PHYSICAL REVIEW VOLUME 99, NUMBER 3 AUGUST 1, 1955

Electromagnetic Properties of the Deuteron. I. Charge Density and Quadrupole Moment*

JEREMY BERNSTEIN AND ABRAHAM KLEIN†‡
Harvard University, Cambridge, Massachusetts

(Received April 27, 1955)

The Tamm-Dancoff method is applied to the calculation of the electrostatic properties of the deuteron. The state vector is assumed to contain amplitudes for at most two mesons in the field, but the possible presence of nucleon-antinucleon pairs is ignored. A formula applicable to the calculation of any multipole moment is derived and used to compute the leading exchange corrections, of order g^2 and g^4, respectively, to the usual expression for the quadrupole moment. For a suitably chosen hard-core wave function the ratio of successive terms is about one tenth, and the g^2 term is itself only a few percent of the total effect.

I. INTRODUCTION

In this paper we shall present a Tamm-Dancoff calculation of the mesonic contributions to the charge density and quadrupole moment of the deuteron in which we shall explicitly include the presence of as many as two mesons in the field. We have chosen to do a Tamm-Dancoff calculation, despite the well-known difficulties in properly taking into account nucleon "self" effects, for two reasons. Firstly, the method can be used to give a reasonable account of the low-energy properties of the two-nucleon system.[1] The finite explicitly two-body contributions arising from the equal times reduction of the covariant equation are also given by our formalism.[2–4] The effects of self-interactions, the treatment of which requires a covariant formalism, are logically separable from the exchange terms treated in this work and will be discussed in a future paper.

Our decision to calculate charge density effects rather than magnetic moments rests on the oft-noted fact that the expressions for charge density phenomena, like the quadrupole moment, are less sensitive than the magnetic effects in a given order to the behavior of the wave function in the region of interaction. The validity of our approach rests upon the rapid convergence of the effects calculated; it is, therefore, a satisfactory result of our work that the one-meson exchange effects amount to a few percent while the two-meson effects are about a tenth as much.

The heart of the calculation consists in determining the effective one-meson and two-meson charge density operators associated with the nucleon current.[5] In Sec. II, we develop the formal apparatus for doing this and discuss the relationship between the charge density operator and the protons probability density. In Sec. III, we present the quadrupole moment formulas and the numerical evaluation of these with appropriately chosen wave functions. In Sec. IV, we discuss the more general aspects of our results; in the appendices, recoil effects are discussed and the Tamm-Dancoff approach as used in this paper is related to the method of canonical transformations and to the covariant two-body equation.

II. FORMALISM

In principle, all of the information about the static electromagnetic effects in the two-nucleon system can be obtained from the Schrödinger equation:

$$(H^0+H'+H^{\mathrm{el}})\Phi=W\Phi, \tag{1}$$

where

$$H^{\mathrm{el}}=-e\int d\mathbf{x}\bar{\psi}(\mathbf{x})\tfrac{1}{2}(1+\tau_3)\gamma_\mu A_\mu{}^e(\mathbf{x})\psi(\mathbf{x}), \tag{2a}$$

and

$$H'=-g\int d\mathbf{x}\bar{\psi}(\mathbf{x})\gamma_5\tau_i\psi(\mathbf{x})\phi_i(\mathbf{x}). \tag{2b}$$

Here $\bar{\psi}$ and ψ are the usual nucleon field operators, and $A_\mu{}^e$ is an external electromagnetic field.

Our program is to obtain explicit expressions for the electromagnetic and nuclear energies defined by Eq. (1) after Φ has been expanded in a basis of free-particle

* A preliminary account of this work was presented at the Washington meeting of the American Physical Society, April, 1954 [Phys. Rev. **95**, 655(A) (1954)].

† Harvard Society of Fellows.

‡ Now at the Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania.

[1] K. Brueckner and K. M. Watson, Phys. Rev. **92**, 1023 (1953).

[2] For a treatment of electromagnetic effects with the covariant equation, including recoil effects but not radiative corrections, see S. Deser, Phys. Rev. **92**, 1542 (1953). The very large (of the order of 50%) mesonic correction which Deser finds in order eg^2 is due partly to his taking the asymptotic form of the deuteron wave function seriously down to the origin and partly to an incorrect coordinate-space reduction of his energy operator. Our treatment also differs from his in the treatment of recoil effects. See Appendix B for a discussion of this point.

[3] Another interesting and quite different calculation has been given by F. Villars, Phys. Rev. **86**, 476 (1952) who uses covariant perturbation theory (canonical transformations) to estimate the mesonic effects on the deuteron moments. He finds that the quadrupole effect is of the order of a few percent and in his approach is given entirely by terms which have a vanishing adiabatic limit. One evident drawback with using canonical transformations, as pointed out by Villars himself, is that the potential to which it leads will not bind the deuteron [M. M. Lévy, Phys. Rev. **84**, 441 (1951)] and hence it is not quite consistent to evaluate the expressions with a phenomenological wave function based on two nucleon binding. The essentials of this approach are described in Appendix C.

[4] A. Sessler, Phys. Rev. **96**, 793 (1954) has given a calculation of charge density effects to order eg^2 which is quite similar to our work. We are pleased to acknowledge an informative correspondence with Dr. Sessler. See also I. Sato and K. Itabashi, Progr. Theoret. Phys. (Japan) **12**, 100 (1954).

[5] The meson current effects depend on the nucleon coordinates via the antisymmetric operator $(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)$ and hence, as has often been remarked, yield vanishing expectation values for any two-nucleon state.

states. Then, in the spirit of ordinary perturbation theory, we shall evaluate the exchange contributions to the electrostatic energy with the solution of Eq. (1) taken in the absence of the external field, A_μ^e.

