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Cross Section for Pair Production in Al, Cu, Cd, and Pb at 2.615-MeV γ-Ray Energy

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Cross sections for pair production by 2.615-MeV γ rays incident on Al, Cu, Cd, and Pb have been found by subtracting tabulated values of the photoelectric cross section from measured values of total cross sections. The latter were determined to be 129.8, 137.0, 147.6, and 181.9 mb per electron, respectively, for the above elements. The statistical uncertainty is $\pm 0.2\%$. Subtraction of photoelectric cross sections gives 0.0642 ($\pm 5\%$), 0.343 ($\pm 2.3\%$), 0.991 ($\pm 1.4\%$), and 3.55 ($\pm 0.8\%$) b per atom for pair production. The present result in Pb is 38.9% higher than the Bethe-Heitler Born approximation result and is about 14% higher than Jaeger-Hulme non-Born theoretical results but only 6% higher than the recent, exact non-Born calculation (neglecting screening) by Øverbø, Mork, and Olsen. Differences between our values for Al, Cu, and Cd and the calculated values of these authors are even smaller, namely, 0.5% for Al and Cd, and 3% for Cu. It is pointed out that pair-production cross sections can be found from Born-approximation values by multiplication by $a + bZ$ for $0 < Z < 50$, with $a = 0.97 \pm 0.01$ and $b = 4 \times 10^{-3}$, and by a factor $1 + \alpha Z^2$ for $Z \geq 50$ with $\alpha = (5.8 \pm 0.1) \times 10^{-5}$.

1. INTRODUCTION

Several authors have made experimental and theoretical investigations of pair production in the fields of a nucleus[1] and of a bound or free electron.[2–4] The main interest has been the validity of the early Bethe-Heitler theory[5] for the energy and Z dependence of the cross section.

Experiments carried out in the past at 2.615-MeV γ-ray energy are those of Colgate,[6] Hahn, Baldinger, and Huber,[7] Dayton,[8] and Meriç,[9] and lately those of Titus and Levy.[10] The most precise calculation in the past was that of Jaeger.[11] Recently, Øverbø, Mork, and Olsen have published the results of a new computation based on an "exact non-Born" calculation neglecting screening.[12]

The results of an experiment at 2.615 MeV are presented and compared with the theoretical values and with other experimental data.

2. EXPERIMENTAL METHOD

The measurements of the cross section for pair production have been made directly by detecting the electron-positron pair produced by γ rays[10] by observing the coincidence between the resulting γ annihilation quanta,[8,13] and by measuring the total attenuation coefficient.[6,14–17]

In the author's experiment, which is described in detail elsewhere,[18] a highly filtered and collimated beam of γ rays was used for measurement of the total attenuation coefficient from which the photoelectric contribution was subtracted. The detector consisted of two thin-wall β-ray counters in coincidence. Great care was taken to prevent scattered radiation from reaching the detector and thus complicating the results.

The average values of μ_{tot} for each element were derived from the slope of a large logarithmic plot of true γ-ray intensities transmitted as a function of absorber thickness. Therefore, the error (due to the variation of the absorbers from their nominal thickness) was allowed for, but could not be avoided entirely due to the inhomogeneity within one absorber. Moreover, each element was assumed to have one particular crystalline state. Thus, the possible variation of density for one type of absorber was neglected.

A. Total Absorption Cross Section

A series of measurements in four different materials, Al, Cu, Cd, and Pb, gave the following values for the total absorption cross section per electron in mb: 129.80 ± 0.26, 134.04 ± 0.27, 147.60 ± 0.29, and 181.89 ± 0.36 (using $r_0^2 = 7.9398 \times 10^{-30}$ cm^2).

These results do not agree with the theoretical value for the total absorption coefficient given by Heitler[19]; for example, $\mu_{\text{expt}} = 0.493$ cm^{-1} for Pb (as against his 0.477 cm^{-1}), and $\mu_{\text{expt}} = 0.1017$ cm^{-1} for Al (as against 0.104 cm^{-1}). But they appear to be in very good agreement with the latest tabulated data given by Hubbell and Berger.[20] The numerical values of μ_{tot} in b/atom at 2.615 MeV obtained from their table are 1.67, 3.95, and 14.46, compared to the measured values of 1.68, 3.97, and 14.92 in Al, Cu, and Pb, respectively. This comparison was made by the interpolation of the curves of μ_{tot} versus energy at 2.615 MeV by using latest tabulated data for Al, Cu, Mo, Sn, and Pb as a function of energy. The values for cadmium were derived from a second interpolation, in atomic number, at 2.615 MeV.

B. Pair-Production Cross Section

We assume that the total attenuation is due to Compton collisions (noncoherent scattering only), photoelectric absorption, and pair-production phenomena (i.e., Rayleigh coherent scattering is not taken into account). Thus, the measured σ_{tot} is the noncoherent σ_{tot} and consists of $\sigma_{C(KN)}$, τ_{pe}, and Φ_{pair} (nucleus). Thus, the pair-production cross section for Al, Cu, Cd, and Pb can be obtained by subtracting the sum of σ_C and τ_{pe} from μ_{expt}. The values of σ_C and τ_{pe} are given in Table I and discussed below. The error in the last column is based only on 0.2% statistical uncertainty in μ due to the counting rate. The cross sections equal 0.0642, 0.343, 0.991, and 3.55 b/atom for Al, Cu, Cd, and Pb, respectively. The corresponding values for Φ_{pair}/Z^2 are 380.0, 407.9, 430.4, and 527.6 μb.

C. Photoelectric Absorption Cross Section

The values recommended by Hubbell *et al.*[21] were used. These are based on a number of recent experiments.[22–27] τ_{pe} for Cd was estimated first by applying the formula

$$\tau_{pe} = f(E) Z^{4.5} \times 10^{-9} / E(\gamma) \, , \quad \text{b/atom}$$

to elements ranging from carbon to uranium for the energy range 0.5–10 MeV. Here $f(E)$ is an energy-dependent factor, Z is the atomic number of the absorber, and $E(\gamma)$ is the energy in MeV of the γ rays interacting with the material. This set of curves was then interpolated for a different Z at a fixed energy. Figure 1 shows the curve τ_{pe} as a function of Z for 2.615 MeV, the value for $Z = 48$ being equal to 0.105 b/atom. This corresponds to $\tau_{pe} = 2.18$ mb per electron; the corresponding quantities for Al, Cu, and Pb are given in Table I, column 2. It should be noted that these values, though much smaller than the values given by Grodstein,[28] are still large when compared with the experimental values found by Titus.[22] Grodstein used the Hall equation with corrections and adjustment, which gives smaller values for τ_{pe} than that of the unadjusted Hall equation; yet her tabulated values are still larger than the values adopted by Hubbell and Berger. The value of 1.13 b/atom we adopt for lead is in very good agreement with the values of 0.93 b/atom found by Bleeker[27] for 2.754-MeV γ rays. However, recent information from Pratt[29] indicates that the photoelectric data may need further revision, possibly as much as 10% in the same region.

