Intermediate Coupling in Odd-Odd Nuclei

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An intermediate-coupling model is applied to the structure of lithium-6 and nitrogen-14. A ground-state wave function is selected which reproduces the observed ground-state moment data. A two-body potential, with spin-orbit and tensor forces included, is sought which will have the proper wave function as its lowest state and which will reproduce the observed level splitting of the lower excited states of the nucleus. The usual variational method is used, the potential is assumed to have a Yukawa radial dependence, and the wave functions are assumed to have an harmonic oscillator radial dependence. Only the p-shell nucleons are included. This procedure leads to a consistent, but not unique, result for lithium requiring a strong tensor force for satisfactory solutions. When the additional requirement of the near vanishing of the carbon-14–nitrogen-14 beta-decay element is imposed, and the procedure repeated for the nitrogen nucleus, the results are much less satisfactory. Very large and physically unmeaningful values of the potential parameters appear to be needed to obtain agreement with experimental data.

THE application of intermediate coupling to the p-shell polyads has been discussed in detail by Inglis[1] for the case in which the internucleon potential is central. Elliott[2] and Regge[3] have investigated intermediate coupling in lithium-6 for the case in which a small amount of tensor force is included in the internucleon potential. Their results indicate that in this case one cannot simultaneously produce the energy level scheme and the ground-state data. Jancovici and Talmi[4] have shown that a very strong tensor force can account for the observed carbon-14–nitrogen-14 beta decay. The present investigation shows that an internucleon potential with a strong tensor force provides a reasonably consistent scheme for describing lithium-6, within the scope of the intermediate coupling model with configuration mixing neglected.

The approach usually followed in applying the model is to start from a specific two-body potential and to obtain from it the level structure and the ground-state wave function. This investigation has proceeded in the reverse order. A wave function is obtained from the ground-state data. A potential is then sought which will produce this wave function for its lowest state, and which will also give the low-lying level structure for lithium-6.

A. LITHIUM

The ground state of lithium-6 is a $J=1$, even parity state with a magnetic moment of 0.822 nm and a static electric quadrupole moment less than $\frac{1}{2}$ millibarn in absolute value. The sign of the quadrupole moment is uncertain. The most general wave function which can be formed for this state from the p nucleons is

$$\Psi = a\,{}^3S_1 + b\,{}^3D_1 + c\,{}^1P_1. \tag{1}$$

Three conditions including normalization are available to fix the coefficients: the wave function must reproduce the static magnetic dipole and electric quadrupole moments. These conditions determine the limits between which the coefficients will lie. The coefficients are not uniquely determined because of the uncertainties in the observed value of the quadrupole moment, and in the value of the mean square displacement of the p-shell proton from the center-of-mass of the nucleus. The value $\frac{2}{3}(1.42\times10^{-13}A^{\frac{1}{3}}\text{ cm})^2$ has been used for the latter quantity. This requires the S and D states of the wave function to have the same sign. The calculations show that the coefficients are not very sensitive to this quantity. The sign of the contribution of the P state to the wave function is not fixed since only c^2 appears in these conditions. The results of the calculations are

$$0.8515 \leqslant |a|^2 \leqslant 0.8557;\quad 0.0030 \leqslant |b|^2 \leqslant 0.0113;\quad 0.1330 \leqslant |c|^2 \leqslant 0.1455. \tag{2}$$

Since lithium-6 is a self-mirror nucleus, one can argue that the usual meson exchange magnetic moments which are symmetrical cannot contribute.[5]

The potential is assumed to have the form suggested by the analysis of the two-body problem by Hall and Powell.[6] In addition, a specific spin-orbit force is assumed in which the spin-other-orbit forces are neglected. Thus,

$$V_{np} = -V_0\left[(1-g+gP^\sigma)\frac{e^{-r/\alpha_c}}{r/\alpha_c} + \gamma\frac{e^{-r/\alpha_t}}{r/\alpha_t}S_{np}\right] \times(1-\epsilon+\epsilon P^r) - V_0\zeta\textstyle\sum_i \mathbf{l}_i\cdot\mathbf{s}_i, \tag{3}$$

where ζ and γ are dimensionless parameters for the spin-orbit coupling and for the tensor coupling respectively, g and ϵ are the fractions of spin exchange and space exchange, respectively, α_c and α_t are the central and tensor ranges respectively, and V_0 is the depth of the potential. P^r is the Majorana space exchange operator and P^σ is the spin exchange operator.

[1] D. R. Inglis, Phys. Rev. **87**, 915 (1952); Revs. Modern Phys. **25**, 390 (1953).
[2] J. P. Elliott, Proc. Roy. Soc. (London) **A218**, 345 (1953).
[3] T. Regge, Nuovo cimento **11**, 285, (1954).
[4] B. Jancovici and I. Talmi, Phys. Rev. **95**, 289 (1954).
[5] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Press, Cambridge, 1953).
[6] H. H. Hall and J. L. Powell, Phys. Rev. **90**, 912 (1953).

