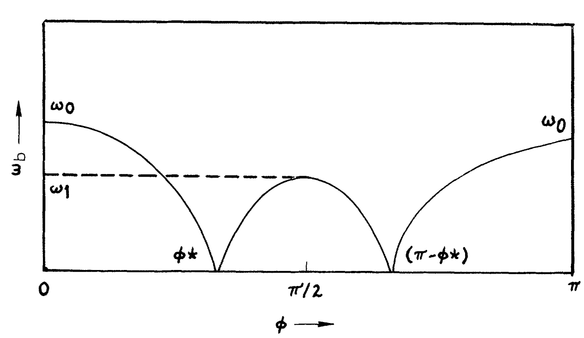

FIG. 3. Schematic diagram showing "bounce" frequency of particles as a function of their angular orientation with respect to the field.

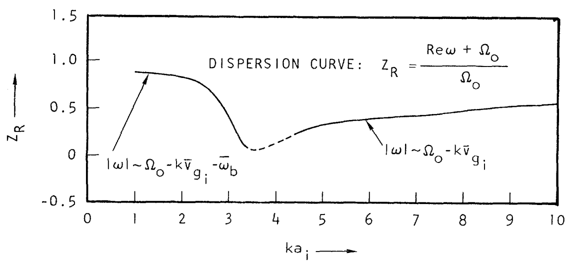

FIG. 4. Dispersion of the drift-cyclotron mode.

i.e., the drift wave frequency is equal to the Doppler-shifted cyclotron frequency. The inclusion of the $\bar{\omega}_b$ term in the denominator now alters the resonance condition roughly as follows:

$$|\omega| \approx \Omega_0 - k\bar{v}_{g_i} - \bar{\omega}_b \approx kv_{c_i}/(1+k^2\lambda_D^2)$$

$$\approx \Omega_0(1 - ka_i^2/\bar{R}_c - a_i/L) ,$$

where $\bar{R}_c$ is the average radius of curvature and L is the connection length.

The experimental situation corresponds to numbers which satisfy the following:

$$\Omega_0 > \bar{\omega}_b > \omega , \quad ka_i \sim 1 , \quad (\Omega - \Omega_0)^2/\bar{\omega}_b^2 \sim 1 ,$$

and $\bar{R}_c \sim L$. Thus, $\bar{\omega}_b$ and $k\bar{v}_{g_i}$ are comparable and the Doppler shift would be expected to be twice as large.

If we also include the first term inside the parentheses in Eq. (6), which we neglected above, we notice that its contribution will become important for ω given by

$$|\omega| \approx \Omega_0 - k\bar{v}_{g_i} .$$

Thus, in the complete solution of our dispersion relation, we may expect to see if there are two different branches of oscillation for the mode.

NUMERICAL RESULTS

In order to investigate this, we obtain a complete numerical solution of Eq. (9). We transform the complex transcendental equation that is given by $F=0$, ω complex, k real, into a pair of coupled nonlinear first-order differential equations for $\mathrm{Re}\,\omega$ and $\mathrm{Im}\,\omega$, taking k as the independent variable. To do that, we write $dF=0$, from which we obtain

$$\frac{d\lambda}{dX} = -\left(\frac{\partial F}{\partial X}\right)_{\lambda=\text{const.}}\left[\left(\frac{\partial F}{\partial \lambda}\right)_{X=\text{const.}}\right]^{-1} ,$$

where $\lambda = (\omega + \Omega_0)/\Omega_0$ (a complex quantity), and $X = ka_i$. From this, we have

$$\frac{d(\mathrm{Re}\lambda)}{dX} = -\left(\frac{\partial A}{\partial X}\frac{\partial A}{\partial \lambda} + \frac{\partial B}{\partial X}\frac{\partial B}{\partial \lambda}\right)\left[\left(\frac{\partial A}{\partial \lambda}\right)^2 + \left(\frac{\partial B}{\partial \lambda}\right)^2\right]^{-1} ,$$

remembering that $\partial/\partial\lambda \equiv \partial/\partial(\mathrm{Re}\lambda)$, and where $F = A + iB$. The idea is to solve this pair of coupled equations starting out from some estimated values of $\mathrm{Re}\lambda$ and $\mathrm{Im}\lambda$ at a certain value of k.[9]

