Hall Theory in *n*-Type Germanium*

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A STUDY of the galvanomagnetic properties of the eight-[111] and six-[100] ellipsoidal models, appropriate respectively for the energy surfaces in *n*-type germanium and silicon, has been in progress for some time in the Lincoln Laboratory.[1] Using the assumption of energy-independent scattering time τ which has already been employed to explain cyclotron resonance,[2] theoretical calculations have been carried out. Preliminary comparison of the angular and magnetic field B dependence of the magnetoresistance reveals essential agreement with the energy-dependent τ theories,[3,4] although admittedly the constant τ assumption is restrictive. The latter has the advantage, however, of greatly reducing the computational effort required in arriving at explicit results.

The behavior of the Hall coefficient R_H in *n*-germanium is as follows: Plots of R_H *vs* B have two asymptotic values which are independent of orientation—the saturation (high-field) limit is just $R_\infty = 1/Nqc$, while in the low-field limit $R_0 = [3K(K+2)/(2K+1)^2]R_\infty$. In between these limits, an orientation-dependent minimum occurs which becomes more pronounced as the current density J and magnetic field directions are chosen with higher Miller indices. The occurrence of such minima for τ characteristic of lattice scattering was pointed out earlier.[3] However, in the present description, it is easier to show this for diverse orientations; and moreover, for a given orientation of **J** and **B**, a universal plot can be made for R_H *vs* $\omega\tau$, where

$$\omega = \frac{qB}{\bar{m}^* c} \cdot \frac{3K}{2K+1}, \quad K = m_1/m_2,$$

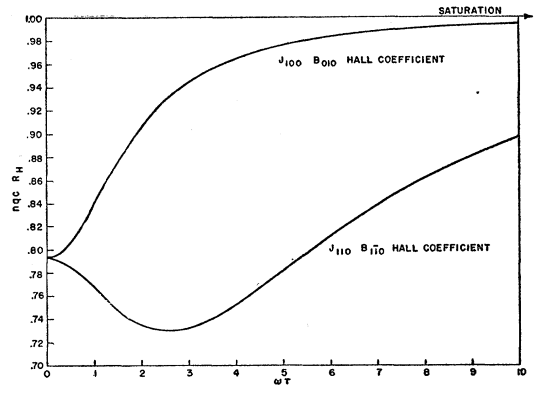

FIG. 1. Behavior of Hall coefficient plotted in terms of the universal parameter $\omega\tau$ which simultaneously depicts the effect of magnetic field and scattering time: (a) The case $\mathbf{J}_{100}$, $\mathbf{B}_{010}$; (b) the case $\mathbf{J}_{110}$, $\mathbf{B}_{1\bar{1}0}$.

$\bar{m}^*$ being the average effective mass defined in terms of the mass tensor components m_1 and m_2, by

$$3/\bar{m}^* = 1/m_1 + 2/m_2.$$

Favorable resolution of the minimum requires the proper combination of symmetry for **J** and **B** along with the proper range of $|B|$ and τ, being generally abetted by small τ associated with high temperatures and impure samples.

Figure 1 illustrates typical behavior of the universal plots. The arrangement $\mathbf{J}_{100}$, $\mathbf{B}_{010}$ exhibits the behavior shown in Fig. 1(a); the minimum here is coincident with the R_0 value. Experiment indeed indicates a minimum very close to $B=0$.[5] The situation for $\mathbf{J}_{110}$, $\mathbf{B}_{1\bar{1}0}$ is more amenable for the observation of the minimum which as Fig. 1(b) indicates occurs at $\omega\tau \sim 2.5$ with an R_H value a few percent below R_0. Again experiment seems to be essentially in line with this.[5] So far, it appears that the experimental data fall somewhere between the predictions of the constant and energy-dependent τ theories; the measured value at liquid nitrogen temperatures of R_∞/R_0 for $\sim$10 ohm-cm material at room temperature lies between 1.27 and 1.08 in these respective theories, for K about 19.[6]

I would like to thank Miss M. Clare Glennon of the Lincoln Laboratory Computer Group for carrying out the calculations and W. M. Bullis and W. Krag for permission to refer to their research still in progress.

