Fortunately it was on nickel ($Z=28$), the highest-Z element studied, that we could make the most precise energy measurement (partly because the calibration energy was practically equal to the $L\alpha$ energy). Attributing all the shift to the $2p$ level, we can say that the energy of the $2p$ state of $Z=28$ is affected by the specific pion-nucleus interaction no more than 12 kev, or 1.6%.

VIII. ACKNOWLEDGMENTS

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Energy of Electrons or Photons from Their Cascade Showers in Copper*

W. E. HAZEN
Ecole Polytechnique, Paris, France
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The photographs obtained by the Ecole Polytechnique cloud-chamber group have been used for the study of cascade showers in copper plates produced by electrons of known momentum. The best constant for obtaining primary energy from the total number of track segments is given. The uncertainty in primary energy as determined by this method is found from the experimentally observed distributions. The results are applied to the interpretation of the heavy *S*-particle observed at the Massachusetts Institute of Technology.

1. INTRODUCTION

THE main interest in photon-electron cascade showers stems from their use as a tool. In the case of air showers, for example, one uses the theoretical results to estimate the energies of the incident primaries. In the case of particle interactions, one uses the shower-producing property as a qualitative means of identification of electrons or photons and tries to determine the energy from the size of the shower. Here we are dealing with showers in solids, in general, and, in the case of multiplate cloud chambers, the plate material is usually such that the available calculations are not very useful. The Monte-Carlo method[1] gives a better approximation than calculations but it is severely limited by the difficulty of handling the scattering problem. Thus, one must return to purely experimental results and use the theory for a guide or for minor corrections.

D'Andlau[2] has made a detailed study of the showers at particular depths with electron primaries of known energies. Among other things these results enable one to know the probability of mistaking an electron for a heavier particle when observing the penetrating power.

In the study of the radiation emitted by *S*-particles in multiplate cloud chambers, one is frequently faced with the question of estimating the energy of the initiators of electron-photon cascades.[3] Again, one must turn to experimental results. Previous work utilized π^0 mesons from nuclear disintegrations[3,4] but the statistics were not adequate to allow determination of a probability distribution.

The photographs of nuclear disintegrations obtained by the Ecole Polytechnique group[5] with their double cloud-chamber arrangement give the opportunity to observe the production of cascade showers by electrons of known momentum. Thus, one can observe the showers produced by electrons of known energy and from these data one can deduce the probability distribution in initiating energy for a shower of given size.

2. METHOD

(a) Selection of Data

The photographs of the lower chamber, which contained copper plates of one-centimeter thickness, were scanned for individual cascade showers. Showers that were selected were sufficiently distant from other events so that possible confusion with other radiation would only lead to uncertainties small compared with the inherent statistical uncertainty. The corresponding photographs of the upper chamber were inspected for cases in which the initiating particle could be identified unambiguously. Correspondence between tracks in the upper chamber and showers in the lower chamber could be determined to within ~1 cm on the reprojection table of the laboratory. Since multiple scattering and

* This study was made while on leave from the University of Michigan as a Fulbright research scholar.
[1] R. R. Wilson, Phys. Rev. **86**, 261 (1952).
[2] C. A. D'Andlau, Nuovo cimento **12**, 859 (1954).
[3] Bridge, Courant, DeStaebler, and Rossi, Phys. Rev. **95**, 1101 (1954); H. Courant, Phys. Rev. **94**, 797(A) (1954).
[4] P. A. Bender, thesis, Washington University, St. Louis, Missouri, 1955 (unpublished).
[5] Gregory, Lagarrigue, Leprince-Ringuet, Muller, and Peyrou, Nuovo cimento **11**, 292 (1954).

inelastic scattering contributed uncertainties of the order of 1 or 2 centimeters, depending on the energy, it was unnecessary to use a more accurate method for checking correspondence. The electronic radiation in the upper chamber originated from the π^0 component of the penetrating showers that were generated in material above the top chamber. Thus, in general, a cascade of electrons, positrons, and photons, accompanied by mesons and protons, emerged from the material above the chamber. In the selected events, the axis of the cascade shower was inclined toward the right or left. Therefore, the gamma rays were expected to be well separated from the electron or positron that was bent downward by the magnetic field and passed into the bottom chamber. Since the multiplicity of particles in the upper chamber was usually rather low, there was a consequent low probability of mistaking the identity of the initiator of the shower in the lower chamber. The major cause of uncertainty was the multiple scattering in the bottom wall of the upper chamber. However, since the scattering material constituted 0.25 radiation units at a distance of 40 cm above the bottom chamber, the rms lateral displacement would only be $300/E_0$ (Mev) in centimeters at the bottom chamber. Also, the direction of the shower axis was usually well enough defined to allow a check on correct identification. For example, a γ ray from the generator would have quite a different direction from that of an electron or positron because of the deflection of the charged particles in the magnetic field.

