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PHYSICAL REVIEW A VOLUME 1, NUMBER 4 APRIL 1970

# Cu $K\alpha/K\beta$ X-Ray Production from Proton and Oxygen Bombardment*

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$K_\alpha$ and $K_\beta$ x rays produced in the bombardment of Cu with protons of 6- to 10-MeV incident energy and with $O^{4+}$ ions of 15- to 19-MeV incident energy have been measured using a 4-mm-thick cooled Si(Li) detector with an instrumental resolution of 280 eV. The $K_\alpha/K_\beta$ transition ratios are observed to be constant over the energy range studied with a value of $7.8 \pm 0.1$ for proton excitation and $7.0 \pm 0.1$ for the oxygen excitation, compared to a previous experimental value of 7.33 for electron excitation and a calculated value of 8.2. The cross section for x-ray production at 90° to the beam direction for 7-MeV protons on Cu is $23.9 \pm 3.6$ b/sr and for 15-MeV oxygen on Cu is $0.88 \pm 0.27$ b/sr. The $K_\beta$ peak produced by oxygen bombardments is observed to shift upwards in energy by $97 \pm 15$ eV relative to the $K_\beta$ peak produced in proton bombardments.

## I. INTRODUCTION

When a target is bombarded with charged particles, atomic and nuclear collisions occur. In nuclear collisions, large- and small-angle scattering result, as well as nuclear reactions, whereas in atomic collisions, ionization and excitations of the target atom occur. The recombination of an inner-shell electron can be witnessed by observing the characteristic x ray emitted in the process of filling a $K$- or $L$-shell vacancy. A standard device for observing the charged particles in nuclear reactions is the Si(Li) detector operated at room temperature or at dry-ice temperature. The energy resolution obtained with these detectors varies from 6 to 30 keV.[1] A resolution of 2.5 keV can be obtained for 1.1-MeV electrons.[1] A recent advance in the state-of-the-art use of Si(Li) detectors has been the detection of x rays with an energy resolution as low as 250 eV.[2] These detectors have low capacitance (small area and relatively thick) and are used with a preamplifier that has its first stage and the Si(Li) detector cooled to nearly liquid-nitrogen temperature.

We have used this type of x ray spectrometer[2] to measure the characteristic $K_\alpha$ and $K_\beta$ x rays[3] emitted from a copper target when bombarded with protons of energies between 6 and 10 MeV and with $O^{4+}$ ions of energies between 15 and 19 MeV.[4,5] The charged particles from nuclear events, mainly elastic and inelastic scattering – although lower in intensity by two orders of magnitude – are stopped in a low-$Z$ foil in front of the detector in order to reduce the background.

In this paper we present the preliminary results of this study plus a discussion of other beam-foil experiments using Si(Li) detectors to observe the characteristic x rays.

## II. EXPERIMENTAL FACILITIES

The University of Texas tandem Van de Graaff was used to produce monoenergetic beams of protons and oxygen ions. A beam of $H^-$ (or $O^-$) ions was produced in the duoplasmatron ion source[6] utilizing a barium-oxide-coated tungsten filament and accelerated to the terminal of the Van de Graaff where the ion is stripped to $H^+$ (or $O^{n+}$) by a thin carbon foil. The positive ions are then accelerated away from the terminal and subsequently analyzed by a 90° bending magnet. Many charge states of oxygen are produced in the stripping process and are given a different acceleration energy. The desired charge state is selected by the analyzing magnet. The proton beam was normally kept at 0.05 $\mu$A and the oxygen beam at 0.1 $\mu$A. The beam was focused by a magnetic quadrupole through a $\frac{1}{8}$-in. tantalum aperture, impinged on a Cu foil in a 2-in. cubic scattering chamber which was followed by a beam dump that is 12 ft long and 6 in. in diam. Two copper foils were used, a thick foil of 0.000125 in. thickness and a thin foil which was a factor of 17.9 times thinner. The chamber contained a $\frac{1}{2}$-in.-diam window covered with 0.5-mil Mylar. The Si(Li) detector was placed outside this window at a distance of 5.5 in. from the target. Measurements were also made with the detector directly against the Mylar window placed on a 20-in.-diam chamber.