A standard variation method is used, in which the coefficients of the L-S states of the wave function are used as variation parameters. This method gives the usual sets of secular equations and determinants. The matrix elements for the spin-orbit force are given by Condon and Shortely.[7] The matrix elements for the internucleon potential can be evaluated by methods given by Talmi[8] or Elliott.[2]

Only one $J=3$ state can be formed from the two p nucleons, hence the matrix of this state has only one element and the energy is given in terms of the parameters of the potential. This energy is then combined with the observed splitting between the $J=3$ state and the other levels of the nucleus to eliminate the eigenvalues from the remaining matrices. The matrices then become relations among the potential parameters. The secular equations for the ground state become a set of equations in the potential parameters, when the coefficients Eq. (2) are used in place of the variation parameters. These relations have been numerically solved for the case when the central potential has a range of 1.18×10^{-13} cm.

It is possible to match the observed splitting between the $J=3$ state and the ground state only when the P state has the same sign as the S and D states. For the parameters used in this calculation, it is possible to retain the form of the Hall and Powell potential only when the potential is a Serber type, if the observed level splitting between the ground state and the $J=0$ state is to be matched. The requirement to maintain the form of the Hall and Powell potential is not a rigid one, and hence calculations have been performed also for a potential having the same exchange properties as the Christian and Hart[9] potential but with the addition of a spin-orbit force exactly as in Eq. (3). This potential has a Serber central part and a tensor part with 63 percent space exchange. The results of the calculations are given in Table I for the Hall and Powell form, and in Table II for the Christian and Hart form.

The electromagnetic transitions within lithium-6 have been examined with the wave functions which result from these potentials. Qualitative agreement with the results of Day and Walker,[10] that the $M1$ transition from the $J=0$ state to ground is overwhelming in comparison with the $E2$ transition from the $J=3$ state to ground, is obtained. The ft value for the helium-6–lithium-6 beta decay has also been calculated with these wave functions. The Christian and Hart form of the potential gives $\log ft=2.99$ in reasonable agreement with the observed $\log ft=2.94\pm0.04$. The value of the $\log ft$ for the Hall and Powell form of the potential is somewhat higher.

These calculations show that the intermediate-coupling model of the nucleus can provide a consistent, but not unique, scheme for describing lithium-6 if the interaction potential has a strong tensor contribution. It should be noted that the position of the $J=2$ level is unsatisfactory for its calculated position, is always lower than its observed position. This tendency is in agreement with the results of Jancovici and Talmi, and Regge, and probably is due to the neglect of configuration interaction.

TABLE I. Parameters of the modified Hall-Powell potential for lithium. The symbols are defined in the text after Eq. (3).

	V_0(Mev)	α_t(10^{-13} cm)	ζ	γ	g
$a^2=0.8515$	71.35	1.18	−9.80	11.92	0.28
$b^2=0.0030$	43.98	1.355	−6.04	11.72	0.38
$c=0.3815$	25.13	1.53	−3.47	16.62	0.45
	13.24	1.705	−1.83	23.12	0.73
$a^2=0.8557$	44.18	1.18	−6.41	17.29	0.37
$b^2=0.0113$	26.01	1.355	−3.80	18.22	0.58
$c=0.3647$	13.26	1.53	−1.97	24.25	1.25
	4.80	1.705	−0.76	49.09	1.23

B. NITROGEN-14

Since nitrogen-14 has a neutron and a proton hole in the p shell, it may be treated in a manner quite similar to that used for lithium-6. However the sign of the spin-orbit coupling parameter and the sign of the electric quadrupole moment operator must be reversed for the p-shell holes.

One would expect that the intermediate-coupling model would be less successful in describing nitrogen, since the higher configurations should mix more strongly with the p-shell configuration in nitrogen than they apparently do in lithium-6. Certainly the presence of low-lying states of odd parity in nitrogen gives qualitative evidence of this increased importance of higher configurations. Therefore, by considering both nuclei to be described by the same two-body p-shell interaction, one may hope to obtain information about the relative importance of configuration mixing for the two nuclei.

Inglis[1] has pointed out that the strength of the spin-orbit coupling appears to increase as one goes from the beginning of the first p shell to the end beyond it. Beyond O^{16}, a fully developed j-j coupling appears to have set in. This increase in the spin-orbit force will

TABLE. II. Parameters of the modified Christian and Hart potential for lithium. The symbols are defined in the text after Eq. (3).