In the expansion of the state vector, using a free-particle basis, we shall restrict ourselves to the form

$$\Phi = a_0\Phi_0 + a_1\Phi_1 + a_2\Phi_2, \tag{3}$$

where the Φ_i are the solutions of the free-particle Schrödinger equation involving two nucleons and i mesons, and the a_i are the amplitudes for the ith meson state. We have omitted all nucleon-pair amplitudes.[6] The relations among the a_i can be determined by a variational principle. Thus, if

$$W = (\Phi, (H_0 + H^I)\Phi)/(\Phi,\Phi), \tag{4}$$

where H^I is the total interaction Hamiltonian, then putting Eq. (3) into Eq. (4) we see at once that

$$\begin{aligned} W(a_0,a_1,a_2) = [&|a_0|^2\epsilon_0 + |a_1|^2\epsilon_1 + |a_2|^2\epsilon_2 \\ &+ |a_0|^2 H_{00}{}^I + |a_1|^2 H_{11}{}^I + |a_2|^2 H_{22}{}^I \\ &+ a_0{}^* a_1 H_{01}{}^I + a_1{}^* a_0 H_{10}{}^I \\ &+ a_1{}^* a_2 H_{12}{}^I + a_2{}^* a_1 H_{21}{}^I] \\ &\times [|a_0|^2 + |a_1|^2 + |a_2|^2]^{-1}, \end{aligned} \tag{5}$$

integrations over momentum variables being understood and where the ϵ_i are free-particle kinetic energies. The correct choice of the a_i is given by the condition

$$\partial W/\partial a_i = 0, \tag{6}$$

and hence

$$\begin{aligned} (W - \epsilon_0 - H_{00}{}^I)a_0 &= H_{01}{}^I a_1, \\ (W - \epsilon_1 - H_{11}{}^I)a_1 &= H_{10}{}^I a_0 + H_{12}{}^I a_2, \\ (W - \epsilon_2 - H_{22}{}^I)a_2 &= H_{21}{}^I a_1. \end{aligned} \tag{7}$$

By carrying out a formal elimination of the other a_i in terms of a_0 we obtain the following integral equation,

$$\begin{aligned} &(W - \epsilon_0 - H_{00}{}^{\text{el}})a_0 \\ &= [H_{01}'(W - H^0 - H^{\text{el}})^{-1} H_{10}' \\ &\quad + H_{01}'(W - H^0 - H^{\text{el}})^{-1} H_{12}'(W - H^0 - H^{\text{el}})^{-1} \\ &\quad \times H_{21}'(W - H^0 - H^{\text{el}})^{-1} H_{10}']a_0. \end{aligned} \tag{8}$$

Since the physical consequences of the theory cannot depend upon how Φ is normalized, we set $\|a_0\| = 1$. Hence upon expanding the electromagnetic effects to first order and taking the scalar product of Eq. (8) with a_0, we find

$$W = \langle H^0\rangle + \langle V'(W)\rangle + \langle V^{\text{el}}(W)\rangle, \tag{9}$$

[6] The justification of the neglect is twofold: quite generally, the work of many authors [for example, see A. Klein, Phys. Rev. **95**, 1061 (1954) where references to previous literature are given] indicates that pair effects associated completely with the nuclear interaction are in fact suppressed. On the other hand, those effects in which the pair is either created or annihilated by the external field and thus not necessarily suppressed [see, N. M. Kroll and M. Ruderman, Phys. Rev. **93**, 233 (1954)] give smaller contributions to the electrostatic effects than the leading terms by the ratio of meson to nucleon mass.

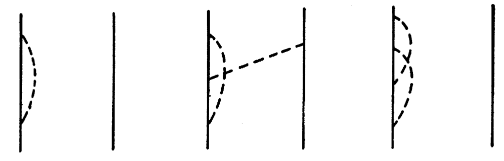

FIG. 1. Diagrams for parts of $\langle V'\rangle$ which involve self-interactions. Solid lines represent nucleons, dashed lines are mesons, and the sense of time can be taken as running up the page.

where, with $G_0 = (W - H^0)^{-1}$, we have

$$\langle V'(W)\rangle = \langle H_{01}' G_0 H_{10}'\rangle + \langle H_{01}' G_0 H_{12}' G_0 H_{21}' G_0 H_{10}'\rangle, \tag{10a}$$

and

$$\begin{aligned} \langle V^{\text{el}}(W)\rangle = &\langle H_{00}{}^{\text{el}}\rangle + \langle H_{01}' G_0 H_{11}{}^{\text{el}} G_0 H_{10}'\rangle \\ &+ \langle H_{01}' G_0 H_{12}' G_0 H_{22}{}^{\text{el}} G_0 H_{21}' G_0 H_{10}'\rangle \\ &+ \langle H_{01}' G_0 H_{11}{}^{\text{el}} G_0 H_{12}' G_0 H_{21}' G_0 H_{10}'\rangle \\ &+ \langle H_{01}' G_0 H_{12}' G_0 H_{21}' G_0 H_{11}{}^{\text{el}} G_0 H_{10}'\rangle. \end{aligned} \tag{10b}$$

Of course, Eqs. (10a) and (10b) contain many divergent contributions. Typical samples of these are indicated in Fig. 1 and in Fig. 2. Since estimates of such effects with the present model are necessarily cruder than that of the finite contributions, we shall not attempt to give them here. However, since these terms are easily recognized, we can split them off and lump them together as $S(W)$ leaving a finite residue which we call $\langle V'(W)\rangle_c$ and $\langle V^{\text{el}}(W)\rangle_c$. Hence Eq. (9) becomes

$$W = \langle H^0\rangle + \langle V'(W)\rangle_c + \langle V^{\text{el}}(W)\rangle_c + \langle S'(W)\rangle + \langle S^{\text{el}}(W)\rangle. \tag{11}$$

We now turn to the essential business of extracting the two-body electrostatic moments from Eq. (11) and learning what effect the $\langle S(W)\rangle$ have upon these. To this end, we recall that the electrostatic energy of the two-body system in the presence of an external potential $A_0{}^e(\mathbf{r})$ can always be written in the form,

$$\int \langle \rho_{\text{op}}(\mathbf{r})\rangle A_0{}^e(\mathbf{r}) d\mathbf{r}, \tag{12}$$

where $\rho_{\text{op}}(\mathbf{r})$ is the effective charge density operator which will include the effects which arise from meson exchange. We may expand $A_0{}^e(\mathbf{r})$, about the center of mass of the system, taken at $\mathbf{r} = 0$, and thus obtain the

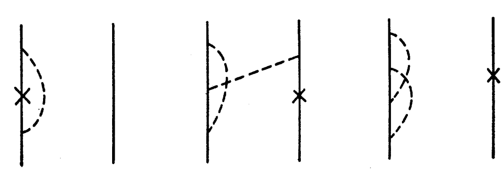

FIG. 2. Diagrams for parts of $\langle V^{\text{el}}\rangle$ which involve self-interactions. The notation is the same as in Fig. 1 with the addition of a cross to indicate an interaction with the external field.