TABLE I. Determination of absolute cross section ϕ_{pair} in millibarn per electron.

Z	$\sigma_{compton}$ Klein-Nishina	$\tau_{photoelectric}$	$\sigma_{C(KN)} + \tau_{pe}$	ϕ_{pair} (mb/e)
13	124.84	0.019	124.86	4.94 ∓ 0.26
29	124.84	0.37	125.21	11.83 ∓ 0.27
48	124.84	2.18	127.02	20.66 ∓ 0.29
82	124.84	13.78	138.62	43.27 ∓ 0.36

D. Compton Cross Section

The Compton cross section σ_C is calculated from the Klein-Nishina formula assuming simple Compton effect with the free electron at rest. Corrections for small-angle scattering (double Compton effect) are neglected here. The order of magnitude of these corrections, as well as of those for Rayleigh scattering and for electron binding and motion,[30] will be given later.

3. COMPARISON OF RESULTS WITH PREVIOUS THEORETICAL AND EXPERIMENTAL VALUES

The calculated values for Φ/Z^2 are the following: Jaeger and Hulme's[31] value of $\Phi_{pair} = 2.5$ b/atom for lead for Th C'' γ rays, using a Bethe-Heitler calculation based on the Born approximation gives $\Phi/Z^2 = 372$ μb. The value given by Heitler[19] in terms of $\Phi/\overline{\Phi}$ is equal to 0.64, and corresponds $\Phi/Z^2 = 370.88$ μb, where $\overline{\Phi} = r_0^2/$"137" Φ_{pair}(Born), calculated by Hough[32] for lead, gives $\Phi = 2.55$ b/atom, which corresponds to $\Phi/Z^2 = 379.24$. The average value of the latter obtained from his formula of Φ_{BH} for $Z = 6$, 13, 29, 50, 82, and 92 is equal to 378.38 μb. The value obtained by Davisson[33] and by Yamazaki and Hollander[34] from their interpolated values of $\Phi/\overline{\Phi}$ is $\Phi/Z^2 = 377$ μb.

A recent "exact-Born" calculation by Maximon[35, gives $\Phi/Z^2 = 378.27$ μb for 2.62 MeV ($5.127 mc^2$). The ratio of our value (380.00) for aluminum at $5.117\, mc^2$ to this theoretical value is 1.004_3, which is in good agreement with the ratio 1.006 found by Dayton.[8] (Although the differences are much less than $\frac{1}{2}$% in both cases, such excellent agreement could be considered a coincidence, as the over-all experimental error is, in general, greater.) The corresponding relative values for Cu, Cd, and Pb were found to be equal to 1.0783, 1.1371, and 1.3959, respectively. The deviation from the Bethe-Heitler calculation at high atomic number is evident. Since the difference is nearly 40% in lead, stronger Z-dependence of Φ_{pair} than previously suggested seems to give better agreement with the observed values.

For comparison we have expressed our measured values in terms of $\Phi/\overline{\Phi}$. This can be obtained by dividing the experimental Φ/Z^2 by $\overline{\Phi}/Z^2$, which is a constant for a given Z and equal to 579.4 μb for Pb (using $r_0^2 = 7.9390\times 10^{-26}$ cm^2 and $1/\alpha = 137.0388$). Previously, a value of 579.5 μb was adopted for this ratio by the author[18] as against a value of 571.0 μb from Heitler.[19] An account of the results is given in Table II.

As is seen, the exact calculation of Jaeger[11] using a precise wave equation, which takes into account the interaction of electrons with the field of the nucleus, gives a higher cross section for lead by about 25%. This corresponds to $\Phi_{\text{pair}} = 3.09$ b/atom, and is about 14% lower than our experimental result in lead. A more recent exact (except for screening) theoretical calculation by Øverbø, Mork, and Olsen,[12] which has come to the author's attention, tion, is included in Table II. For lead, their value of Φ_{pair} at $5.2mc^2$ is 3.40 b/atom, corresponding to 3.34 b/atom at $5.117mc^2$ (i.e., at 2.615 MeV), which is only 6% different from our present result. This difference could be well within the error limit introduced by the uncertainty of the photoelectric absorption cross section, which would be more pronounced in lead. In fact, the difference between Øverbø, Mork, and Olsen's result and our reduces to 0.5% for Al and Cd and 3% for copper, which could be well within the limit of over-all experimental and theoretical error. (See Fig. 2.)

The various results are compared in Table III. The comparison of the relative cross sections is shown in Fig. 3. As is seen, the trend of the curve of Titus *et al.*[10] for Φ_{pair} is much more similar to that of the curve by the author than that of Dayton, but their value appears to be systematically low for all elements when compared to ours and to Dayton's as well. If one assumes that the Born approximation holds for small Z and that the cross section varies with Z^2, one would expect Φ_{Born}/Z^2 to be constant, and the value from the

