transition is assumed. A mixing ratio of 1.8–3 percent for $E2/M1$ or 36 percent for $M3/E2$ fits the measured distribution according to assignments of 3+ or 4+, respectively, for the 2.42-Mev level.[11] Of the two possibilities, the estimated α_K listed in Table IV seems to favor the $E2-M1$ mixture. No assignment other than 3+ or 4+ is consistent with the experimental evidence. Assignment of 3+ or 4+ is in fair agreement with the $\log ft$ values of $\beta 3$ given in Table II.

The two angular correlations involving the $0.328-1.60$ and the $0.328-0.490$ Mev gamma-ray pairs, although subject to uncertainty in magnitude, serve to corroborate the foreoing interpretation. The theoretical anisotropies to be expected for these correlations, when the spins of the levels are successively 0–2–4–3, are *negative* for both and are both given by[11]

$$W(\theta)=1-0.140P_2.$$

For the case 0–2–4–4, *positive* anisotropies are expected with the following correlation applying to both pairs:

$$W(\theta)=1+0.197P_2.$$

[11] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).

In the first case (0–2–4–3), an assignment of $M1$ or $E2+M1$ for the 0.328-Mev transition is possible from the experimental correlation. In the second case (0–2–4–4), the negative experimental anisotropies can only be obtained if this transition is of a mixed $E2-M1$ character. Rose's[12] theoretical K conversion coefficients for the $E2$ and $M1$ cases (Table IV) are too close in value for the estimated value of α_K to distinguish between the two possibilities. The K/L ratio would seem to favor the $M1$ assignment and the corresponding spin of 3+ for the 2.42-Mev level. It may be further mentioned that the measured intensities of the gamma rays originating from this level are in keeping with either a 3+ or 4+ configuration. Although the possibility of 4+ is not eliminated by these experiments, the configuration 3+ seems more in keeping with all of the data of Table IV.

ACKNOWLEDGMENTS

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[12] Rose, Goertzel, Spinrad, Harr, and Strong, Phys. Rev. **83**, 79 (1951).

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Supermultiplets and Spin Dependent Forces*

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A tentative explanation for a group of favored negatron transitions from nuclei with $N-Z=3$ is proposed on the basis of a deviation from the supermultiplet formalism due to spin-dependent forces. The experimental evidence is exhibited and discussed.

1. INTRODUCTION

THE supermultiplet formalism[1–4] provides, thus far, the most natural theoretical explanation of the striking empirical difference between favored and unfavored allowed β transitions. The empirical fact that the unfavored decays have transition matrix elements (squared) that are, on the average, about a hundred times smaller than those of the favored decays is in qualitative accord with the supermultiplet formalism insofar as it predicts that the only nonvanishing matrix elements are for transitions between states in the same supermultiplet.

Exceptions to the theory can be put into two classes. First, there exist some transitions, such as $P^{30}\rightarrow Si^{30}$, that should, according to the theory, be favored but the empirical evidence indicates that they are not. Second, there exists a small class of β transitions that should be unfavored and yet show remarkably low comparative half-lives. In particular, the supermultiplet formalism in the approximation of spin independence for nuclear forces does not permit superallowed *negatron* decay for odd-A nuclei with $A>3$. There is considerable experimental evidence from the decay of nuclei with $T_z=\frac{3}{2}$ to excited states of nuclei with $T_z=\frac{1}{2}$ that is contrary to such a restriction. It is the purpose of this note to exhibit this evidence and to propose an explanation. The point of view adopted is that the spin dependence of nuclear forces is the most likely reason for a breakdown of the usual restrictions of the supermultiplet formalism. It is shown that the experimental observations of fast negatron decay to excited states of stable odd-A nuclei with $A\lesssim 25$ can be qualitatively

* Supported by the U. S. Atomic Energy Commission.
[1] E. P. Wigner, Phys. Rev. **51**, 106 (1937).
[2] E. P. Wigner, Phys. Rev. **51**, 947 (1937).
[3] F. Hund, Z. Physik **105**, 202 (1937).
[4] E. P. Wigner, Phys. Rev. **56**, 519 (1939).