equation

$$\int \langle\rho_{\mathrm{op}}(\mathbf{r})\rangle A_0{}^e(\mathbf{r})d\mathbf{r}=\int \langle\rho_{\mathrm{op}}(\mathbf{r})\rangle[A_0{}^e|_0+\mathbf{r}\cdot\nabla A_0{}^e|_0+\tfrac{1}{2}\mathbf{rr}:\nabla\nabla A_0{}^e|_0+\cdots]d\mathbf{r}, \quad (13)$$

or symbolically,

$$\langle V^{\mathrm{el}}(W)\rangle=\sum_i\langle V_i{}^{\mathrm{el}}(W)\rangle\lambda_i, \quad (14)$$

where

$$\lambda_i=(i!)^{-1}(\partial_x+\partial_y+\partial_z)^i A_0{}^e(\mathbf{r})|_0. \quad (15)$$

One can then define the 2^ith static moment tensor by means of the terms contained in $\partial W/\partial\lambda_i|_{\lambda=0}$. However, using Eq. (11), one obtains

$$\partial W/\partial\lambda_i|_0=[\langle\partial V'(W)/\partial W\rangle_c\partial W/\partial\lambda_i]_{\lambda=0}+[\langle\partial S'(W)/\partial W\rangle\partial W/\partial\lambda_i]_{\lambda=0}+[\langle V_i{}^{\mathrm{el}}(W)\rangle_c]_{\lambda=0}+[\langle S_i{}^{\mathrm{el}}(W)\rangle]_{\lambda=0} \quad (16)$$

or, again understanding the limit $\lambda_i=0$,

$$\partial W/\partial\lambda_i=[\langle V_i{}^{\mathrm{el}}(W)\rangle_c+\langle S_i{}^{\mathrm{el}}(W)\rangle]\times[1-\langle\partial S'/\partial W\rangle-\langle\partial V'/\partial W\rangle_c]^{-1}. \quad (17)$$

On the other hand, the electrostatic energy must also be given by

$$(\Phi,H^{\mathrm{el}}\Phi)/(\Phi,\Phi), \quad (18)$$

and if we expand Φ and H^{el} and remember that $\|a_0\|=1$, we learn that

$$\partial W/\partial\lambda_i=[\langle V_i{}^{\mathrm{el}}\rangle_c+\langle S_i{}^{\mathrm{el}}\rangle]\times\left[1+\sum_{i=1}^{\infty}\int|a(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}_1,\cdots\mathbf{k}_i)|^2\times d\mathbf{p}_1d\mathbf{p}_2d\mathbf{k}_1\cdots d\mathbf{k}_i\right]^{-1}. \quad (19)$$

Since the $\int|a(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}_1,\cdots\mathbf{k}_i)|^2$ are the relative probabilities for finding i mesons in the field, we are thus led to the theorem[7]:

$$\sum_{i=1}^{\infty}\int|a(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}_1,\cdots\mathbf{k}_i)|^2d\mathbf{p}_1d\mathbf{p}_2d\mathbf{k}_1,\cdots d\mathbf{k}_i=-\langle\partial S'/\partial W\rangle-\langle\partial V'/\partial W\rangle_c\equiv P. \quad (20)$$

However, as can be seen from Eq. (12b), $\langle V_i{}^{\mathrm{el}}\rangle_c$ is itself, in the general case, an infinite series in g^2. Thus if we choose $\lambda_i=(2!)^{-1}(\partial_x+\partial_y+\partial_z)^2A_0{}^e$ we are led in the standard way to the following infinite series for the quadrupole moment,

$$Q=\sum_{J=0}^{\infty}\langle Q^J\rangle[1+P]^{-1}, \quad (21)$$

where $\langle Q^0\rangle$ can be identified with the usual expression for the quadrupole moment as expressed in terms of the bare proton charge density.

[7] See also remarks by A. Klein, Proceedings of the Fourth Annual Rochester Conference (University of Rochester Press, Rochester, 1954), p. 44.

Until now we have been dealing with P and $\langle S^{\mathrm{el}}(W)\rangle$ as if they were perfectly finite. However, under the assumption that in a properly renormalized theory P would be small compared to unity, we would expand $[1+P]^{-1}$ and hence note the additivity of *all* self-effects to those arising from exchange interaction. Leaving it at that, we shall henceforth devote ourselves solely to interaction effects.

In ordinary quantum mechanics one defines the charge density in terms of the probability density of the proton coordinate and the quadrupole moment in terms of this charge density. In our calculation, however, the probability density of the proton in momentum space can be taken as

$$\rho(\mathbf{p}_1)=\sum_{i=0}^{\infty}\int|a(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}_1,\cdots\mathbf{k}_i)|^2d\mathbf{p}_2d\mathbf{k}_1,\cdots d\mathbf{k}_i, \quad (22)$$

from which it follows that the sum in Eq. (21) has the form

$$\sum_{J=0}^{\infty}\langle Q^J\rangle=\int\rho(\mathbf{r})r^2\times(3\cos^2\theta-1)d\mathbf{r}, \quad (23)$$

where $\rho(\mathbf{r})$ is the Fourier transform of Eq. (22). For the remainder of this section we illustrate how $\rho(\mathbf{r})$ may be computed directly from the nuclear interaction, in a manner analogous to the use of Eq. (20).

To this end, we shall consider in parallel the expressions in momentum space arising from Figs. 3(a) and 3(b). Inserting the details of the coupling we may write as the contribution to the energy from Fig. 3(a)

$$\frac{g^2}{(2\pi)^3}\int\frac{d\mathbf{p}_1d\mathbf{p}_2d\mathbf{k}}{2\omega(\mathbf{k})}a^*(\mathbf{p}_1-\mathbf{k},\,\mathbf{p}_2+\mathbf{k})\times\langle\gamma_0\gamma_5\rangle_1\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\langle\gamma_0\gamma_5\rangle_2\times[W-\epsilon(\mathbf{p}_1-\mathbf{k})-\epsilon(\mathbf{p}_2)-\omega(\mathbf{k})]^{-1}a(\mathbf{p}_1,\mathbf{p}_2), \quad (24a)$$

where $\langle\ \rangle$ means that we are to take the matrix element of the enclosed Dirac operator between free spinors. From Fig. 3(b) we have

$$\frac{eg^2}{(2\pi)^3}\int\frac{d\mathbf{p}_1d\mathbf{p}_2d\mathbf{k}d\mathbf{q}}{2\omega(\mathbf{k})}a^*(\mathbf{p}_1-\mathbf{k}+\mathbf{q},\,\mathbf{p}_2+\mathbf{k})\times\langle A_0{}^e(\mathbf{q})\Lambda_+(\mathbf{p}_1-\mathbf{k})\gamma_0\gamma_5\rangle_1\times\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\langle\gamma_0\gamma_5\rangle_2\{[W-\epsilon(\mathbf{p}_1-\mathbf{k}+\mathbf{q})-\epsilon(\mathbf{p}_2)-\omega(\mathbf{k})]\times[W-\epsilon(\mathbf{p}_1-\mathbf{k})-\epsilon(\mathbf{p}_2)-\omega(\mathbf{k})]\}^{-1}a(\mathbf{p}_1,\mathbf{p}_2). \quad (24b)$$