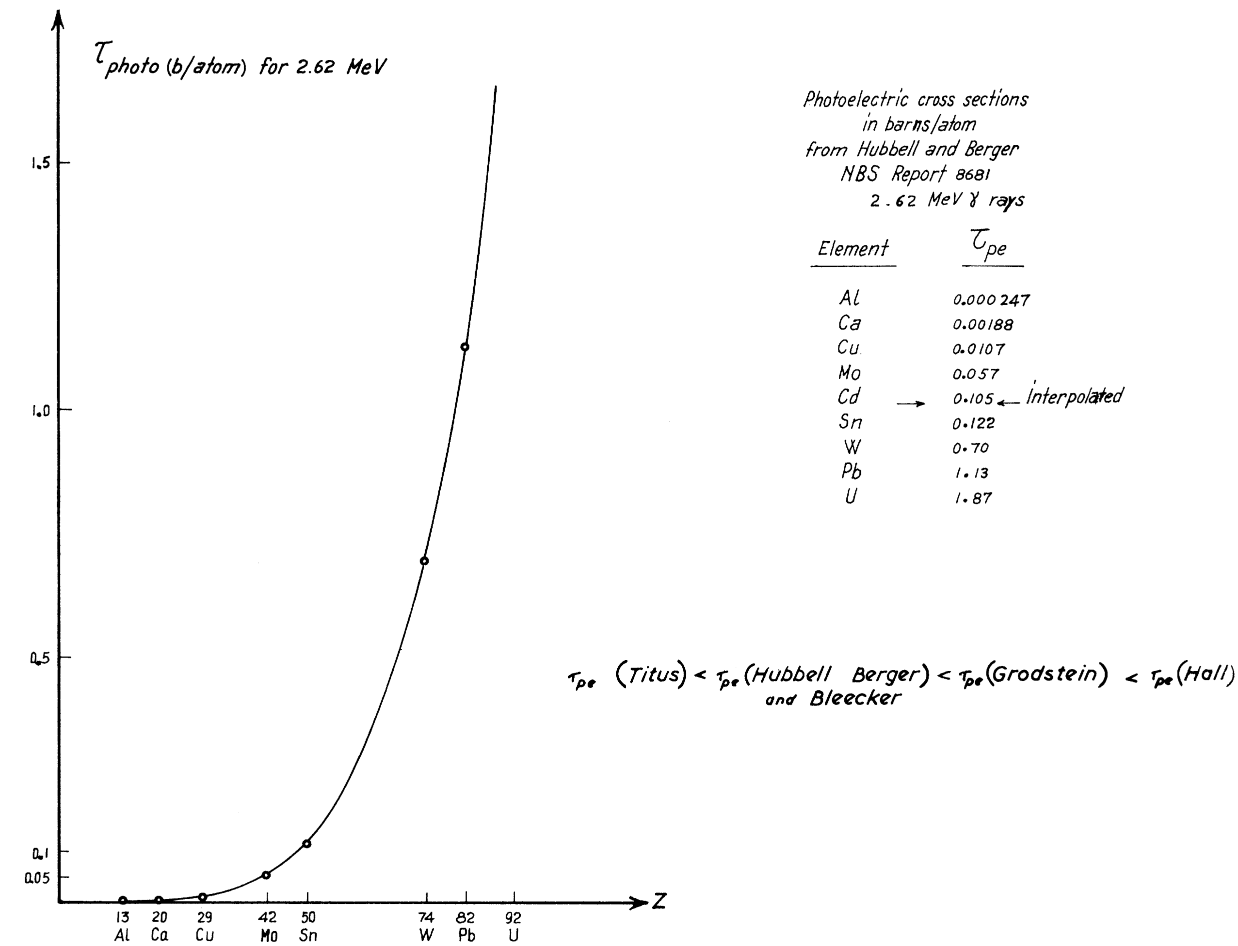

FIG. 1. Photoelectric cross section in b/atom as a function of atomic number Z for 2.62-MeV γ-ray energy according to the formula and other numerical data given by Hubbell and Berger.

TABLE II. Comparison of $\phi/\overline{\phi}$: present work and theoretical values.

Element	Barkan (measured)	Hough (Born)	Maximon (exact Born)	Heitler (tabulated)	Jaeger-Hulme (calculated)	Øverbø, Mork, and Olsen
Al	0.656	0.654	0.653	0.640	0.642	0.659
Cu	0.704			···	···	0.684
Cd	0.742			···	···	···
Sn	···			···	···	0.738
Pb	0.911			0.730 (referred to Jaeger-Hulme)	0.795	0.851

"exact-Born" calculation to be nearly the same for Al. This seems to suggest an adjustment which implies a correction factor of 378.27/349.11 =1.083, i.e., about 8% on their result. The adjusted values are given in the second column of Table IV. If one notes the plot of the two sets of data of Titus *et al.*[10] in Fig. 4, one adjusted and one unadjusted, and compares the former with the

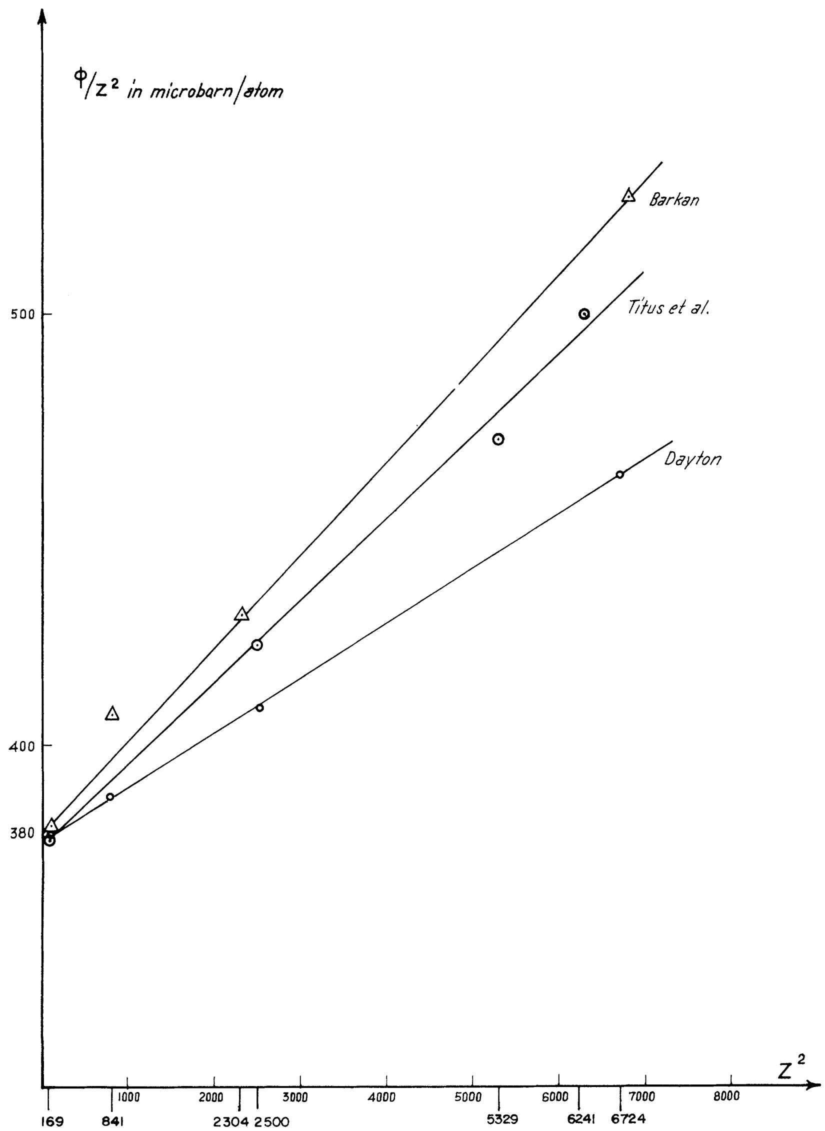

FIG. 2. Variation of the experimental pair-production cross section Φ/Z^2 as a function of Z^2, according to Titus *et al.*, Dayton, and the present author.