	V_0(Mev)	α_t(10^{-13} cm)	ζ	γ	g
$a^2=0.8515$	71.35	1.18	−9.80	11.92	0.19
$b^2=0.0030$	43.98	1.355	−6.04	11.72	0.26
$c=0.3815$	25.13	1.53	−3.47	16.62	0.31
	13.24	1.705	−1.83	23.12	0.47
$a^2=0.8557$	44.18	1.18	−6.41	17.29	0.25
$b^2=0.0113$	26.01	1.355	−3.80	18.22	0.37
$c=0.3647$	13.26	1.53	−1.97	24.25	0.66
	4.80	1.705	−0.76	49.09	1.74

[7] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, Cambridge, 1951), p. 268.
[8] I. Talmi, Helv. Phys. Acta **25**, 185 (1953); Phys. Rev. **89**, 1065 (1953).
[9] R. S. Christian and E. W. Hart, Phys. Rev. **77**, 441 (1950).
[10] R. B. Day and R. L. Walker, Phys. Rev. **85**, 582 (1952).

necessitate an increase in the tensor force also as these two forces tend to oppose each other.

As in lithium, the ground state is a positive parity, $J=1$ state, so the most general wave function that can be constructed is given by Eq. (1). Once again the Russell-Saunders components are normalized functions formed out of the single p holes. The coefficients can be determined from normalization and from the experimental value of the magnetic moment, 0.404 nm and the experimental value of the electric quadrupole moment, 0.01 barn.

The very long half-life of the carbon-14–nitrogen-14 beta decay provides additional information about the structure of the ground state and the first excited state of nitrogen-14. Upon the assumption of charge independence of forces the first excited state of nitrogen-14 is a member of the same supermultiplet as the ground state of carbon-14, and so it should have the same wave function apart from a small Coulomb effect. Since this state has $J=0$, the general wave function can be written

$$\Psi = x\,{}^1S_0 + y\,{}^3P_0. \tag{4}$$

The ft value of the beta decay between carbon and nitrogen is given by

$$ft = \frac{K}{\left|\int \boldsymbol{\sigma}\right|^2} = \frac{K}{6(xa - yc/\sqrt{3})^2}, \tag{5}$$

where K is an average beta-decay coupling strength. Now the denominator of this expression must be very small to give a lifetime in agreement with experiment. For convenience, it will be set equal to zero, and the coefficients of the carbon-14 ground state and the first excited state of nitrogen-14 will be taken to be

$$x = y \cong 0.71. \tag{6}$$

This also fixes the sign of the P-state coefficient for the ground state of nitrogen-14 to be positive. Using a value of the mean square displacement of the p proton hole from the center of the nucleus of $(2.79\times10^{-13}\text{ cm})^2$, one obtains for the coefficients of the ground-state wave function of nitrogen-14, the values

$$a=0.2872;\quad b=0.8190;\quad c=0.4974. \tag{7}$$

These values are more sensitive to the value of the mean square displacement than were the values for lithium. However, the experimental quadrupole moment is more certain for nitrogen. As has been shown by Jancovici and Talmi,[4] this choice of the carbon-14 wave function is consistent with the oxygen-14–nitrogen-14 positron decay because the Coulomb repulsion will cause a slight change in the ground state of oxygen-14.

The near vanishing of the carbon-14–nitrogen-14 beta-decay matrix element might seem to imply a near vanishing of the gamma-ray element between the first excited level of nitrogen and the ground state. Experimentally this gamma transition has been observed to to be quite strong.[11] But for an allowed beta decay as this one is,[12] there is a selection rule which prevents the orbital angular momentum from changing. The gamma transition which proceeds by magnetic dipole radiation, on the other hand, will have contributions from states differing by one unit of orbital angular momentum. Thus, when the wave functions which have been obtained above, are used to calculate the gamma transition probability for this process, one obtains a result in qualitative agreement with experiment.

One can proceed to determine the parameters of potential assumed to be of the same form as Eq. (3) by an analogous procedure to that described for lithium. One inserts the coefficients of the ground state and first excited state wave function into the matrices for the $J=1$ and $J=0$ levels. When the observed splitting between the first excited state and the ground state of nitrogen is inserted, one obtains relationships among the parameters of the potential. When these relationships are satisfied, the static moment data of the ground state of nitrogen-14, the observed splitting between the first excited state and the ground state, and the observed half-life of the beta decay will be reproduced. An auxiliary condition, that the observed splitting between the first excited $J=1$ level and the ground level be reproduced, can also be imposed.

Preliminary numerical solutions of these equations have been performed, with reasonable values for the ranges and depth of the potential. These indicate that the condition imposed from the beta-decay half-life severely limits the freedom of choice of parameters compared with the lithium situation. For reasonable values of the ranges and depth of the potential, the numerical values of the spin-orbit parameter and of the tensor parameter which have been found necessary to satisfy the relationships are so large as to make difficult any physical interpretation of the results. This tends to confirm the difficulties that Jancovici and Talmi[4] experienced in fitting the level structure.

Although the calculations performed so far have not exhausted the possibility that a reasonable solution exists, one can speculate that neglecting the interactions among the other particles in the p shell and neglecting higher configuration interactions may be entirely unjustified in nitrogen. The severe requirements imposed by observation may require a more realistic model, for even qualitative agreement.

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[11] T. Lauritsen and F. Ajzenberg, Revs. Modern Phys. (to be published).

[12] Pohm, Waddell, Powers, and Jensen, Phys, Rev. **97**, 432 (1955).