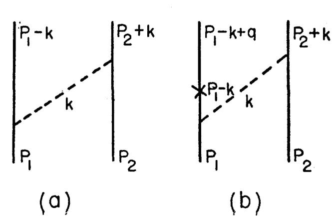

FIG. 3. Contributions to $\langle V'\rangle_c$ and to $\langle V^{\mathrm{el}}\rangle_c$ in the order of g^2 and eg^2 respectively.

If in Eq. (23b) we perform the same expansion as in Eq. (15), then the contribution to the quadrupole energy is obtained by replacing $-A_0{}^e(\mathbf{q})$ by

$$\tfrac{1}{2}\sum_{i,j}(\partial^2/\partial q_i\partial q_j)\delta(\mathbf{q})\partial_i\partial_j A_0{}^e|_0. \quad (25)$$

It is also useful at this stage to introduce center of momentum and relative coordinates in the amplitudes in Eq. (23) which is done under the approximation that the deuteron is unperturbed, i.e., we write typically

$$a(\mathbf{p}_1,\mathbf{p}_2)=\delta(\mathbf{P})a(\mathbf{p})(2\pi)^{\frac{3}{2}}V^{-\frac{1}{2}}, \quad (26)$$

where

$$\mathbf{P}=\mathbf{p}_1+\mathbf{p}_2, \quad \mathbf{P}=\tfrac{1}{2}(\mathbf{p}_1-\mathbf{p}_2), \quad (27)$$

and V is the system volume. An integration by parts, then, transfers the δ function differentiations on to the other functions of q in Eq. (23b). We shall show in Appendix A that the only term arising from this process which has a nonvanishing adiabatic limit has the form

$$-\frac{eg^2}{(2\pi)^3}\int[\tfrac{1}{4}(\partial^2/\partial p_i\partial p_j)a^*(\mathbf{p}-\mathbf{k})] \times\langle\gamma_0\gamma_5\rangle_1\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\langle\gamma_0\gamma_5\rangle_2[2\omega(\mathbf{k})]^{-1} \times[W-\epsilon(\mathbf{p}-\mathbf{k})-\epsilon(\mathbf{p})-\omega(\mathbf{k})]^{-2} \times a(\mathbf{p})\partial_i\partial_j A_0{}^e|_0 d\mathbf{p}d\mathbf{k}. \quad (28)$$

One need only compare Eq. (28) with Eq. (24a) to understand that in the limit contemplated, $\rho(\mathbf{r})$ is given by $-\partial V'(\mathbf{r})/\partial W$, where the last quantity is the adiabatic limit of the energy derivative of the interaction. Hence, it is sufficient to give the formulas for $\partial V'/\partial W$, from which both the normalization P and the moment terms can be computed.

III. RESULTS

We write the expressions for $\partial V'/\partial W$ at the stage after the spin-matrix elements have been reduced to large components and the adiabatic limit taken. We are then led to a local operator in coordinate space which has the form

$$-\partial V_1'(\mathbf{r})/\partial W =\left(\frac{g}{2M}\right)^2\frac{1}{(2\pi)^3}\int d\mathbf{k}e^{i\mathbf{k}\cdot\mathbf{r}}\frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k}}{\omega^3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \quad (29)$$

$$-\partial V_2'(\mathbf{r})/\partial W =\left(\frac{g}{2M}\right)^4\frac{1}{(2\pi)^6}\int\frac{d\mathbf{k}_1d\mathbf{k}_2}{4\omega_1\omega_2}e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}} \times\{[6(\mathbf{k}_1\cdot\mathbf{k}_2)^2+4(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\times\mathbf{k}_2)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_1\times\mathbf{k}_2)] \times[4(\omega_1{}^3\omega_2)^{-1}+(\omega_1\omega_2)^{-2}] +[2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\mathbf{k}_1\cdot\mathbf{k}_2)^2+3(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\times\mathbf{k}_2)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_1\times\mathbf{k}_2)] \times[8(\omega_1{}^3\omega_2)^{-1}-2(\omega_1\omega_2)^{-2} -4(\omega_1\omega_2)^{-1}(\omega_1+\omega_2)^{-2}]\}. \quad (30)$$

All of the Fourier transforms in Eqs. (29) and (30) are standard except the last term of Eq. (27) which we evaluate under the assumption that

$$4[\omega_1\omega_2(\omega_1+\omega_2)^2]^{-1}\cong(\omega_1\omega_2)^{-2} \quad (31)$$

over the momentum range considered. Setting $x=\mu r$, we then find (with $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2=-3$ and $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2=+1$ for the deuteron),

$$-\partial V_1'(r)/\partial W =(g^2/4\pi)(\mu/2M)^2(2/\pi) \times\{[K_0(x)-K_1(x)/x] +S_{12}[K_0(x)+2K_1(x)/x]\}, \quad (32)$$

$$-\partial V_2'(r)/\partial W =(g^2/4\pi)^2(\mu/2M)^4 \times\{(2/\pi)^2[(43/2)(K_1(x)/x)^2 +19K_0(x)(K_1(x)/x)+6(K_0(x))^2] -(e^{-2x}/x^4)[3x^3+2x^2+8x+4]\} -S_{12}(2/\pi)^2[7(K_1(x)/x)^2+(7/2)K_0(x)(K_1(x)/x)] -S_{12}(e^{-2x}/x^4)[2x^2+5x+4]\}. \quad (33)$$