TABLE III. The comparison of the pair-production cross section obtained by Barkan, Titus *et al.*, and Dayton (ϕ expressed in barn per atom).

Element	Author's work	Titus *et al.*	Dayton
Al	0.0642	0.059	0.0643
Cu	0.343	···	0.327
Cd	0.991	···	···
Sn	···	0.98	1.022
Ta	···	2.32	···
Au	···	2.89	···
Pb	3.550	···	3.125

present results, it can be considered that there is probably sufficient justification for making this correction. If we take into account the value of 381.71 due to Øverbø, Mork, and Olsen for Al and make the adjustment to the results of Titus *et al.* by a factor of 381.71/349.11 = 1.092, it brings their values up by 1%. If we subsequently normalize the present cross section with respect to Øverbø's value, this reduces our result by 1%, and thus the 2% difference which existed in Fig. 4 between the two experimental results vanishes, i.e., the two curves coincide in the middle. This is shown by the dot-dash line on the same figure.

Although the value $\Phi/Z^2 = 349.11\ \mu$b for Al given by Titus *et al.* is still within the experimental error, its lowness could be attributed to their using the calculated photoelectric cross section which, probably based on Grodstein's table, gives a higher value for τ_{pe} at 2.62 MeV by about 20% and thus a relatively small cross section for Al. Measurements of τ_{pe} by Titus[22] did not include Al because the photoelectric cross section for low Z is very small, which probably prevented his making direct measurements on Al.

Before we draw further conclusions from the comparisons that are made, we would like to estimate the magnitude of the error due to the other effects which might be involved, but which have been neglected during the course of the experiment and calculation. The magnitudes of some of these effects, and the final values of the pair-production cross section (experimental) after these corrections have been made, are given in Table V. The errors considered in items 1 and 4–7, which will be described in the Appendix, tend to diminish the total attenuation coefficient, and thus the pair-production cross section deduced from it. In items 2 and 3 the combined correction (electron binding effect on Compton scattering plus Rayleigh scattering) is also negative. Column 6 shows the results when corrections were applied to the relative cross section.

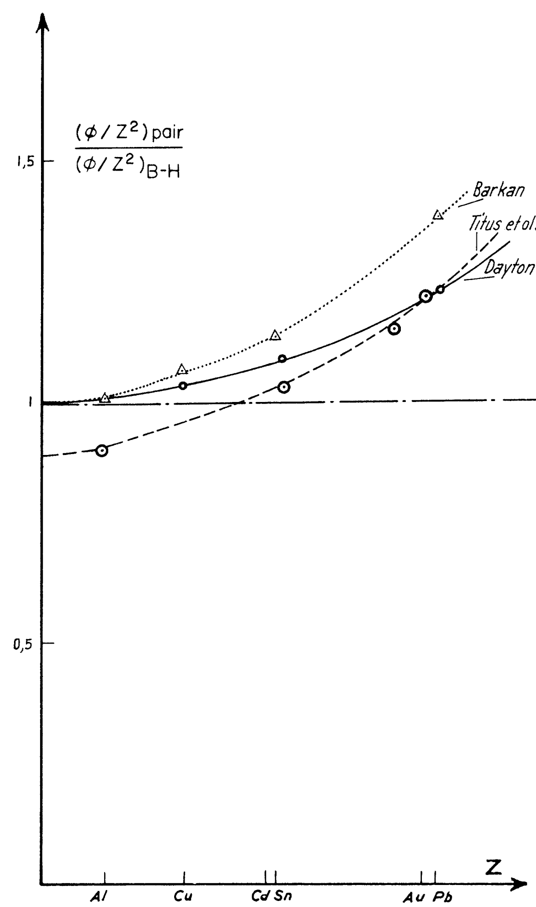

FIG. 3. Comparison of the relative cross sections measured by Titus *et al.*, Dayton, and the present author, in terms of Bethe-Heitler pair-production cross section calculated in the "exact Born" approximation.

4. CONCLUSION AND DISCUSSION

The data of Titus *et al.*,[10] though consistent with

TABLE IV. Comparison of the relative pair-production cross section by Barkan, Titus *et al.*, and Dayton (ϕ_{exp}/Z^2 in terms of $\phi_{exact\ Born}/Z^2$). All values are normalized for $\phi_{exact\ Born} = 378.27\ \mu$b. No correction was applied to Dayton's *original measured values* of $\sigma_{pair} \equiv \phi Z^2/\phi_{Bethe\ Heitler}$ prior to renormalization. The corrected values for various errors for each column will be given later with the magnitude of the relevant factor for each correction.

Element	Author's work	Titus *et al.* (adjusted)	Dayton
Al	1.004_3	1.000	1.006_5
Cu	1.078	···	1.029_7
Cd	1.137	···	···
Sn	···	1.123	1.080_8
Ta	···	1.247	···
Au	···	1.326	···
Pb	1.396	···	1.228_6

TABLE V. Correction estimates and final value of relative $\phi_{\rm exp}$ pair-production cross section.