The pulses from the Si(Li) detector were fed into a Tennelec TC-200 amplifier[7] followed by a Tennelec TC-250 biased amp and subsequently fed into a multichannel pulse-height analyzer in the initial stages of the experiment. In the latter stages of the experiment the pulses were digitized in a 1024-channel analyzer and subsequently fed into a PDP-7 computer.

The cross section was calculated using the measured geometry and the target thickness. An estimated error of ± 15% is given because of the uncertainty in the effective solid angle, detector efficiency, target thickness, and absorption corrections.

The x-ray energy calibration of the system was obtained by observing Mn*K*, Ca*K*, and Cu*K* x rays excited by radioactive sources.

## III. RESULTS AND DISCUSSION

### A. $K_\alpha/K_\beta$ Ratios

A spectrum obtained in approximately 2 min from the bombardment of Cu by 7.00 MeV protons is given in Fig. 1. The $K_\alpha$ and $K_\beta$ lines are clearly resolved and the uncorrected ratio of the yields is $K_\alpha/K_\beta = 7.34$. The corrected $K_\alpha/K_\beta$ ratio is obtained by correcting for the difference in absorption in air, in the Mylar window, and in the thin Be window on the detector for the two x-ray lines. The ratio extrapolated to zero attenuation is $K_\alpha/K_\beta = 7.8 \pm 0.1$ and is observed to be constant within the experimental errors as a function of proton bombarding energy over the range studied here. It is important that the alignment and size of the aperture on the face of the Si(Li) detector be carefully determined. The construction of the detector provides a central active area with a very thin window surrounded by an area with a dead layer which increases in thickness rapidly with increasing distance from the center of the detector. This dead layer of undrifted silicon causes considerable attenuation of the x rays. The difference in the energy of the $K_\alpha$ and $K_\beta$ lines is sufficient to cause a significant difference in their attenuation. Thus, the $K_\alpha/K_\beta$ ratio is reduced if part of the detector dead area is exposed to the target.

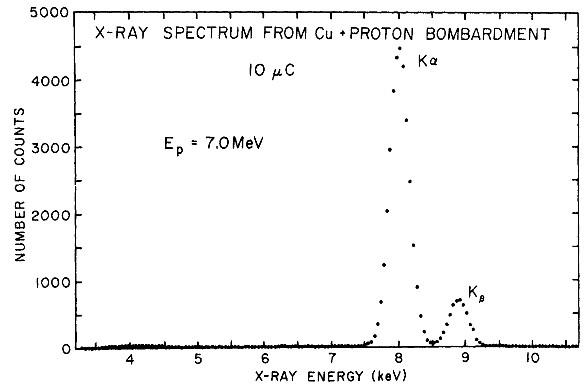


FIG. 1. X-ray spectrum for 7-MeV protons on Cu taken with a 4-mm Si(Li) detector. A total proton charge of 10 $\mu C$ was accumulated in approximately 2 min running time. The resolution at full width at half-maximum is approximately 280 eV.

This problem was studied using an $^{55}$Fe x-ray source and apertures of various diameters. The center of the detector was located by plotting the count rate as a function of aperture position for a very small aperture. Once the center was located the count rate as a function of aperture diameter between 1 and 8 mm was plotted. The count rate increased linearly with area up to a maximum diameter of 5 mm. The $K_\alpha/K_\beta$ ratio was also obtained as a function of the aperture diameter and found to be fairly constant for diameters of less than 5 mm. We thus used an aperture centered on the detector and 3.2 mm in diam.

The energy spectra for oxygen ions on Cu are similar to but not identical to the spectra for proton bombardment. The differences will be discussed in Sec. III C. The experimentally observed ratio, after corrections for attenuation, due to $O^{3+}$ on Cu is $K_\alpha/K_\beta = 7.0 \pm 0.1$. This ratio is also observed to be independent of bombarding energy over the range studied.