In terms of the standard expression for the deuteron ground state, which we identify with a_0,

$$\psi(\mathbf{r})=(4\pi)^{-\frac{1}{2}}r^{-1}[u(r)+2^{-\frac{3}{2}}w(r)S_{12}]\chi_{1,1}, \quad (34)$$

we obtain by means of Eqs. (32) and (33) the following contributions to the quadrupole moment, separated according to order in g^2 and orbital angular momentum state:

$$Q_{1S}=\frac{g^2}{4\pi}\frac{2}{\pi}\left(\frac{1}{2M}\right)^2\frac{1}{5} \times\int dru^2(r)x^2[K_0(x)+2K_1(x)/x], \quad (35)$$

$$Q_{1S-D}=-\frac{g^2}{4\pi}\frac{2}{\pi}\left(\frac{1}{2M}\right)^2\frac{1}{5\sqrt{2}} \times\int druwx^2[K_0(x)+5K_1(x)/x], \quad (36)$$

$$Q_{1D}=\frac{g^2}{4\pi}\frac{2}{\pi}\left(\frac{1}{2M}\right)^2\frac{1}{20} \times\int drw^2x^2[5K_0(x)+13K_1(x)/x], \quad (37)$$

$$Q_{2S}=-\left(\frac{g^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^4\frac{1}{\mu^2}\frac{1}{5}\int dru^2(r)x^2\{(2/\pi)^2 \times[7(K_1(x)/x)^2+(7/2)K_0(x)K_1(x)/x] +(e^{-2x}/x^4)[2x^2+5x+4]\}, \quad (38)$$

$$Q_{2S-D}=\left(\frac{g^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^4\frac{1}{\mu^2}\frac{1}{5\sqrt{2}}\int druwx^2\Big\{\left(\frac{2}{\pi}\right)^2\times[(71/2)(K_1(x)/x)^2+12K_0(x)(K_1(x)/x)+6(K_0(x))^2]+(e^{-2x}/x^4)\times[-3x^3+2x^2+2x+4]\Big\}, \quad (39)$$

$$Q_{2D}=-\left(\frac{g^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^4\frac{1}{\mu^2}\frac{1}{20}\int drw^2x^2\Big\{\left(\frac{2}{\pi}\right)^2\times[(127/2)(K_1(x)/x)^2-2K_0(x)(K_1(x)/x)+6(K_0(x))^2]+(e^{-2x}/x^4)\times[-3x^3+10x^2+22x+20]\Big\}. \quad (40)$$

We must also record the normalization terms arising from $-\langle\partial V'/\partial W\rangle_c$,

$$N_{1S}=\left(\frac{g^2}{4\pi}\right)\left(\frac{2}{\pi}\right)\left(\frac{\mu}{2M}\right)^2\int dru^2(K_0-K_1/x), \quad (41)$$

$$N_{1S-D}=\left(\frac{g^2}{4\pi}\right)\left(\frac{2}{\pi}\right)\left(\frac{\mu}{2M}\right)^2 4\sqrt{2}\times\int druw(K_0+2K_1/x), \quad (42)$$

$$N_{1D}=-\frac{g^2}{4\pi}\left(\frac{2}{\pi}\right)\left(\frac{\mu}{2M}\right)^2\int drw^2(K_0+5K_1/x), \quad (43)$$

and in fourth order

$$N_{2S}=\left(\frac{g^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^4\int dru^2\Big\{\left(\frac{2}{\pi}\right)^2\times[(43/2)(K_1/x)^2+19K_0(K_1/x)+6K_0^2]+(e^{-2x}/x^4)[3x^3+2x^2+8x+4]\Big\}, \quad (44)$$

$$N_{2S-D}=-\left(\frac{g^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^4 4\sqrt{2}\int druw\Big\{\left(\frac{2}{\pi}\right)^2\times[7(K_1/x)^2-(7/2)K_0K_1/x]+(e^{-2x}/x^4)[2x^2+5x+4]\Big\}, \quad (45)$$

$$N_{2D}=\left(\frac{g^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^4\int drw^2\Big\{\left(\frac{2}{\pi}\right)^2\times[(71/2)(K_1/x)^2+12K_0(K_1/x)+6K_0^2]+(e^{-2x}/x^4)[-3x^3+2x^2+2x+4]\Big\}. \quad (46)$$

It will be noticed that the operators in Eqs. (44), (45), and (46) are too singular to permit the integrations to be performed down to the origin. Hence, in performing the numerical evaluation of our theory we made use of a phenomenological wave function based on a hard core, which gives a good fit to the bound-state data.[8] The results are given in Table I. We have chosen $(g^2/4\pi)=10$, even though the experimental evidence indicates a slightly larger constant, in order to facilitate comparison with previous work.[2,4] The g^2 terms, which are perfectly convergent at the origin, have, for comparison's sake, been calculated using a Hulthén wave function. One should keep in mind when considering our meson correction terms that the measured Deuteron moment is 2.73×10^{-27} cm².

If, from Eq. (21) we take as the total mesonic correction to the quadrupole moment,

$$\Delta Q_{\text{tot}}=\langle Q^1\rangle+\langle Q^2\rangle-Q(P_1+P_2), \quad (47)$$

then from Table I, $\Delta Q_{\text{tot}}=-0.539\times10^{-28}$ cm², which is the order of two percent of the measured moment.

IV. FINAL REMARKS

As in all treatments of the two-nucleon problem which make use of the adiabatic approximation, the validity of the results depends upon the justification of the use of the hard-core wave function. With this proviso, however, it appears that the Tamm-Dancoff method provides a means of computing mesonic contributions to charge density effects which is rapidly convergent. In fact, the rate of convergence for these effects, as shown in Table I, is considerably more favorable than that for the nuclear potential itself, since the second moment of $\partial V'/\partial W$ is less singular in the interaction region than V' itself.

TABLE I. Numerical results for quadrupole moment and normalization integrals. The coupling constant $g^2/4\pi$ has been chosen equal to 10.

Wave function	Order	Quadrupole moment ($\times10^{28}$ cm²)	Normalization integrals ($\times10^2$)
Core: $S-S$	eg^2	0.974	−0.289
$S-D$		−0.435	5.136
$D-D$		0.115	−0.778
Hulthén $S-S$	eg^2	1.177	−3.797
$S-D$		...	...
$D-D$		...	...
Core: $S-S$	eg^4	−0.270	0.742
$S-D$		−0.050	−0.395
$D-D$		0.350	0.176
Core:	Total eg^2	0.636	4.069
	Total eg^4	0.028	0.523

[8] J. M. Blatt and M. H. Kalos, Phys. Rev. **92**, 1563 (1953). We are grateful to Dr. Kalos for giving us the use of his wave function tables and to Dr. Sessler for correspondence relevant to the numerical integrals performed.

It is to be noted that the same techniques employed in calculating the quadrupole moment can be used to estimate the deuteron charge density radius. Here, mesonic contributions may play a role in the interpretation of high-energy electron-deuteron scattering experiments when these become accurate enough to give a precision measurement of this quantity. One may also employ the formalism of the paper for transition problems where meson currents play the dominant role at high energies (over 100 Mev). Such work is in progress.