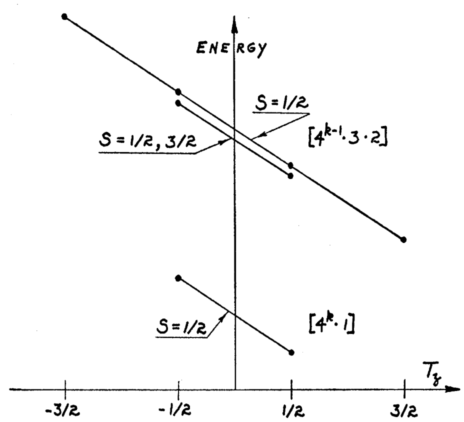

FIG. 1. Supermultiplet structure for nuclei with $A=4n+1$ or $A=4n+3$ in the approximation of spin-independent forces.

accounted for by interpreting the final states of the fast transitions as possessing components of quartet intrinsic spin states belonging to the same supermultiplet as the initial state but lower in energy because of spin-dependent forces.

2. SUPERMULTIPLET SPLITTING

In order to exhibit some of the effects of spin-dependent forces on the supermultiplets, we examine first the supermultiplet structure of odd-A nuclei under the assumption of spin-independent forces. Figure 1 gives a schematic diagram of two supermultiplets for nuclei with $A=4n+1$. The notation $[4^k \cdot 1]$ represents a core in which each space orbital has two neutrons and two protons and outside the core is one nucleon. The slope of the lines to the right is representative of Coulomb effects and accounts for positron decay within a supermultiplet after the neutron-proton mass difference has been overcome by the Coulomb energy difference. This slope to the right takes place for odd-A nuclei with $A>3$. Thus the usual criterion for superallowed or favored-allowed β transitions (namely transitions within a supermultiplet) restricts these fast β transitions to positron decay or electron capture for

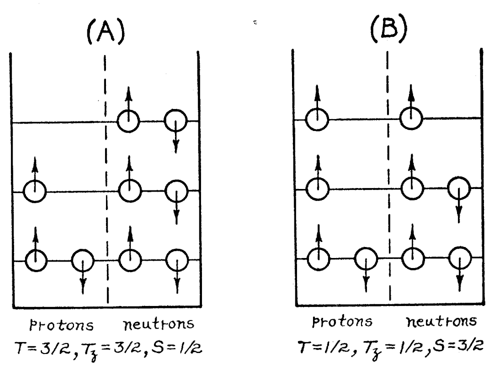

FIG. 2. Detailed structure of (A) a doublet and (B) a quartet intrinsic spin state of the $[4^{k-1}\cdot 3\cdot 2]$ supermultiplet.

odd-A nuclei with $A>3$, at least in the approximation of spin independence for nuclear forces.

As illustrated in Fig. 1, the $[4^{k-1}\cdot 3\cdot 2]$ supermultiplet contains charge multiplets with $T=\frac{1}{2}$ and $T=\frac{3}{2}$. While the states belonging to the $T=\frac{3}{2}$ charge multiplet are restricted to doublet intrinsic spin values ($S=\frac{1}{2}$), quartet values ($S=\frac{3}{2}$) are possible for $T=\frac{1}{2}$. Under the assumption of spin independence, the states ($T=\frac{3}{2}$, $T_z=\frac{1}{2}$, $S=\frac{1}{2}$), ($T=\frac{1}{2}$, $T_z=\frac{1}{2}$, $S=\frac{1}{2}$), and ($T=\frac{1}{2}$, $T_z=\frac{1}{2}$, $S=\frac{3}{2}$) belonging to the $[4^{k-1}\cdot 3\cdot 2]$ supermultiplet[5] correspond to a degenerate energy eigenvalue which exceeds the energy eigenvalue of the ($T=\frac{3}{2}$, $T_z=\frac{3}{2}$, $S=\frac{1}{2}$) state belonging to the same supermultiplet[6] by the Coulomb energy difference minus the neutron-proton mass difference. If, however, the detailed structure of the states is examined, it becomes clear that the known tendency for neutron and proton to favor the triplet intrinsic spin value is sufficient to account qualitatively for a lowering in energy of the $S=\frac{3}{2}$ states with respect to the $S=\frac{1}{2}$ states.