The ratio as measured by Williams[8] in electron bombardments is 7.33. We have calculated the ratio $(K_{\alpha 1}+K_{\alpha 2})/(K_{\beta 1}+K_{\beta 3})$ from the $E1$-matrix elements[9] $I_K \propto E_K^3 \,|\langle \phi[(1S_{1/2})^{-1}]\, |r|\, \phi[(nlj)^{-1}]\rangle|^2 S_{s1/2,\, lj}$. In this expression, $E_K$ is the x-ray transition energy, $\langle \phi[(1S_{1/2})^{-1}]$ is the wave function of the initial state, and $\phi[(nlj)^{-1}]$ is the wave function for the final state. We have used the radial-wave functions as calculated by Herman and Skilman.[10] The quantity $S_{s1/2,\, lj}$ is a statistical factor[9] which depends on the spins of the initial and final states. Using this expression, the intensity ratios are $K_{\alpha 1}:K_{\alpha 2}:K_{\beta 1}:K_{\beta 3} = 100:49.5:12.2:6.1$, which gives a ratio $K_\alpha/K_\beta = 8.2$. We have neglected the effect of Auger electrons, which will lower the ratio and thus give better agreement with the experimental values obtained in either proton or electron excitation.

In the proton excitation we thus obtain a $K_\alpha/K_\beta$ ratio ~6% higher than those previously measured by electron excitation. The disagreement is outside the errors of the present experiment; however, the errors in the previous experiment are not known. The observed lower $K_\alpha/K_\beta$ ratio in the $O^{3+}$ excitations on the other hand might be understood in terms of multiple inner-shell ionization in the target atom.[11] This is still an open question and needs further investigation.

### B. Cross Sections

Figure 2 contains the cross section as a function of proton bombarding energy for protons on a Cu target in the range of 6 to 10 MeV. The total cross section was obtained from the 90° cross section by assuming an isotropic angular distribution for the $K_\alpha$ plus the $K_\beta$ cross sections. The solid curve in this figure is a best-fit linear curve and is only intended to guide the eye. Recent theoretical calculations by Merzbacher *et al.*[12] of the energy dependence for $K$ x-ray production by protons show an approximate linear dependence in the energy range studied here, but have a turnover at 17 MeV. Recent data on Ni with proton x-ray excitation by Bissinger and Shafroth[13] for proton energies up to 28 MeV exhibit a turnover at approximately 17 MeV. Evidence of the onset of this turnover is not discernible in the present data.

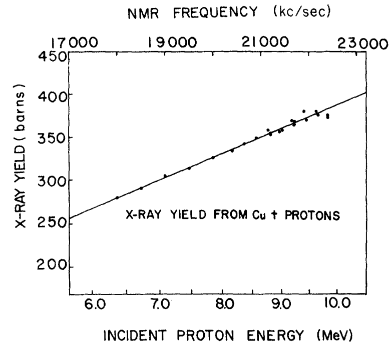


FIG. 2. Excitation function for x rays produced in a proton bombardment on Cu for incident proton energies between 6–10 MeV. The solid curve is intended only to guide the eye. The $K_\alpha$ plus $K_\beta$ cross section is calculated to be $300 \pm 40$ B at 7 MeV.

The excitation function for $K_\alpha + K_\beta$ production from $O^{4+}$ bombardment is given in Fig. 3, in which a straight line has been drawn through the data points. Assuming an isotropic angular distribution the cross section at 15 MeV is $11.1 \pm 3.5$ b, which is smaller than the cross section for 7 MeV proton bombardments by a factor of 27. The large error in this cross section is due primarily to the beam integration of the oxygen ions. The ions can be stripped to higher charge states by the residual gas in the vacuum system (a vacuum of $5\times10^{-5}$-mm Hg was normally kept) and by the target foil, thereby making the number of ions per μC uncertain.