APPENDIX A. RELATION OF CHARGE DENSITY TO QUADRUPOLE MOMENT

In this Appendix, we shall present the details upon which the remarks following Eq. (22) of the text are based.

To this end we consider in detail Eq. (24b), with the substitutions indicated in Eqs. (25), (26), and (27), and with all irrelevant multiplicative factors omitted, as follows:

$$\begin{aligned}\int & d\mathbf{P}d\mathbf{p}d\mathbf{k}d\mathbf{q}\delta(\mathbf{P}+\mathbf{q})a^*(\mathbf{p}-\mathbf{k}+\tfrac{1}{2}\mathbf{q})\\ &\times[u^*(\mathbf{p}+\tfrac{1}{2}\mathbf{P}-\mathbf{k}+\mathbf{q})\Lambda_+(\mathbf{p}+\tfrac{1}{2}\mathbf{P}-\mathbf{k})\\ &\times\gamma_0\gamma_5 u(\mathbf{p}+\tfrac{1}{2}\mathbf{P})]_1[u^*(\tfrac{1}{2}\mathbf{P}-\mathbf{p}+\mathbf{k})\gamma_0\gamma_5 u(\tfrac{1}{2}\mathbf{P}-\mathbf{p})]_2\\ &\times a(\mathbf{p})\delta(\mathbf{P})\partial^2/\partial q_i\partial q_j\delta(\mathbf{q})\partial_i\partial_j A_0^e|_0\\ &\times\{\omega(\mathbf{k})[W-\epsilon(\mathbf{p}+\tfrac{1}{2}\mathbf{P}-\mathbf{k})-\epsilon(\tfrac{1}{2}\mathbf{P}-\mathbf{p})-\omega(\mathbf{k})]\\ &\times[W-\epsilon(\mathbf{p}+\tfrac{1}{2}\mathbf{P}-\mathbf{k}+\mathbf{q})-\epsilon(\tfrac{1}{2}\mathbf{P}-\mathbf{p})-\omega(\mathbf{k})]\}^{-1}. \quad \text{(A.1)}\end{aligned}$$

At this stage one performs an integration by parts with respect to q. One then has four classes of terms to consider. There are terms in which one or more derivatives acts on $\delta(\mathbf{P}+\mathbf{q})$. All of these vanish quite independently of the adiabatic limit since on the one hand single derivatives give rise to vanishing odd integrals and the double derivatives when integrated pick out only the trace of $\partial_i\partial_j A_0^e|_0$, which vanishes since there are no sources of the external field in the nucleus. Next, there are the terms in which one or more derivatives act on an energy denominator. These can all be shown to have vanishing adiabatic limits and represent recoil corrections not evaluated explicitly in the text. The derivatives acting on the spinor $u^*(\mathbf{p}_1-\mathbf{k}+\mathbf{q})$ also give rise to recoil terms dropped in the adiabatic limit.

Hence, we are left with the term in which both derivatives act on the amplitude $a(\mathbf{p}-\mathbf{k}+\frac{1}{2}\mathbf{q})$, Eq. (28) of the text. Transforming this term to coordinate space will readily convince the reader that this is the contribution to the quadrupole energy defined as in Eq. (23). The structure of Eq. (A.1) is general enough to indicate that the argument will go through similarly to any order in g^2. We therefore conclude that in the adiabatic limit it is correct to compute charge density effects from $\partial V'/\partial W$, as we have done.

APPENDIX B. RECOIL CORRECTIONS

The problem of the treatment of nuclear recoil in our work has two aspects. There is the problem of defining a suitable nuclear interaction from which the amplitude $a(\mathbf{p}_1,\mathbf{p}_2)$ is to be computed. In this connection we have adopted the attitude that Eq. (10) has been solved with sufficient accuracy so that $a(\mathbf{p}_1,\mathbf{p}_2)$ actually contains the significant facets of the nuclear motion in intermediate states. There is also the occurrence of recoil effects in the exchange operators themselves. Here we have uniformly employed the adiabatic approximation in order to facilitate our numerical computations. Thus all energy denominators of the form

$$W-\epsilon(\mathbf{p})-\epsilon(\mathbf{p}')-\omega(\mathbf{k}), \quad \text{(B.1)}$$

occurring in $\partial V'/\partial W$, have been set equal to $-\omega(\mathbf{k})$. This has been done under the assumption that

$$[W-\epsilon(\mathbf{p})-\epsilon(\mathbf{p}')]/\omega(\mathbf{k})\ll 1, \quad \text{(B.2)}$$

and since in the deuteron this ratio is of order μ/M, we do not expect modifications of the effective exchange operators based on the inclusion of recoil terms to make substantial alterations in our results.

However, another entirely different approach to recoil terms can be given in the Tamm-Dancoff formalism along the lines of what Deser[2] has done for the covariant equation. In this approach one assumes that the static nuclear potential binds the deuteron, i.e., one solves Eq. (10) of the text in the adiabatic limit, and then one adds both the electromagnetic field and the "recoil potential" as perturbations on the system. If one now applies perturbation theory, it is evident that cross terms between the velocity dependent potential and the electromagnetic field will make a contribution to the exchange moment energies. It is interesting to follow this idea through in detail in our formalism since we shall succeed then in making connection with Deser's results.

To this end we consider a typical contribution to V' as given, for example, by Eq. (23a) of the text, with the structure,

$$\frac{g^2}{(2\pi)^3}\frac{\langle\gamma_0\gamma_5\rangle_1\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\langle\gamma_0\gamma_5\rangle_2}{2\omega(\mathbf{k})[W-\epsilon(\mathbf{p}'-\mathbf{k})-\epsilon(\mathbf{p})-\omega(\mathbf{k})]}. \quad \text{(B.3)}$$

In order to separate the recoil effects in Eq. (B.3) we write

$$\begin{aligned}[W-\epsilon(\mathbf{p}'-\mathbf{k})-\epsilon(\mathbf{p})-\omega(\mathbf{k})]^{-1}\\ =-\omega^{-1}+[W-\epsilon(\mathbf{p}'-\mathbf{k})-\epsilon(\mathbf{p})]\\ \times\{\omega[W-\epsilon(\mathbf{p}'-\mathbf{k})-\epsilon(\mathbf{p})-\omega]\}^{-1}. \quad \text{(B.4)}\end{aligned}$$

Thus by the "recoil potential," V_r', we shall mean all contributions to V' such as Eq. (B.3) with the nucleon energy denominators replaced by expressions like the *last* term of Eq. (B.4).