Figure 2 compares the detailed structure of the ($T=\frac{3}{2}$, $T_z=\frac{3}{2}$, $S=\frac{1}{2}$) state and the ($T=\frac{1}{2}$, $T_z=\frac{1}{2}$, $S=\frac{3}{2}$) state belonging to the $[4^{k-1}\cdot 3\cdot 2]$ supermultiplet. The distinguishing feature between the two states (and for that matter between (B) and any state with $S=\frac{1}{2}$ belonging to the $[4^{k-1}\cdot 3\cdot 2]$ supermultiplet) is that (B) possesses two more pairs of particles in triplet intrinsic spin states than does (A). If we denote the energy difference between a triplet and singlet state for a pair of particles in equivalent space orbits by $\Delta E_s(=)$ and that same difference for a second pair of particles in nonequivalent space orbits by $\Delta E_s(\neq)$, then the energy of the possible negatron decay between the two states of Fig. 2 can be expressed as

$$E_{\beta^-}=\Delta E_s(=)+\Delta E_s(\neq)-\Delta E_c, \tag{1}$$

where ΔE_c is the Coulomb energy difference minus the neutron-proton mass difference. For the purposes of this discussion, ΔE_c can be approximated by the mass difference between the neutral atoms of two mirror nuclei with corresponding nuclear charge.

The term $\Delta E_s(=)$ can be estimated from odd-odd, $N=Z$ nuclei where the energy difference between the $T=0$, ground state and the lowest $T=1$, excited state is known. Although there are no direct measurements of $\Delta E_s(\neq)$, the tendency for odd-odd, $N\neq Z$ nuclei to favor triplet states speaks strongly for its positive value.

The considerations of this section can be applied in the same manner to the $A=4n+3$ decays from $T_z=\frac{3}{2}$ nuclei to $T_z=\frac{1}{2}$ nuclei (for example $C^{15}\rightarrow N^{15*}$). Thus, it can be expected that β^- transitions from nuclei having Z protons and $Z+3$ neutrons will take place to excited states of nuclei having $Z+1$ protons and $Z+2$ neutrons

[5] These states represent excited levels of nuclei such as Be^9, C^{13}, O^{17}, etc.

[6] This state represents the ground level of nuclei such as Li^9, B^{13}, N^{17}, etc.

with $\log ft$ values characteristic of superallowed transitions[7] if

$$\Delta E_c < \Delta E_s(=) + \Delta E_s(\neq).$$

3. EXPERIMENTAL EVIDENCE

Table I lists the experimentally known decays of the type that fit the description given in Sec. 2. Also listed are the decays that are expected to be favored but have not been investigated or are of such small intensity as to be virtually unobservable.

The ft values are all in a slightly larger than image transition range but in a considerably smaller than unfavored range. The decay of F^{21} appears to offer the best chance of observing another favored transition. Its 5s half-life is good assurance that the transition is favored and the energy and intensity of the transition to the excited state of Ne^{21} are expected to be in the observable range. Although the $Na^{25} \rightarrow Mg^{25}$ decay *may* satisfy the conditions necessary to produce a favored transition, it seems unlikely that the intensity of the

TABLE I. Experimental evidence for favored negatron decay in nuclei with $A=4n+1$ or $A=4n+3$.[a]

A	Disintegration	Half-life (seconds)	E_{β^-} (Mev)	$\log fot$	$E_{\beta^-}+\Delta E_c$ (Mev)
9	${}_3Li_6 \rightarrow {}_4Be_5^*$	0.17	7.3?	3.7	8.2
11	${}_4Be_7 \rightarrow {}_5B_6^*$	particle unstable?			
13	${}_5B_8 \rightarrow {}_6C_7^*$	particle unstable?			
15	${}_6C_9 \rightarrow {}_7N_8^*$	2.4	3.5	3.6	5.7
17	${}_7N_{10} \rightarrow {}_8O_9^*$	4.1	3.7	3.8	6.4
19	${}_8O_{11} \rightarrow {}_9F_{10}^*$	29.4	2.9	4.3	5.7
21	${}_9F_{12} \rightarrow {}_{10}Ne_{11}^*$	5			
23	${}_{10}Ne_{13} \rightarrow {}_{11}Na_{12}^*$	40	1.2	3.8	4.7
25	${}_{11}Na_{14} \rightarrow {}_{12}Mg_{13}^*$	62			

[a] For complete references see National Bureau of Standards Circular 499 (U. S. Government Printing Office, Washington, D. C., 1950), its supplements, and Nuclear Science Abstracts, Vol. 6, No. 24B (1952), Vol. 7, No. 24B (1953), Vol. 8, No. 24B (1954), and Vol. 9, No. 6B (1955).

transition would be sufficient to detect even the γ following the decay (see Sec. 4).