The shower parameter that is expected to have the smallest fluctuations for a given primary energy (E_0) is the total track length of electron secondaries (l). The primary energy is determined from $E_0=(dE/dx)l$, where (dE/dx) is the average rate of energy loss of the cascade particles. Fluctuations are then due only to variations in the distribution of differential track length from event to event. These fluctuations have little effect on the calculated value of E_0 since (dE/dx) is not strongly dependent on the differential track length distribution. However, in a multiplate chamber, most of the energy dissipation is inside the plates. One determines the track length in a plate by taking the mean of the number of tracks entering and leaving the plate[6]; the true track length within the plate fluctuates widely about this "observed" value because of the stepwise nature of the changes in number and because of the effects of scattering. For example, if we choose a simple model, neglecting scattering, in which it is equally likely that a track entering a plate stops at any depth in the plate and a track leaving a plate originates at any depth, the probable error in each element of "measured" track length is $t/2$ (where t is the plate thickness). Consequently the probable error in total track length determined by observation of N segments would be $l/2\sqrt{N}$. If scattering were included in the model, the probable error per element would be increased and we might expect a probable error of the order $l/\sqrt{N}$. Thus the *a priori* expectation is that E_0 might be determined with an accuracy of about $E_0/\sqrt{N}$. In general, other methods for determining E_0, such as the number of particles at the maximum, must involve greater uncertainty. It can be shown that there is negligible gain in averaging E_0 determined from N with E_0 determined from other parameters such as the number of particles at the maximum.

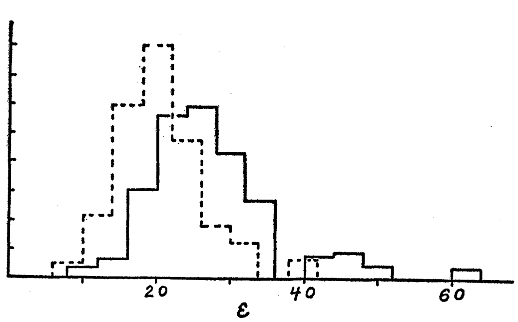

FIG. 1. The distributions in ϵ for two ranges of E_0. The dotted distribution is for $100<E_0<300$; the solid is for $E_0>500$.

TABLE I. Energy per track segment, $\epsilon=E_0/N$.

E range	100 to 200	200 to 300	300 to 500	>500
Number of events	33	25	30	28
Average ϵ	21.0	20.3	26.5	26.7
Average ϵ	20.4		27.8	
$0.67\Delta\epsilon_{rms}$	4.0		6.0	
Average N	10		20	
Expected statistical P.E.	4.3		4.8	

There was a significant probability for loss of energy by radiation by an electron as it traversed the bottom and associated material of the top chamber. The angle of departure of the photon from the path of its parent electron is small (the effect at the bottom chamber is an rms displacement of only $100/E_0$ cm) but the electron is deflected in the fringing magnetic field. The deflection in the fringing field depends on the fraction of energy (f) lost by the electron. When the fractional energy loss by the electron is too large for a given E_0, the electron will arrive at a point too far from the expected point of arrival to be considered as a satisfactory identification between primary and shower. On the

[6] All track segments were counted, including those that might represent back scatterings and those of electrons that scattered noticeably in the gas. The latter may represent less track length in copper than higher energy electrons because they will not penetrate far into the next plate. On the other hand, they have scattered more in the plate from which they emerge and therefore represent more than average path length in the plate of emergence. Consequently, they were given equal weight with higher energy electrons. Back-scattered segments represent less track length in copper than other segments and therefore should be given lower statistical weight, in principle. However, they are not numerous and their identification is difficult in most cases.

other hand, if f is sufficiently small for a given E_0, and E_0 sufficiently large, not only will the electron arrive sufficiently near to the predicted point for satisfactory identification of its shower, but also the photon will arrive sufficiently close to the electron so that the showers are considered as one. A quantitative treatment shows that the region of the f and E_0 plane in which data might be incorrectly interpreted is exceedingly small, affecting no events of $E_0>240$ Mev and giving a maximum uncertainty at $E_0=120$ Mev. At the latter energy, 25 percent of the events might have 20% error, the other events <20%.