By standard second-order perturbation theory the eg^2 contribution to the electrostatic energy due to V_r' and

$A_0^e(q)$ is given by

$$\Delta W=\sum_n \frac{\langle 0|V_r'|n\rangle\langle n|V^{\rm el}|0\rangle+\langle 0|V^{\rm el}|n\rangle\langle n|V_r'|0\rangle}{W_0-W_n}. \quad \text{(B.5)}$$

Here, W_0 refers to the unperturbed deuteron energy and the summation is over continuum states of the two nucleon system. In order to get a manageable expression for Eq. (B.5) we shall ignore interactions in intermediate states and write for the continuum wave functions indexed by momentum variables,

$$\begin{aligned}\phi_n(\mathbf{p}_1,\mathbf{p}_2)&=\phi_{\mathbf{p}',\mathbf{p}''}(\mathbf{p}_1\mathbf{p}_2)\\ &=u(\mathbf{p}')u(\mathbf{p}'')\delta(\mathbf{p}_1-\mathbf{p}')\delta(\mathbf{p}_2-\mathbf{p}''),\end{aligned} \quad \text{(B.6)}$$

where the $u(\mathbf{p})$ are free-particle spinors. If we put in the details we obtain as a typical contribution to Eq. (B.5),

$$\begin{aligned}&\frac{eg^2}{(2\pi)^3}\int\frac{d\mathbf{p}_1d\mathbf{p}_2d\mathbf{k}d\mathbf{q}}{2[\omega(\mathbf{k})]^2}a_0^*(\mathbf{p}_1,\,\mathbf{p}_2+\mathbf{q})\langle\gamma_0\gamma_5\tau_i\rangle_1\\ &\times\langle A_0^e(\mathbf{q})\Lambda_+(\mathbf{p}_2)\gamma_0\gamma_5\tau_i\rangle_2 a_0(\mathbf{p}_1-\mathbf{k},\,\mathbf{p}_2+\mathbf{k})\\ &\times\Bigg[\frac{W-\epsilon(\mathbf{p}_1)-\epsilon(\mathbf{p}_2+\mathbf{k})}{[W-\epsilon(\mathbf{p}_1)-\epsilon(\mathbf{p}_2)][W-\epsilon(\mathbf{p}_1)-\epsilon(\mathbf{p}_2+\mathbf{k})-\omega(\mathbf{k})]}\\ &+\frac{W-\epsilon(\mathbf{p}_1-\mathbf{k})-\epsilon(\mathbf{p}_2)}{[W-\epsilon(\mathbf{p}_1)-\epsilon(\mathbf{p}_2)][W-\epsilon(\mathbf{p}_1-\mathbf{k})-\epsilon(\mathbf{p}_2)-\omega(\mathbf{k})]}\Bigg],\end{aligned} \quad \text{(B.7)}$$

to which must be added three terms of similar structure. The subscript zero on $a_0(\mathbf{p}_1,\mathbf{p}_2)$ means that these amplitudes are solutions of Eq. (10) of the text *after* the adiabatic limit has been taken in the interaction kernel.

If in Eq. (B.7) we were to set all ratios of the form

$$[W-\epsilon(\mathbf{p})-\epsilon(\mathbf{p}')]/[W-\epsilon(\mathbf{p}'')-\epsilon(\mathbf{p}''')]=1 \quad \text{(B.8)}$$

and take the adiabatic limit, Eq. (B.7) would become at once,

$$\begin{aligned}&\frac{-2eg^2}{(2\pi)^3}\int\frac{d\mathbf{p}_1d\mathbf{p}_2d\mathbf{k}d\mathbf{q}}{2\omega^3}\{a_0^*(\mathbf{p}_1,\,\mathbf{p}_2+\mathbf{q})\\ &\times\langle\gamma_0\gamma_5\tau_i\rangle_1\langle A_0^e(\mathbf{q})\Lambda_+(\mathbf{p}_2)\gamma_0\gamma_5\tau_i\rangle_2\\ &\times a_0(\mathbf{p}_1-\mathbf{k},\,\mathbf{p}_2+\mathbf{k})+\cdots\}.\end{aligned} \quad \text{(B.9)}$$

To Eq. (B.9) we add the electrostatic exchange energy computed directly from the adiabatic nuclear potential, i.e., terms of the form

$$\begin{aligned}&\frac{eg^2}{(2\pi)^3}\int\frac{d\mathbf{p}_1d\mathbf{p}_2d\mathbf{k}d\mathbf{q}}{2\omega^3}\{a_0^*(\mathbf{p}_1,\,\mathbf{p}_2+\mathbf{q})\\ &\times\langle\gamma_0\gamma_5\tau_i\rangle_1\langle A_0^e(\mathbf{q})\Lambda_+(\mathbf{p}_2)\gamma_0\gamma_5\tau_i\rangle_2\\ &\times a_0(\mathbf{p}_1-\mathbf{k},\,\mathbf{p}_2+\mathbf{k})+\cdots\}.\end{aligned} \quad \text{(B.10)}$$

The principal point to notice is that the structure of Eq. (B.9) is identical to that of Eq. (B.10), and differs from it only by a factor of -2. If these two equations are now added together, we get as a total expression of the eg^2 electromagnetic energy exactly Eq. (21) of Deser's paper.[2] Hence, the burden of this appendix has been to show under what conditions Deser's covariant calculation and the three-dimensional configuration-space approach reduce to each other, the transitions having been made only by means of the two unverified assumptions contained in Eqs. (B.6) and (B.8).