Element	$\Delta\sigma_C^M$ (Mork correction) $\sigma_C'^{\,a}+\sigma_r^{\,b}$ (%)	Small angle scattering $\sigma_R^{\,c}+\sigma_{\rm bind}^{\,d}$ (%)	Total triplet $\phi_e(\text{bound})+\phi_e(\text{free})$ (%)	Total corrections $\sum_{i=1}^{n}\Delta_i$ (%)	$\phi_{\rm exp}-\sum\Delta_i$
Al	< 0.2	< 0.1	0.3	< −0.5	≈ 1.000
Cu	< 0.2	< 0.1	0.1	< −0.4	1.074
Cd	< 0.2	0.1	0.1	≈ −0.3	1.134
Pb	0.2	0.2	0.07	≈ −0.5	1.389

[a] Double Compton.
[b] Radiative.
[c] Rayleigh scattering.
[d] Bound electron.

those of Dayton,[8] appear to be in better agreement with the present results, as seen in Figs. 3 and 4. In view of this fact, the adoption of $\tau_{\rm pe}$ from Hubbell and Berger is justified, but their pair-production cross sections are still somewhat lower than the present results imply. The revised formula,[21] given for $\Phi_{\rm pair}$, includes some additional correction terms, such as that for screening due to Sörenssen[37,38] and empirical corrections, without which even the further improved Coulomb correction results in a negative cross section near threshold energy. The variation of these corrections as a function of energy is given in Figs. 6 and 7. The estimated values of these corrections at 2.62 MeV are $S(\text{HFS}) = 5Z^2$ μb per atom and $\Delta_{\rm empirical} = -0.139$, respectively.

Taking a Mork-Olsen[39] correction factor of 1.012 and $\Phi_{\text{exact Born}} = 378.27$ μb for 2.62 MeV, Φ/Z^2 is found to be 460.83 μb/atom. Although this is a much better empirical formula than the previous ones (Grodstein, Zerby, etc.), the value obtained from it is still low when compared with some of

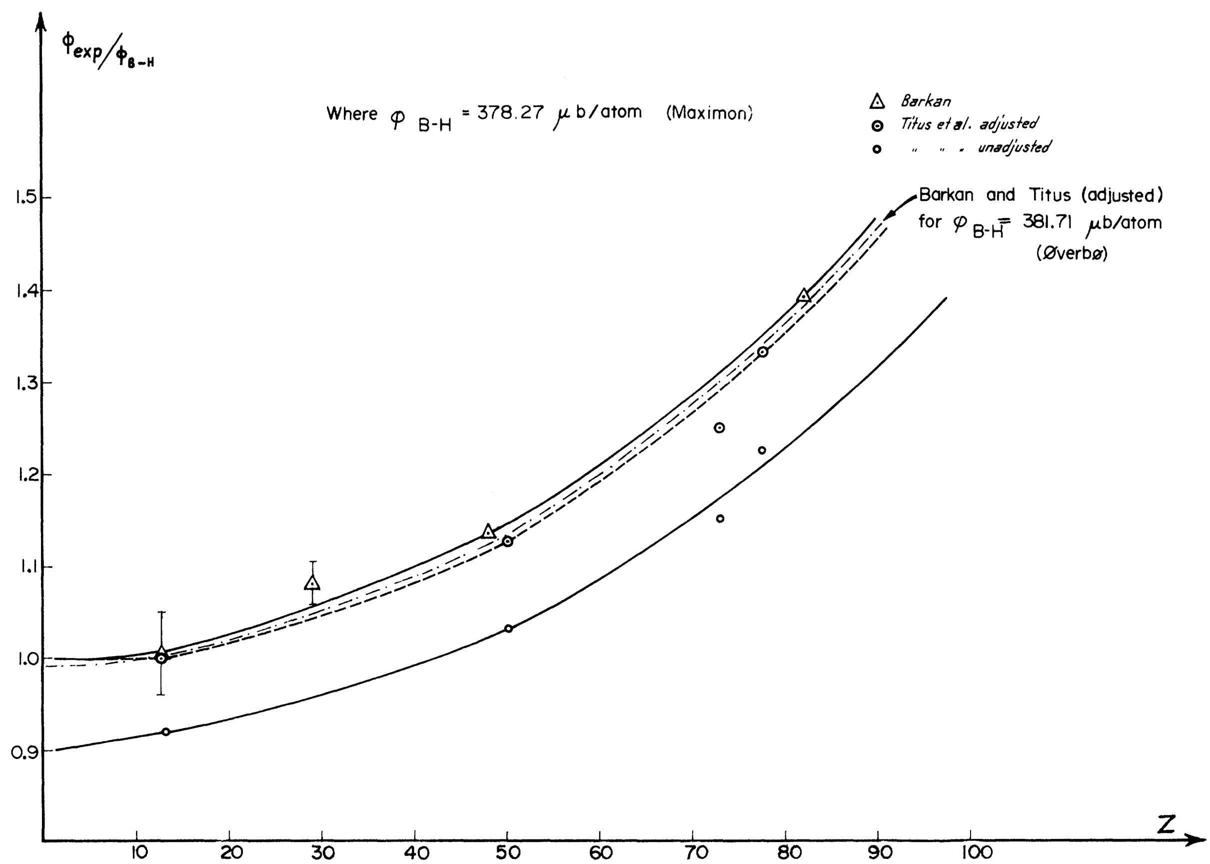

FIG. 4. Comparison of the results of Titus *et al.*, given in adjusted and unadjusted form, with the present work.

the reevaluated cross sections at this energy (including ours). For a general comparison see Table VI. In our opinion this could be remedied by adopting a higher value of $\Delta_{\text{empirical}}$ correction term.

If we consider the experimental τ_{pe} given by Titus[22] to be correct, the pair-production cross section should depend still more strongly upon Z, which would confirm our remarks further.

The least-squares fit to our data gives a coefficient of $\alpha = (5.8 \pm 0.1) \times 10^{-5}$ in an expression of the kind $f(Z) = 1 + \alpha Z^2$ for the Z dependence of relative cross sections. The first term equal to 1 means that exact Born expression holds for Φ_{expt}

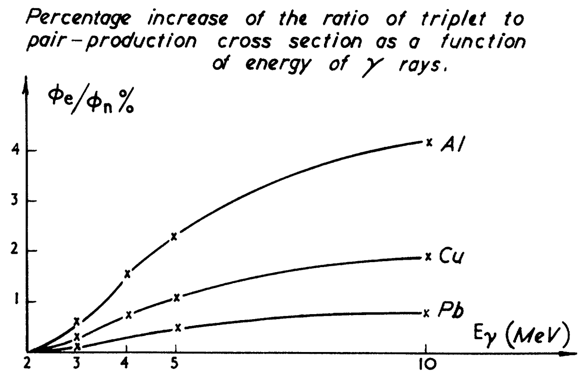

FIG. 5. Percentage decrease on total pair-production cross section due to triplet production, as a function of γ-ray energy in different material.