(b) Analysis

There are two questions that we seek to answer: what is the best value that we can obtain for the energy (E_0) of a shower initiator; and what is the probability distribution in E_0 for a given observed shower? As noted above, the total track length (l) as measured by the observed track segments above and below the plates gives the best measure of E_0, i.e., $E_0=(dE/dx)l=\epsilon_0 l$, where ϵ_0 is the critical energy and l is measured in radiation units.[7] If n_i represents the number of track segments above the ith plate, $l=K\sum n_i t$, where t is the plate thickness and K is a proportionality constant which, as we noted earlier, is probably >1 because of scattering etc. in the plates. Letting $N=\sum n_i$, we have

$$E_0=K\epsilon_0 tN=\epsilon N, \quad (1)$$

where ϵ is the average energy dissipation in a plate per observed track segment. Thus, we wish to determine the best value for ϵ in order to answer the first question at the beginning of the paragraph. Since scattering is a decreasing function of electron energy and the mean energy of the electrons in the shower is a slowly increasing function of E_0, we expect K (and ϵ) to be a slowly decreasing function of E_0. Thus a possible energy dependence of ϵ should be sought.

The second question, that of the distribution in E_0 for a given N, can be answered by selecting showers in narrow bands of N values, correcting all observed values of E_0 to the equivalent values for the average N, and analyzing the resultant E_0 distribution.

3. RESULTS

(a) Determination of $\bar{\epsilon}(=E_0/N)$

The data were divided into E_0 groups, since one expects ϵ to depend on E_0, as noted above. Each datum for $E_0<500$ Mev was given a weight E_0, on the assumption that the main uncertainty derives from the statistical variation in the number of track segments N. The average value of N is proportional to E_0, since ϵ does not vary appreciably with E_0, as we shall see later. For $E_0>500$, the uncertainty in momentum measurement becomes comparable with the statistical uncertainty. Therefore these events were weighted equally.

[7] See, B. Rossi, *High Energy Particles* (Prentice-Hall, Inc., New York, 1952) for the information on shower theory that is assumed.

TABLE II. E_0 distributions for a given observed N.

N	5 to 7	8 to 10	11 to 15	16 to 22
Number of events	16	22	26	22
Average N	6.2	9.2	12.9	19.4
Average E	162	265	290	542
$0.67\Delta E_{rms}/E_{Av}$	37%	28	16	23
$0.67/N^{\frac{1}{2}}$	27%	22	19	15

The results for the four selected energy ranges are shown in Table I. Since no rapid variation of $\bar{\epsilon}$ with E_0 is apparent, the distributions were examined in terms of only two E_0 groups, 100 to 300 Mev and greater than 300 Mev. These distributions are displayed in Fig. 1, and the corresponding parameters are given in Table I. An effective $\bar{N}$ was estimated for each group of data and we see that the breadths of the observed distributions are about what we expect from the statistical fluctuations in N alone. The two values of $\bar{\epsilon}$ differ more than the probable errors of their means. However, since one expects an energy dependence in the opposite direction [see Sec. 2(b)], it is probably most reasonable to assume that we have a fluctuation effect. Therefore, the overall mean of 24±3 is the best value to use for all E_0 from 100 to 1000 Mev.

(b) Uncertainty in E_0

In order to determine the distribution in E_0 for a given N with reasonable accuracy, it was desirable to combine data for neighboring values of N. This grouping is indicated in Table II. Each E_0 was multiplied by $\bar{N}/N$, where $\bar{N}$ is the mean value for each group, and each event was weighted according to the relative frequency of occurrance of events of energy E_0 in the entire body of data.

The results are presented in Table II, where the observed probable errors are compared with the probable error deriving from statistical variations in N. Since the number of events in each group is small, the uncertainty in the probable error is large for any particular group. However, we can combine the results from the four groups if we make some kind of assumption concerning the dependence of the true probable error on shower size. For example, the first approximation would be that the probable error is simply a constant factor times the statistical probable error. We then find that the average factor is 1.25, i.e., that the average ratio of observed to statistical error is 1.25. This sort of result was anticipated from the theory [Sec. 2(b)], namely, that the fluctuations would correspond approximately to the statistics of N events.