### C. $K_\beta$ Energy Shift

It has also been observed in these experiments that a slight shift in the position of the $K_\beta$ peak is

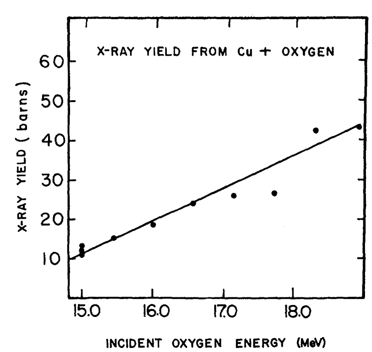


FIG. 3. Excitation function for x rays produced in an oxygen bombardment on Cu for incident oxygen energies between 15 and 19 MeV. The solid curve is intended only to guide the eye. The $K_\alpha$ plus $K_\beta$ cross section is calculated to be $11.1 \pm 3.5$ b at 15 MeV.

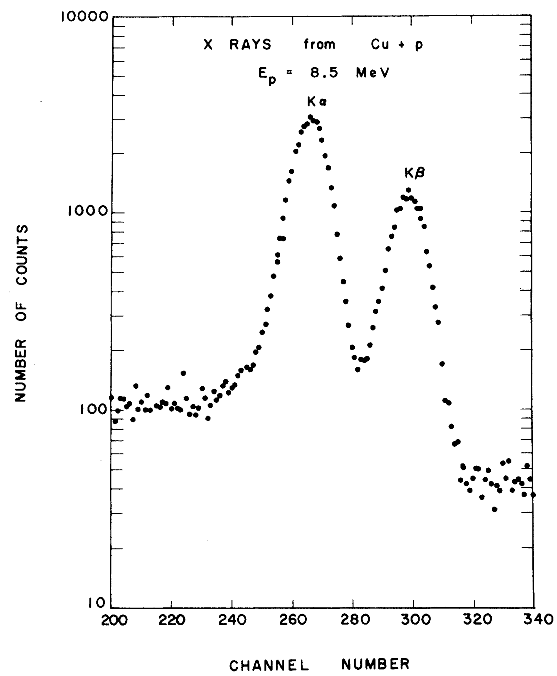


FIG. 4. Energy spectrum for x-ray production in Cu + protons at 8.5 MeV taken with an Al absorber between the target and the detector.

ments. In addition, the $K_\alpha/K_\beta$ ratio from proton excitations is 6% higher than that obtained in electron excitations.

By making use of the high efficiency of the detector, it appears feasible to study nuclear reaction products by observing the characteristic x-ray yields which may accompany such processes. Capture cross sections as well as direct nuclear reaction cross sections might be studied in this manner. Capture of heavy particles ($Z > 2$) could possibly give a reasonably clean region in the spectrum for identifying the x rays from the final atom since they would be at energies above the target $K_\alpha$ and $K_\beta$ x rays. An example of such a reaction is $^{16}O + ^{40}Ca \rightarrow ^{56}Fe$ + x ray. Information on the distribution of $Z$ in fission products would be given directly by observing the x-ray spectra. Radiative-source experiments have recently been done, utilizing Si(Li) detectors for observing the $K$ x rays associated with fission.[15]

Another very interesting aspect of the high-resolution high-efficiency Si(Li) detector is the possibility of observing the energy shift of the $K_\alpha$ and $K_\beta$ lines due to the charge state of the target atom following a collision as a function of incident projectile mass and energy.

produced in the oxygen bombardments compared to the proton bombardments on Cu, as can be seen by comparing Figs. 4 and 5. In addition to the $K_\beta$ energy shift there is a slight broadening of the $K_\beta$ linewidth. A careful study of this problem has been conducted for Cu and Ni, and is reported elsewhere.[11] A net energy shift of $97 \pm 15$ eV is reported for Cu. It is suggested that the energy shift is due to removal of several electrons from the Cu atom prior to the deexcitation by x-ray emission. Calculations of the $K_\alpha$ transition energy as a function of atomic-ionization state were made for $K_\alpha$ transitions by House,[14] and calculations of the $K_\beta$ transition energies for various ionization states are presently being pursued.