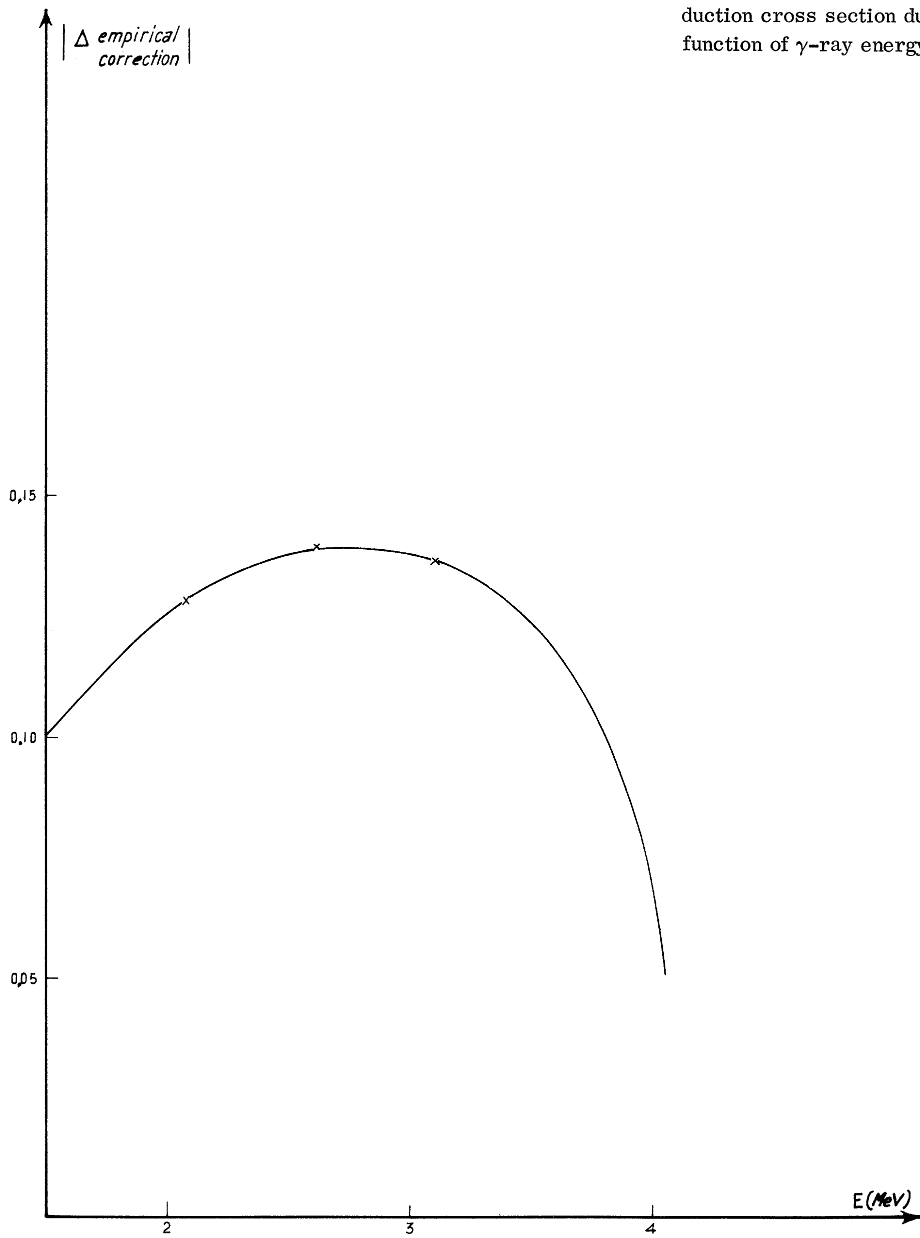

FIG. 6. Variation of the $\Delta_{\text{empirical}}$ correction as a function of energy.

TABLE VI. Relative pair-production cross section for 2.615-MeV γ rays: corrected experimental values expressed in $\phi_{\text{exact Born}} = 378.27Z^2$ μb/atom. $\phi_{(\text{Colgate})}$ of this table are based on the revised values by J. Hubbell using the proper photoelectric absorption cross section, etc. Rayleigh-scattering corrections were assumed to be the same as that in the Colgate calculation. The values of Hahn *et al.* are redetermined. Dayton's values are almost equal to σ/Z^2 given in his paper (Ref. 8, p. 548, last column of Table 2). The values of Titus *et al.* are the adjusted cross sections based on the assumption that the value of ϕ_{BH} (exact Born) holds for small Z and the cross section for Al is unity.

Element / Authors	$_4$Be	$_6$C	$_{13}$Al	$_{26}$Fe	$_{29}$Cu	$_{48}$Cd	$_{50}$Sn	$_{73}$Ta	$_{79}$Au	$_{82}$Pb	$_{83}$Bi	$_{92}$U
Dayton	1.075	0.932	1.006	···	1.031	···	1.081	···	···	1.227	···	···
Colgate	···	1.175	1.064	···	1.025	···	1.047	···	···	1.180	1.205	1.340
Hahn *et al.*	···	···	0.938	1.009	0.990	1.099	1.158	···	···	1.306	1.335	···
Barkan	···	···	1.000	···	1.074	1.134	···	···	···	1.389	···	···
Titus *et al.*	···	···	1.000	···	···	···	1.123	1.247	1.326	···	···	···