For the octopole machine, the following are a representative set of numbers:

$$\Omega_0 \sim 10^7 \text{ Hz} , \quad \bar{\omega}_b \sim 1.5\times10^6 \text{ Hz} ,$$

$$v_i \sim 1.45\times10^7 \text{ cm/sec} , \quad n'/n \sim 0.5 \text{ cm}^{-1} ,$$

$$\lambda_{D_i} \sim 0.5 \text{ cm} , \quad R_1 \sim 12 \text{ cm (taken to be "bad")},$$

$$R_2 \sim -8 \text{ cm (taken to be "good")} .$$

Using these values of the various parameters, we integrate the coupled differential equations starting from a suitably chosen large value of X, where the solution is obtained from Eq. (10). That solution may be written

$$\omega \approx \tfrac{1}{2}(-\Omega_0 + k\bar{v}_{g_i} + \bar{\omega}_b) - \tfrac{1}{2}kv_{c_i}/(1+k^2\lambda_D^2)$$
$$\pm\tfrac{1}{2}\left(\{\Omega_0 + k\bar{v}_{g_i} + \omega_b + [kv_{c_i}/(1+k^2\lambda_D^2)]\}^2 - \frac{(kv_{c_i})^2[k^2\lambda_D^2 - \bar{v}_{g_i}/v_{c_i}(1+k^2\lambda_D^2)]}{\sqrt{\pi}\,|ka_i|(1+k^2\lambda_D^2)^3}\right)^{1/2} . \tag{11}$$

We single out the proper root by choosing

$$-\Omega_0 + k\bar{v}_g + \bar{\omega}_b \approx -kv_{c_i}/(1+k^2\lambda_D^2) ,$$

and the proper sign in front of the square root in order to obtain a "growth" when the following conditions for instability are satisfied:

$$-\Omega_0 + k\bar{v}_g + \bar{\omega}_b + [kv_{c_i}/(1+k^2\lambda_D^2)] = 0 , \tag{12a}$$

and
$$(\bar{v}_{g_i}/v_{c_i})(1+k^2\lambda_D^2) < k^2\lambda_D^2 . \tag{12b}$$

A Runge-Kutta fourth-order method has been used to integrate the pair of coupled differential equations. At every step of integration ($\Delta X \approx 0.1$), the integral $D(\omega, k)$ occurring inside the function F has been carried out with a variable mesh size (ΔX ranging from 0.006 to 0.001) depending on how fast

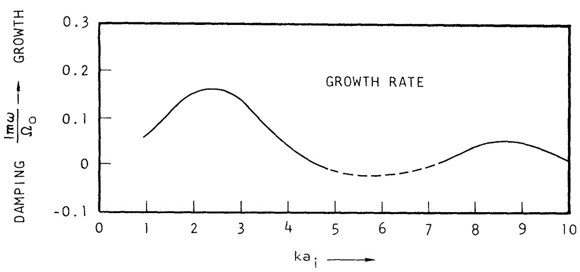

FIG. 5. Growth behavior of the drift-cyclotron mode.

the integrand varied at different values of the independent variable. The convergence of the integral, though not spectacularly fast, assured a respectable accuracy (~ 1 in 10^{-3}) in say, about 500 steps.

The results are shown in Figs. 4 and 5. The plot of growth rate versus $X(=ka_i)$ shows a distinctive peak around $X\approx 2.5$ Note that the corresponding $\mathrm{Re}\,\omega$ versus X plot is rather flat around that point and the magnitude of $\mathrm{Re}\,\omega$ is roughly between -1.3×10^6 and -1.8×10^6. Thus, the calculated frequency is now down by about a factor of 2 from what was computed previously (see Ref. 2). Also, looking at the characteristics of the dispersion curve, it appears as if there are two branches of the oscillation, one characterized by $|\omega| \sim \Omega_0 - k\bar{v}_{g_i}$ with smaller growth rate and the other with $|\omega| \sim \Omega_0 - k\bar{v}_{g_i} - \bar{\omega}_b$ (which turns out also to be $\sim \bar{\omega}_b$) and a higher growth rate.

CONCLUSION

In the previous sections, we have considered the effect of magnetically trapped particles on the mechanism of the drift-cyclotron instability in an octopole device. Our calculations show that the inclusion of trapped particles has definitely improved the quantitative agreement with the experimental results. One may again point out here the various shortcomings of the theoretical treatment. First, in the theoretical calculations, we have used the lowest-order solutions of the orbit equations for particles in a constant magnetic field, only making the gyrofrequency vary with respect to time through its dependence on the variable used to define the length along the line. This is a model and thus cannot be rigorously justified. Second, the model that we have used for the determination of drift velocity represents approximately the actual complicated functional dependence of the drift on the particle velocity vector. For the regions of the octopole where the experimental measurements have been carried out, this model turns out to be fairly realistic. We may also point out that there are various experimental errors involved in the measurement of quantities like the density, density gradient, etc. In spite of all these difficulties, the agreement of the calculated results with experimental observations is surprisingly good.

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