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Measurement of Electron Momentum by Positron Annihilation*†

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IN order to explore the possibility of investigating electronic states in solids by means of positron annihilation we have measured, with good resolution, the angular distribution of gamma rays from two-quantum positron annihilations[1–4] in various metals. The experimental arrangement utilizes two scintillation counters placed behind vertical slits in lead blocks located two meters on either side of a vertical strip of the material being studied. This strip is bombarded by positrons from two Na^{22} sources which are placed on opposite sides of it and which are shielded from direct view of the fixed counter. The movable counter

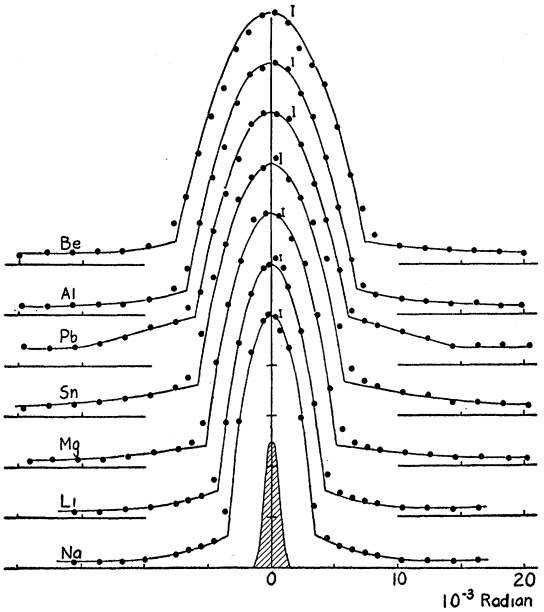

FIG. 1. The shaded curve is the calculated resolution. The central parabolas are computed from density, atomic weight, and valence, assuming an ideal Fermi gas at zero temperature. The abscissas of the sharp corners correspond to computed maximum momenta.

rotates about a vertical axis passing through the sample. The slits are 2 mm wide and the sample has, in each case, an effective width of 1 mm as seen from the counter positions. We estimate the resolution curve to have a width at half-maximum of about 1.3 milliradians.

The experimental results are plotted as a function of the angular deviation from 180°. This deviation, expressed in radians, is equal to the z component of the momentum of the annihilating pair, measured in units of mc. All curves are normalized to the same height at the center. The standard deviation, shown only for the highest point of each curve, varies as the square root of the counting rate. The background at high angles is explainable in terms of random coincidences. A correction for the finite resolution of the instrument would not appreciably alter the position of the points.

The curves of Fig. 1 consist of two parts, a central parabola and a large-angle tail. The parabolas are the computed z-momentum distribution of electrons of a Fermi gas having the same density as that of free electrons in the metal, assuming the number of free electrons per atom equal to the number of the corresponding column in the periodic table. The curves of the tails have been traced through the experimental points.

It is interesting that the momentum distribution of the annihilating pairs should correspond so closely to that of a Fermi gas. This fact seems to imply (a) that the positrons annihilate at rest, (b) that they seldom annihilate with the electrons of the atomic cores, (c) that this type of experiment does not reveal significant local effects of the lattice upon the electrons, (d) that the annihilation probability is independent of the electron velocity. It is also interesting to note that Al, Sn, and Pb show alkali-like angular distributions in spite of the presence of overlapping bands. This may indicate that we are sampling electrons only in interstitial regions.

For the elements of Fig. 2, the decomposition of the curves into two parts is more ambiguous since the tails blend more smoothly with the central peaks. However, a treatment similar to that of Fig. 1 gives reasonable results.

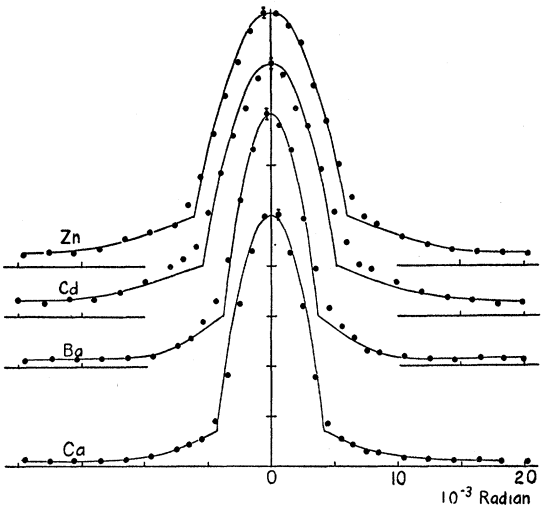

FIG. 2. Other examples of the angular distribution of annihilation radiation. See caption of Fig. 1.

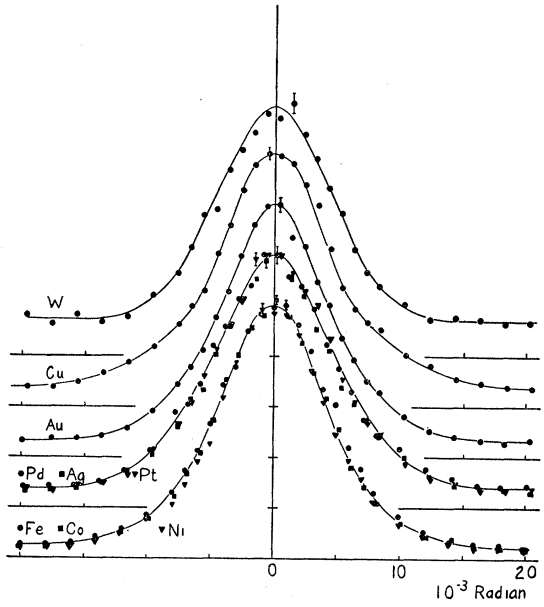

FIG. 3. Angular distributions from transition and noble metals. Curves are experimental.