4. CONCLUSIONS

(a) Determination of E_0

For showers at normal incidence in assemblies of one centimeter copper plates, the best value for the energy of the shower initiator is given by $E_0=24N$ (Mev), where N is the total number of track segments in the shower. If asymmetric effects of particle scattering for inclined showers are neglected, we should expect to have, from (1), $E_0=[36(\epsilon_0/20)t/\cos\theta]N$, where ϵ_0 and t are the critical energy and plate thickness (radiation units), and θ the angle of incidence. We have here assumed that scattering effects have the same fractional effect for all materials and hence the validity of the above equation is limited to elements with Z near copper.

The probable error in E_0, when E_0 is determined by this method, is about $[0.15^2+(0.67\times1.25)^2/N]^{\frac{1}{2}}$, where the first term is a pessimistic estimate of the uncertainty in ϵ and the second term depends on the statistical uncertainty deriving from the finite number of track segments.

In cases where the observable longitudinal (or lateral) development is limited by geometry, only a lower limit to E_0 can be established within reasonable limits (the limits given above). If the shower leaves the chamber before an apparent maximum occurs, one might determine a lower energy limit by using the size of a shower at its maximum as a parameter. However, there is no real gain in information since one must then accept the large uncertainty deriving from the large statistical fluctuations in the size of showers at a given depth. Therefore, a lower energy limit based on total number of observed segments will give the narrowest limits for E_0.

(b) Comparison with Shower Theory

The observed track length in copper is found to be $E_0/36$ radiation units from the present data whereas the total track length is approximately $E_0/\epsilon_0=E_0/20$ from consideration of conservation of energy. Thus, only 0.55 of the track length is detected with this technique. Presumably, the major loss is due to scattering within the plates at low energies and we can assume a sharp low energy cut-off as a first approximation. Richards and Nordheim[6] find that 0.55 of the track length is generated by electrons while their energy $>0.3\epsilon_0$ in air or $0.4\epsilon_0$ in lead. These figures are for energies lower than the validity of the detailed track length calculations, but, since the lengths for $E=0$ are known from conservation of energy, only reasonable interpolations are involved in obtaining $0.3\epsilon_0$ and $0.4\epsilon_0$. Thus, for copper, we estimate $0.35\epsilon_0$ ($=7$ Mev) for the equivalent cutoff. This is a physically reasonable figure since electrons of this energy or less scatter with rms angles of the order unity in distances <0.1 radiation unit, with the result that their path lengths in the plates are much longer than inferred from the method of observation.

(c) Results in Lead

During the early stages of their experiment[4] the Ecole Polytechnique group had alternate lead and carbon plates in the lower chamber. A survey of these photographs yielded nine suitable events with correlated electron primaries and electron cascades. The lead plates (7 mm) were 1.22 radiation units thick and the carbon plates (15 mm) 0.09 radiation unit. The collision loss in these carbon plates is about equal to the critical energy in lead.

Only the segments above the lead plates were counted since it is easier to make an approximate correction for the carbon than to try to establish its effect in detail. The result is $\epsilon=50$ Mev per track segment or $\epsilon=40$ Mev/segment for plates of unit thickness. A first-order correction for the carbon plates, taking only collision loss into account and using calculated track lengths in lead,[6] gives 25 Mev/segment for lead plates of unit thickness. This result agrees with a study by Bender of showers produced by π^0 mesons.[4]

(d) Application to the Heavy S-Particle of MIT[5]

If we make a calculation for the total energy of the initiators of the cascade showers that appear to be due to the decay or capture of the heavy particle observed and discussed by MIT, we find $\sum E_0=1630$ Mev, in agreement with the determination of the MIT group. This is a minimum in the sense that momentum is not conserved, as noted by the MIT group, and that two of the showers probably leave the chamber before all their energy is dissipated. As a result of the present work, we can add that the probable error, deriving from a pessimistic estimate of the uncertainty in the constant ϵ and the observed fluctuations in N is about 20%. Thus it seems quite unlikely that the mass energy of the S-particle was as low as 500 Mev, if one assumes that it was a decay event.

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