## IV. SUMMARY

In summary, the $K_\alpha$ and $K_\beta$ x-ray cross sections produced by bombarding Cu with energetic heavy-charged particles are measured with a high-efficiency, 280-eV resolution Si(Li) detector. The $K_\alpha/K_\beta$ ratio is determined and found to be different for the cases of bombarding Cu with protons and Cu with oxygen. Closely related to this difference is an observed energy shift and broadening of the $K_\beta$ transition line produced in oxygen bombard-

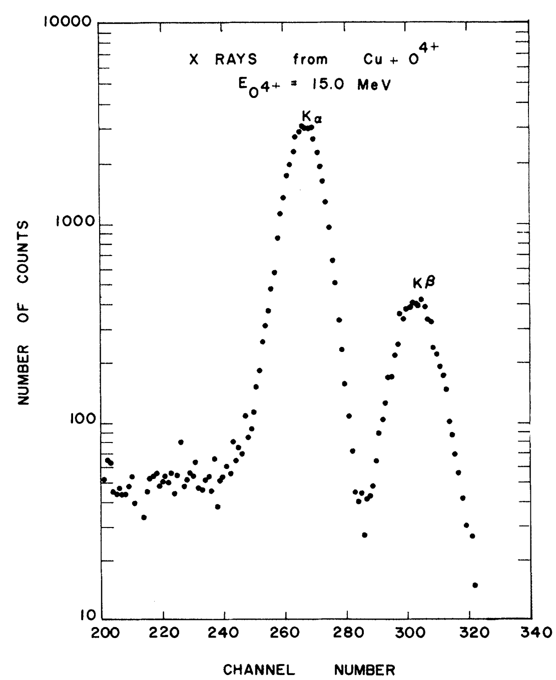


FIG. 5. Energy spectrum for x-ray production in Cu + $O^{4+}$ at 15.0 MeV.

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# Properties of Dilute Solutions of $^3$He in Liquid $^4$He at Low Temperatures

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Using an effective interaction or reaction matrix obtained from a modified Brueckner theory, various low-temperature properties are calculated or estimated for dilute solutions of $^3$He in liquid $^4$He. The system is regarded as a low-density Fermi liquid with $^3$He quasiparticles created by $^3$He atoms in superfluid $^4$He. The single-particle energy spectrum is given by an effective mass, and an effective interaction between the $^3$He quasiparticles is derived. The calculations are done for two different two-body potentials: an Yntema-Schneider potential given by Brueckner and Gammel, and a Frost-Musulin potential given by Bruch and McGee. The Landau $f$ function is estimated from the reaction matrix, and the coefficients of the expansion of the Landau $f$ function in terms of Legendre polynomials are calculated. The estimated values are in reasonably good agreement with experimental results. The exclusion-principle sum rule is also roughly satisfied by the lowest-order coefficients. Low-temperature properties, such as the compressibility, the quasiparticle effective mass or specific-heat ratio, and the magnetic susceptibility, are estimated; results are in fair agreement with experimental values. The various properties are also given as functions of the $^3$He concentration in the solution, and the maximum solubility of $^3$He in liquid $^4$He is estimated in good agreement with the experimental value. Also, transport coefficients – i. e., viscosity, thermal conductivity and spin diffusion – are estimated after identification of the reaction matrix with the scattering amplitude in the formulas developed by Abrikosov and Khalatnikov and by Hone. The agreement with experimental results is surprisingly good, considering that the results depend very much on the value chosen for the effective mass.

## I. INTRODUCTION

Measurements by Edwards *et al.*[1] and Anderson *et al.*[2,3] show that $^3$He atoms in dilute solutions behave like a normal Fermi liquid, as predicted by Landau and Pomeranchuk,[4,5] and that many of the low-temperature properties of $^3$He in solution should be qualitatively similar to those of pure liquid $^3$He. The experiments also indicate that there is a weak and predominantly attractive effective interaction between $^3$He atoms in dilute solutions in liquid $^4$He. From additional theoretical work,[6–17]