APPENDIX C. METHOD OF CANONICAL TRANSFORMATIONS

In this Appendix, we want to explore the question of the existence of exchange effects in the method of canonical transformations. Thus we consider the Schrödinger equation,

$$(H^0+H'+H^{\rm el})\Phi=W\Phi, \quad \text{(C.1)}$$

where H^0 is the free-meson Hamiltonian,

$$H'=-g\sum_k\sum_i(2V\omega_k)^{-\frac{1}{2}}\tau_\lambda^{(i)}(\gamma_0\gamma_5)^{(i)}e^{i\mathbf{k}\cdot\mathbf{x}_i}\times(a_{k\lambda}+a_{-k\lambda}^\dagger), \quad \text{(C.2)}$$

and

$$H^{\rm el}=-e\sum_i(\gamma_0\gamma_\mu)^{(i)}A_\mu(x_i)\tfrac{1}{2}(1+\tau_3^{i}), \quad \text{(C.3)}$$

that is, we ignore nucleon recoil in intermediate states *ab initio*. Equation (C.1) is then transformed by means of a sequence of canonical transformations designed to uncouple components of Φ belonging to different numbers of mesons. The first of these is chosen according to the equation

$$[S^{(1)},H^0]=-H', \quad \text{(C.4)}$$

and is given by

$$S^{(1)}=g\sum_k\sum_i(2V\omega_k^3)^{-\frac{1}{2}}\tau_\lambda^{(i)}(\gamma_0\gamma_5)^{(i)}e^{i\mathbf{k}\cdot\mathbf{x}_i}\times(a_{k\lambda}-a_{-k\lambda}^\dagger). \quad \text{(C.5)}$$

The electromagnetic effects to this order are then evaluated by taking the expectation value of

$$\exp(S^{(1)})H^{\rm el}\exp(-S^{(1)}) \quad \text{(C.6)}$$

in the two-nucleon, no-meson amplitude. However, $S^{(1)}$ has the very special property that it can be written as a sum of commuting operators belonging to distinct nucleons, i.e.,

$$S^{(1)}=S_1^{(1)}+S_2^{(1)}, \quad \text{(C.7)}$$

and

$$[S_1^{(1)},S_2^{(1)}]=0. \quad \text{(C.8)}$$

Hence Eq. (C.6) can be written

$$\exp(S_2^{(1)})\exp(S_1^{(1)})H^{\rm el}\exp(-S_1^{(1)})\exp(-S_2^{(1)}). \quad \text{(C.9)}$$

However,

$$H^{\mathrm{el}}=\sum_i H_i^{\mathrm{el}}, \quad \text{(C.10)}$$

with

$$[H_i^{\mathrm{el}},S_j^{(1)}]=0, \quad i\neq j. \quad \text{(C.11)}$$

It is now evident from Eq. (C.9) that Eq. (C.6) contains no exchange terms. In agreement with Villars,[3] we would have to include nucleon recoil to obtain a nonvanishing result to this order.

If we now construct the next term in the sequence of canonical transformations it can be shown that no decompositions of the form of Eqs. (C.7) and (C.9) is possible. Hence, there are adiabatic exchange effects in the theory, but they are at least of order eg^4.

PHYSICAL REVIEW VOLUME 99, NUMBER 3 AUGUST 1, 1955

Neutron-Electron Interaction

GEORGE SALZMAN*
Physics Department, University of Illinois, Urbana, Illinois

(Received March 21, 1955)

The effective neutron-electron interaction is calculated with the cut-off pseudoscalar meson theory and neglect of nucleon recoil. Values of −7.1 and −8.6 kev are obtained for the meson contribution with two different shapes for the cut-off function. The nucleon contribution is ambiguous in the cut-off theory since the shape of the core charge is undetermined. It is shown that if the no recoil theory with cutoff is interpreted to be the limit, as the nucleon mass becomes infinite, of a relativistic theory with a cutoff, then the contribution of the anomalous magnetic moment (or Foldy) term is not contained in our calculation. Since the Foldy term alone accounts for the major part of the experimental interaction, the sum of pion and nucleon charge contributions in this theory, if correct, should be at most of the order of a few hundred ev, or an order of magnitude less than the pion part. Finally, a brief calculation is made to illustrate the fact that, if spread out in a plausible way, the nucleon core charge could effect the needed cancellation.

1. INTRODUCTION

RECENT experiments at Brookhaven[1] give $-eV_0=-3.86\pm0.37$ kev for the effective neutron-electron interaction energy, where the effective potential V_0 is assigned a radius equal to the classical electron radius, and $-e$ is the electron charge. Although the weak coupling approximation to relativistic pseudoscalar meson theory with pseudoscalar coupling is in qualitative agreement[2] with this value if the coupling constant is fitted with the neutron anomalous magnetic moment μ_N, this theory must be considered unreliable because of its inability[3] to give simultaneously the experimental ratio of μ_N/μ_P, where μ_P is the proton anomalous magnetic moment. The neutron-electron interaction and the neutron anomalous magnetic moment are closely related; one involves the interaction of a slow neutron with a static electric field, the other with a static magnetic field.

Our approach to the nucleon meson problem is related to that of Sachs,[4] who also neglects nucleon recoil. The two treatments differ in that here a specific interaction is assumed, whereas his more general analysis is independent of such details. For the present purposes however, a more important difference lies in our belief that a theory without nucleon recoil should explain only the "large components" part of the neutron-electron interaction. The parameters of this theory were thus fitted to a set of data which is not equivalent to the set of data used by Sachs.

These facts, and recent efforts to describe nucleon meson phenomena by a no recoil cut-off pseudoscalar meson theory,[5,6] make it of interest to examine the neutron-electron interaction in this theory, because it gives the nucleon anomalous magnetic moments fairly well[6,7] without having any adjustable parameters. The parameters were previously determined in pion-nucleon scattering and photomeson production calculations. The mesonic part of the electrostatic neutron-electron interaction is as well defined in this theory as are the nucleon anomalous magnetic moments.

In Sec. 2, the approximation is described and the usual expression for $-eV_0$ is obtained [Eq. (2)]. In Sec. 3, the meson charge density surrounding the neutron is calculated by second-order perturbation theory, and shown to be negative definite independent of the choice of cut-off function. It then follows that the mesonic contribution to $-eV_0$ is negative. This is intuitively expected from a picture of the neutron dissociating according to $N\rightleftharpoons P+\pi^-$. In Sec. 4, an expression for the pion contribution to $-eV_0$ is obtained [Eq. (9)] which applies for the particular meson

* Now at the University of Rochester, Rochester, New York.
[1] Hughes, Harvey, Goldberg, and Stafne, Phys. Rev. **90**, 497 (1953).
[2] B. Fried, Phys. Rev. **88**, 1142 (1952). References are given here to earlier calculations.
[3] L. Foldy, Phys. Rev. **87**, 675 (1952); **87**, 693 (1952).
[4] R. Sachs, Phys. Rev. **87**, 1100 (1952); **95**, 1065 (1954).
[5] G. Chew, Phys. Rev. **94**, 1748 (1954); **94**, 1755 (1954).
[6] G. Chew, Phys. Rev. **95**, 1669 (1954).
[7] M. Friedman, Phys. Rev. **97**, 1123 (1955).