4. CORRELATION OF EMPIRICAL EVIDENCE WITH PROPOSED INTERPRETATION

It is of interest now to correlate the values given in Table I with the proposed interpretation of these experimental results. Figure 3, shows a plot of $E_{\beta^-}+\Delta E_c = \Delta E_s(=)+\Delta E_s(\neq)$ against Z of the decaying nucleus, and also a plot of ΔE_c against Z of the decaying nucleus. This diagram demonstrates quite clearly why favored β^- transitions are not observed for $A \gtrsim 25$. The Coulomb energy difference becomes large enough to compete favorably with the energy gained by the quartet component of the final state. An extrapolated value for $E_{\beta^-}+\Delta E_c$ shows that the favored transition from Na^{25} would be of the order of 200 or 300 kev, and in view of the known complex β^- decay of Na^{25} with a highest energy component of $\sim$3.7 Mev, it is expected that the

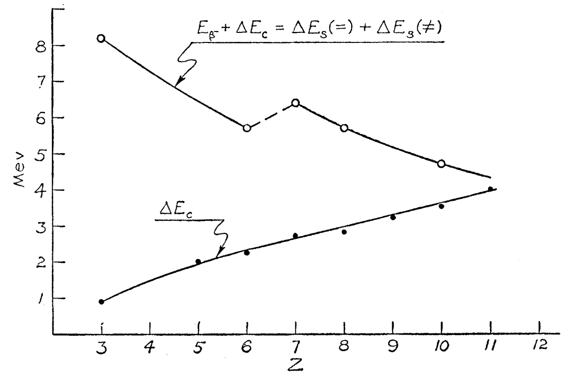

FIG. 3. Plot of $E_{\beta^-}+\Delta E_c$ and ΔE_c *vs* Z of the decaying nucleus.

favored transition would take place at most in only a few hundredths of a percent of the decays. It is thus not surprising that the favored transition from Na^{25} is, as yet, unobserved.

It is too much to hope that the spin-dependent forces present in the nucleus are such as to permit both L and S to be good quantum numbers. We know, in fact, that this is not the case from the example of the deuteron. It is, however, reasonable to investigate the simple interpretation represented by Eq. (1) in order to determine the degree of approximation that it represents.

In Fig. 4 the values of $\Delta E_s(\neq)$ are plotted against Z of the decaying nucleus. The values of $\Delta E_s(\neq)$ have been obtained by identifying $\Delta E_s(=)$ with the energy difference between the $T=1$ and $T=0$ states of the "proper" odd-odd, $N=Z$ nucleus. It turns out that the "proper" odd-odd, $N=Z$ nucleus for both $A=4n+1$ and $A=4n+3$ decays has $A=4n+2$. For comparison a plot of $\Delta E_s(=)$ for the odd-odd, $N=Z$ nuclei is also included.

The values of $\Delta E_s(\neq)$ given in Fig. 4 are surprisingly large, but they show the same tendency to decrease with A as do the $\Delta E_s(=)$ values, and the striking feature is that $\Delta E_s(\neq)-\Delta E_s(=)$ remains at a roughly constant value of about 3 Mev. The apparent strong correlation between $\Delta E_s(=)$ and $\Delta E_s(\neq)$ speaks favorably for the model proposed. While the magnitude of

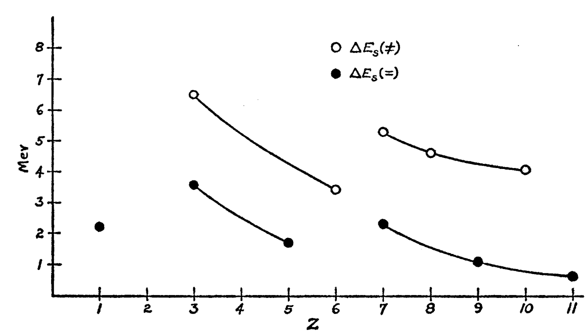

FIG. 4. Plot of $\Delta E_s(\neq)$ *vs* Z of the decaying nucleus and $\Delta E_s(=)$ *vs* Z of the appropriate $N=Z$, nucleus.