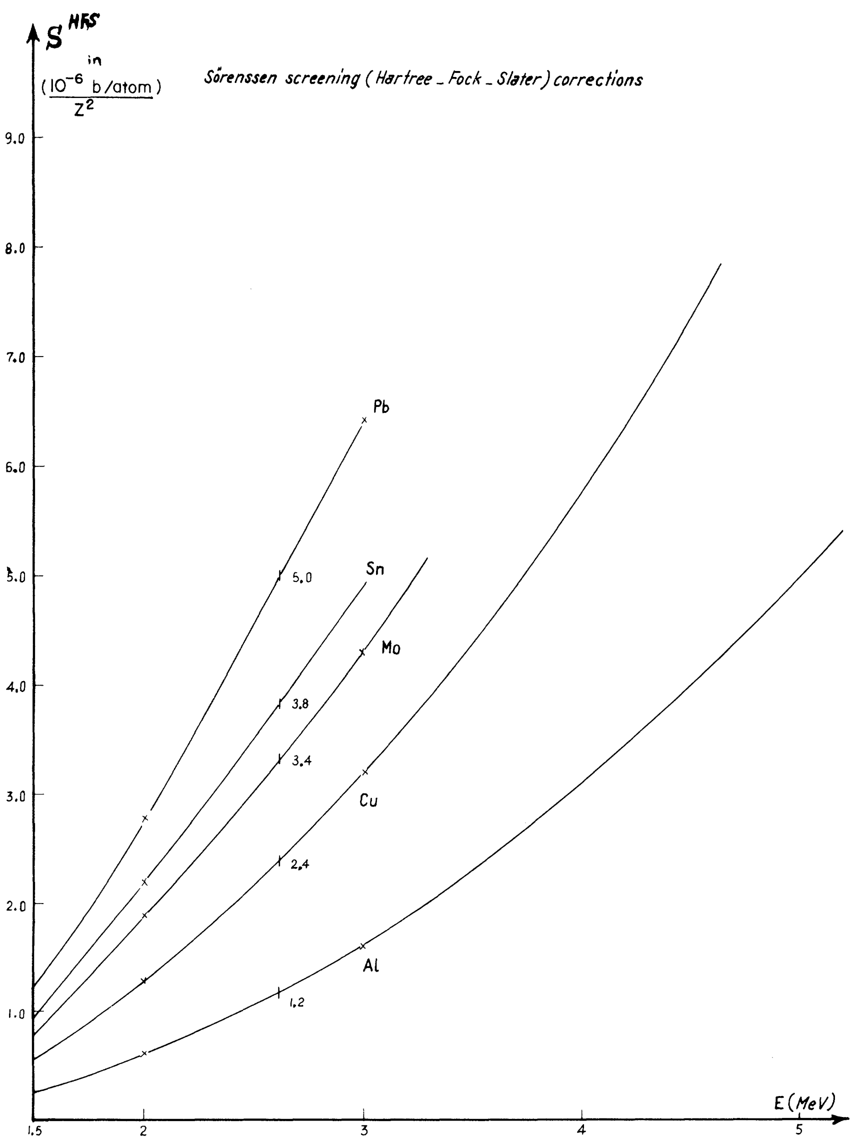

FIG. 7. Magnitude of Sörenssen screening correction as a function of different material at a given energy.

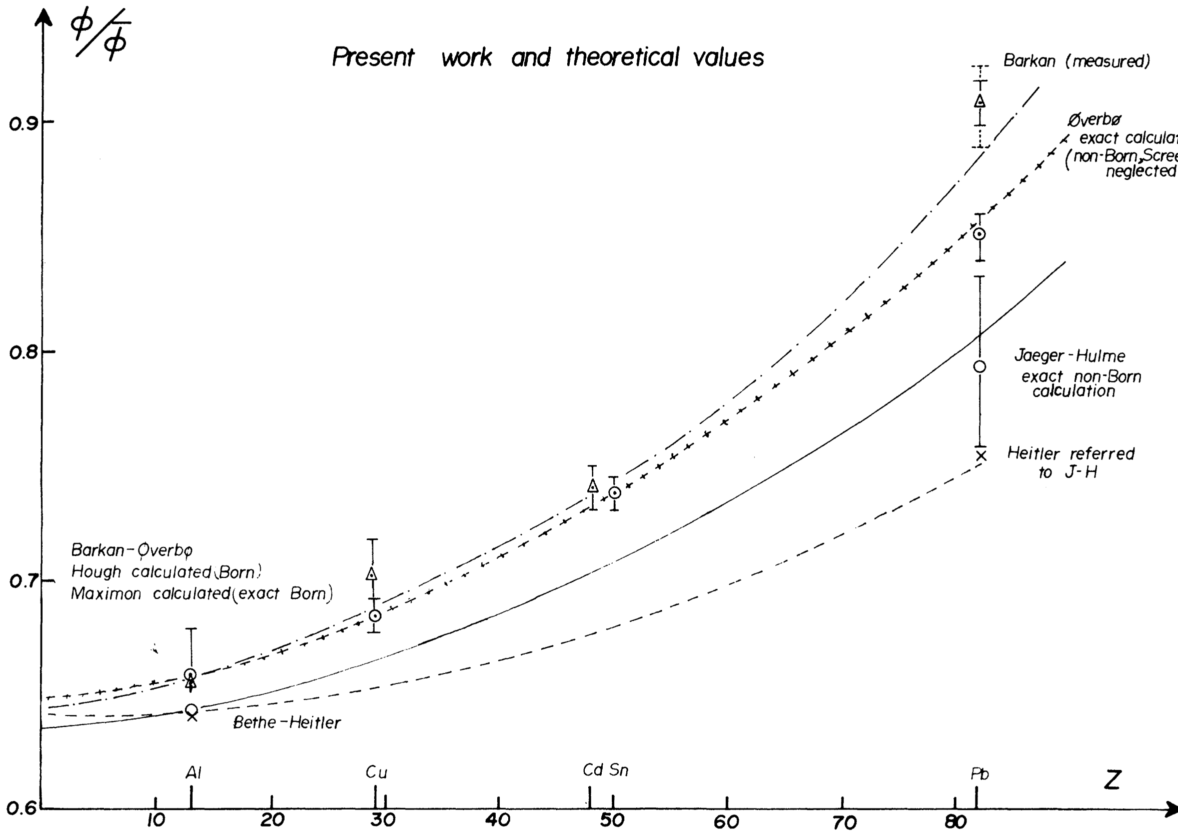

FIG. 8. Comparison of the present experimental results with theoretical calculations.

for small Z. The corresponding values of α for Titus *et al.*[10] and Dayton[8] are 5×10^{-5} and 3.4×10^{-5}, respectively. When we calculate Φ_{expt} over $\Phi_{\mathrm{exact\ Born}}$ on the assumption that $\alpha=5.8\times10^{-5}$, we get good agreement between calculated and measured values except for copper. This might suggest that apart from the experimental error (which certainly exists) there might be a possibility of expressing the Z dependence of the cross section at 2.62 MeV as linear in the form $\Phi_{\mathrm{relative}}=a+bZ$ for $0<Z<50$ (with $a=0.97\pm0.01$ and $b=4\times10^{-3}$), and as quadratic in the form of $\Phi_{\mathrm{relative}}=1+\alpha Z^2$ for $Z>50$ (with $\alpha=5.8\times10^{5}$). To be certain, one will have to carry out more measurements on light elements, for which there is a substantial lack of experimental and theoretical data at 2.62 MeV.