In Fig. 3, transition metals and noble metals, the analysis in terms of parabolas is not possible because of the large size of the tails. Since the cores are close to each other in these elements, one is tempted to associate these tails with the electron of the atomic cores.[5] To the accuracy of this experiment Fe, Co, and Ni are indistinguishable; the same is true of Pd, Pt, and Ag.

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Transition Temperatures of He^3-He^4 Solutions

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We are currently engaged in an experimental study of the superfluidity of dilute solutions of He^3 in He^4, including pure He^4. Measurements of the behavior of a torsion pendulum immersed in the liquids, as previously carried out[1] yield information on the normal fluid density, viscosity, and critical velocity. This technique, together with certain refinements in the measurement of temperature and of pendulum oscillation periods, permits us to make quite accurate determinations of the λ points. Transition temperatures of individual solutions are obtained by locating the discontinuity in slope of the torsion period *versus* temperature, which is equivalent to finding the temperature at which the normal fluid density becomes equal to the total density.

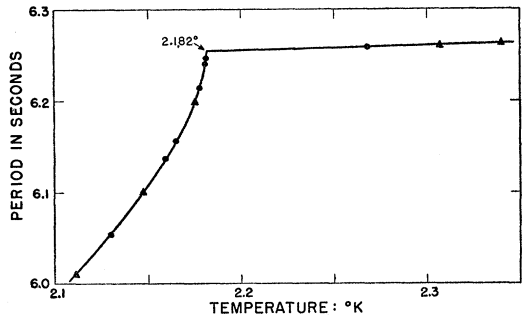

FIG. 1. Behavior of pendulum period in the vicinity of the λ point: pure He^4.

TABLE I. The λ temperatures of solutions.

Concentration x%	λ Temperature °K
Pure He^4	2.182±0.0005
3.65±0.2	2.127±0.002
6.0 ±0.2	2.096±0.002
9.2 ±0.2	2.045±0.002

Liquids under study are condensed in a glass cell surrounded by a bath of liquid He^4. Vapor pressures of the bath and of the cell liquid are read on manometers filled with butyl sebacate, whose density and thermal expansion coefficient have been carefully measured in a large volume pycnometer. The oil manometer readings are expressed in terms of standard mercury level differences, using a conversion factor composed of the oil density, the density of mercury at 20°C,[2] and the gravitational accelerations at Los Alamos and at sea level. Estimated accuracy of the manometry is ±0.2 mm oil, corresponding to ±0.014 mm Hg. Vapor pressures are converted to temperatures on the 1948 "agreed" liquid helium scale.[3] Bath temperatures are stabilized by an electronic regulator of the Sommers type,[4] to within 0.1 millidegree in the vicinity of the λ point. The oscillating system is composed of a fine torsion wire suspending a stack of 50 closely spaced Al disks centered in the glass cell. The pendulum has a period of 5.2680 seconds *in vacuo* and a period of 6.250 seconds in He I near the λ point. Times of several oscillations are measured with the aid of an electronic chronograph to about 1 millisecond accuracy. Sensitivity and accuracy of the method are suggested by Fig. 1, which shows the behavior of the period in the vicinity of the transition, for the case of pure He^4.

When solutions of He^3 in He^4 are condensed in the cell, molar concentration $x=N_3/N_3+N_4$ (where N_3, N_4 are the respective isotopic concentrations) is obtained from the measured vapor pressures and interpolation of Sommers'[5] smoothed data. Table I presents the transition points of pure He^4 and three solutions.

We find 38.00±0.05 mm Hg as the vapor pressure of pure He^4 at the λ point. This value compares with the pressures 38.3±0.2 mm quoted by Schmidt and Keesom[6] and 38.10±0.02 mm quoted by Long and Meyer.[7] On the temperature scale used here, these vapor pressures correspond to 2.185°K and 2.183°K, respectively. Both of these investigations consisted of detecting a vapor pressure difference between a liquid helium bath and helium condensed in a cell at the bottom of the bath. The disappearance of a detectable pressure difference is attributed to the abnormally high heat conductivity of He II. This technique appears to be less direct than the present method.