[7] Since $\Delta T \neq 0$, only Gamow-Teller matrix elements need be considered.

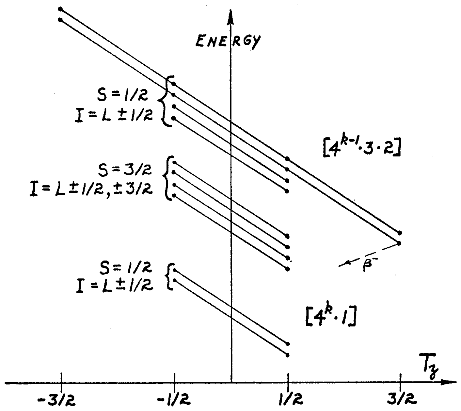

FIG. 5. Schematic representation of proposed supermultiplet splitting for nuclei with $A=4n+1$ or $A=4n+3$.

$\Delta E_s(\neq)$ strains belief, it is of some interest to speculate on the possible origin of such a difference between the two quantities. Considering the general form for a potential of the exchange type between pairs of particles, the terms that could give rise to the behavior exhibited in Fig. 4 must satisfy at least two requirements. (i) They must distinguish between singlet and triplet intrinsic spin states. (ii) They must distinguish between equivalent and nonequivalent space orbits in the sense of the supermultiplet formalism. Because of the $\sim$3 Mev difference involved, it seems reasonable to add to requirement (ii) the condition that the distinction between equivalent and nonequivalent orbits should be apparent in the exchange character and not just in the function of distance between the two particles. Two forces satisfy the above requirements—*the Heisenberg force* and *the tensor force with a Majorana exchange character*. The ordinary Wigner and Majorana forces are ruled out because they don't distinguish between triplets and singlets. The Wigner tensor force and the Bartlett force are ruled out because they don't distinguish between even and odd relative angular momentum states except in their dependence on the distance between particles.

Perhaps a more likely interpretation of the large values of $\Delta E_s(\neq)$ in Fig. 4 is the presence of an additional term in Eq. (1) which assists in lowering the final states of these fast β^- transitions. If in first approximation [the L-S approximation of Eq. (1) is schematically represented in Fig. 5] the supermultiplet is split due to spin-dependent forces, it is reasonable to assume that the same forces will couple doublet to quartet states producing further lowering of a $T_z=\frac{1}{2}$ state belonging to the $[4^{k-1}\cdot 3\cdot 2]$ supermultiplet. Such an interpretation is bolstered to some extent by Feenberg's calculations[8] of the beta-decay transition probabilities within the $[4^{k-1}\cdot 3\cdot 2]$ and $[4^{k-1}\cdot 2\cdot 1]$ supermultiplets. While the transition probabilities for Li^9, C^{15}, O^{19}, and Ne^{23} strongly suggest the predominance of quartet components in the final states of the fast β^- transitions, the decay of N^{17} makes apparent the need for a mixture of doublet and quartet components. However, in the absence of specific information as to whether the coupling of doublet and quartet states is such as to produce constructive or destructive interference terms in the transition probabilities, any specification of the final state is highly speculative.

We are in effect then forced to modify the model to permit the mixing of states within the same supermultiplet. This is neither surprising nor objectionable. A further modification of the model is of course required to explain the existence of unfavored-allowed transitions between different supermultiplets. This modification demands some mixing between different supermultiplets, but far less than that required to produce pure $j-j$ coupling if the empirical distinction between favored and unfavored transitions is to be explained. If the splitting of supermultiplets is according to the model proposed here, it is most likely that the admixtures to the $[4^k\cdot 1]$ and $[4^k\cdot 3]$ supermultiplets are the quartet states of the $[4^{k-1}\cdot 3\cdot 2]$ and $[4^{k-1}\cdot 2\cdot 1]$ supermultiplets respectively. An investigation is underway to determine if admixtures appropriate for the unfavored transitions will improve both the calculated magnetic moments for the $T_z=\frac{1}{2}$ nuclei and the image transition probabilities.

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[8] Eugene Feenberg, following paper [Phys. Rev. **99**, 71 (1955)].