It can be concluded that at present there is still an uncertainty of about 10% in our knowledge of the pair-production cross section in the field of a nucleus at 2.62 MeV for all available experimental data, which will remain until agreement between different experimenters is significantly improved and supported by a thorough *theoretical* investigation in this energy range. We find, in that context, Øverbø, Mork, and Olsen's results very encouraging.

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APPENDIX

The total attenuation coefficient is considered, so far, to consist only of the sum of cross sections for three basic types of interactions, namely, σ_C, τ_{pe}, and Φ_{pair}, and the former was assumed to be calculated by the Klein-Nishina formula, i.e., simple Compton effect with free electron at rest. Thus corrections for interactions such as coherent scattering with bound electrons and Rayleigh scattering, etc., were not taken into account. We shall now consider the different types of interactions and estimate their order of magnitude for the 2.62-MeV γ-ray energy range.

(i) For radiative correction and double Compton scattering, information is based mostly on theoretical estimates. The recent quantitative results due to Mork indicate a correction (now called the Mork correction) to be added to the Klein-Nishina cross section. This is denoted by $\Delta\sigma_{CM}$. Thus,

$$\sigma_C = \sigma_{KN} + \Delta\sigma_{CM} = \sigma_{KN} + \sigma_C' + \sigma_r \quad ,$$

where σ_C' is the double-Compton scattering cross section, and σ_r is the radiative cross section. This correction is estimated to be less than 0.2% for 2.615 MeV, and 0.25 and 1% at 4 and 100 MeV, respectively.

(ii) The electron binding effect[30] tends to reduce the observed differential cross section in comparison with the differential Klein-Nishina cross section. This therefore implies a negative correction for the Compton scattering cross section at small angles, but a positive correction at large angles. The net effect on the integrated cross section is therefore negligible; its sign is such as to increase the observed total attenuation coefficient in a "small-angle" experimental setup used in the present work. This correction will be treated along with Rayleigh scattering.

(iii) Rayleigh scattering is a much larger correction for a well-collimated beam experiment where the detector subtends a small angle. In the present setup the half-angle θ of the cone was about 2°, and this angle contains at least 75% of the Rayleigh-scattered photons. The true value of this half-angle is known[20] to be 2°30′ for 2.62-MeV γ rays in lead, and still smaller for lighter absorbers, according to

$$\theta = 2 \arcsin (0.0133 Z/E) \quad ,$$

where E is in MeV (see also Moon[41] and Evans[42]).

Thus, to a first approximation we are justified in using the integral Rayleigh scattering cross section for correcting the total attenuation coefficient ($d\sigma_R \approx \frac{3}{4}\sigma_R$). This would also give a positive correction, but in fact the pair-production cross section remains nearly unchanged, because in view of the detector angle, $\frac{4}{3}$ of this correction must then be subtracted from the corrected μ_{tot} to obtain the remainder. Thus, the resulting correction to Φ_{pair} is about $-\frac{1}{4}\sigma_{R+\text{binding}}$ and therefore is negative. It amounts to 0.8% for Pb, 0.4% for Cd, and much less for Cu and Al at the energy of 2.62 MeV, if one interpolates from the tabulated data of Ref. 20.

(iv) Pair production in the field of an electron or triplet production, Φ_{et}. This cross section is considered basically as the total cross section, which is the sum of the pair production in the field of a bound electron and free electron. Though Φ_e is small in magnitude, it implies a negative correction to the results on the total attenuation coefficient and thus diminishes it further. The numerical value of Φ_e[43] is calculated from[20]

$$\Phi_e = 1.01\, \Phi_e^{BG} \Delta(\text{triplet}) ,$$

where Φ_e^{BG} denotes the Borsellino-Ghizzetti cross section, Δ (triplet) is the Mork-Votruba correction, and 1.01 is the Mork-Olsen radiative correction. This formula is valid from threshold to 10 MeV; it is justifiable for 2.62-MeV γ rays. Φ_e is evaluated from the ratio Φ_e/Φ_t which is almost equal to Φ_e/Φ_n (where Φ_t is the total triplet production cross section, and Φ_n is the pair-production cross section in the field of a nucleus).

The ratio Φ_e/Φ_n is expressed in terms of η/Z, where η is the factor related to Φ_{tot} by the relation

$$\Phi_{pair}(\text{total}) = Z(Z+\eta)\Phi_n/Z^2 = \Phi_e + \Phi_n \quad .$$

The percentage decrease in the total pair-production cross section at 2.62-MeV γ-ray energy is determined for Al, Cu, and Pb by using the tabulated value of η, and is found to be about 0.3, 0.1, and 0.07% (see Fig. 5), respectively – that is, almost negligible.

(v) The correction due to impurities in the absorbing material is also neglected, since they are estimated to affect the result by less than 0.5% even in the case of lead, which might contain a still heavier component such as bismuth in amounts to 10%.[44] Because of this, the probable impurities on the order of less than 0.5% in lighter elements must also be negligible.

(vi) Another correction that is neglected in this analysis is the absorption of γ rays in the air; the linear attenuation coefficient in air is 10^4 times smaller than in lead at 2.62-MeV energy.

(vii) The correction due to the contribution of annihilation radiation, resulting from the recombination of pairs, created is not taken into account. This contribution is estimated to be of the order of 4–7% theoretically[45] and 6% experimentally[46] in this energy range, and may be expected to increase the result for the photoelectric cross section substantially, thus reducing the value of Φ_{pair}, especially in lead. However, since the annihilation radiation will be due mainly to two-quantum annihilation at rest, with energy 0.511 MeV, the electrons produced by it would almost be completely stopped, in our case, by the aluminum absorbers between the two counters, and therefore could not contribute to coincidence counting rate. The hard component of annihilation radiation (single-quantum annihilation)[47] is also negligible because of its small cross section and the small angle subtended by the detecting system, which further reduce the probability of detection. Other interactions that are not considered at all in analyzing the data are (1) nuclear resonance scattering, (2) nuclear Thomson scattering, and (3) Delbrück scattering.

The remaining interactions that are not considered in this analysis, because of the energies involved, include the following: (a) nuclear coherent scattering (Mössbauer) – effective at very low energy and important in a very narrow resonance range depending upon nuclear energy level; (b) photo-nuclear effect – threshold >5 MeV, important above 10 MeV; (c) nuclear Compton scattering – scattering by nucleons for energies > 100 MeV; and (d) meson production and nucleon pair production – possible at very high energies.

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