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Z-Dependence of Bremsstrahlung*

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The relative amounts of bremsstrahlung produced by the elements Cu, Ta, Pb, and U for incident electrons of energy 24 and 34 Mev were measured by using the $Cu^{63}(\gamma,n)Cu^{62}$ reaction as the photon detector. Because of the shape of the cross-section function, photons were detected in a comparatively narrow energy band centered at 17.5 Mev. The electron beam was monitored by a Faraday cup placed immediately behind the radiator and detector foils. Corrections were applied to account for ionization and radiation losses of the electrons, and for pair production and Compton losses of the photons.

The experimentally-determined "thin-target" cross sections relative to copper for Ta, Pb, and U, respectively, were 5.681 ± 0.13, 6.959 ± 0.16, and 8.221 ± 0.19 per atom, at 24 Mev; and 5.583 ± 0.08, 6.770 ± 0.09, and 8.172 ± 0.12 at 34 Mev. These values are from five to thirteen percent lower than the corresponding ratios calculated from the Bethe-Heitler theory (including screening and radiation produced in collision with atomic electrons). The results are consistent with a deviation from theory of $(1.54\pm0.2)\times10^{-3}\,Z^2$ percent for 24-Mev electrons, and $(1.38\pm0.14)\times10^{-3}\,Z^2$ percent for 34-Mev electrons.

INTRODUCTION

THE theory of bremsstrahlung of relativistic electrons as developed by Bethe and Heitler[1] has had numerous experimental tests which prove that it is qualitatively correct,[2,3] but some of which indicate quantitative deviations of the order of ten percent for high-Z elements.[4,5] Similar deviations have been well established in the related phenomenon of pair production and have been attributed to the failure of the Born approximation used in the theories.[6–9] Recently, Bethe, Davies, and Maximon[10,11] have published bremsstrahlung and pair-production theories based on the use of Furry wave functions, which they estimate to be correct to order mc^2/ϵ where ϵ is the energy of the final electron in bremsstrahlung or that of the less energetic electron in pair production. Their integral pair-production cross section agrees well with experiment for photon energies greater than 80 Mev and "not unreasonably" at 17.6 Mev.

Their theories indicate an important difference between bremsstrahlung and pair production such that in the limit of complete screening the bremsstrahlung cross section agrees with the Born-approximation result whereas the pair-production cross section does not. In the opposite limit of no screening, however, the new theories indicate that both the bremsstrahlung and pair-production cross sections are the same fraction lower than the result based on the Born approximation. The new integral bremsstrahlung cross section for the case of intermediate screening has not been published, but it is expected to be lower than the Born-approximation value.

The present paper came about as a result of recent measurements of the absolute (γ,n) cross sections of copper[12] and carbon.[13] It was desirable in these measurements to use a radiator having the maximum photon yield and a minimum ionization loss. This requirement is satisfied with a high-Z element; however, the deviation of the absolute bremsstrahlung cross section is expected to increase with Z. Berman and Brown[12] used a copper radiator for this reason, assuming that the deviation would not differ markedly from that which had been observed previously for pair production. Barber *et al.*[13] used both tantalum and copper radiators, and observed a discrepancy of 7 ± 2 percent in the evaluation of the $C^{12}(\gamma,n)C^{11}$ cross section based on the Bethe-Heitler theory, the tantalum yielding an apparently lower cross section.

The reaction $Cu^{63}(\gamma,n)Cu^{62}$ was chosen as a photon monitor for the present experiment because the cross-section curve has a sharp maximum at 17.5 Mev and has less than five percent of its total area above 24 Mev.[12] The primary electron energies were chosen as 24 and 34 Mev in order to gain information about the effect of energy dependence and yet keep the corrections for the degradation of energy in the finite radiator foil small.

EXPERIMENTAL TECHNIQUE

The experimental equipment and techniques employed in this experiment are identical to those described by Berman and Brown in their determination of the $Cu^{63}(\gamma,n)Cu^{62}$ absolute cross section,[12] with the exception that various materials—Cu, Ta, Pb, and U—were successively employed as radiators. In brief, the

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1 H. A. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146**, 83 (1934).
2 J. W. Dewire and L. A. Beach, Phys. Rev. **83**, 476 (1951).
3 Powell, Hartsough, and Hill, Phys. Rev. **81**, 213 (1951).
4 L. H. Lanzl and A. O. Hansen, Phys. Rev. **83**, 959 (1951).
5 C. D. Curtis, Phys. Rev. **89**, 123 (1953).
6 A. I. Berman, Phys. Rev. **90**, 210 (1953).
7 C. R. Emigh, Phys. Rev. **86**, 1028 (1952).
8 R. L. Walker, Phys. Rev. **76**, 527 (1949); **76**, 1440 (1949).
9 J. L. Lawson, Phys. Rev. **75**, 433 (1949).
10 H. A. Bethe and L. C. Maximon, Phys. Rev. **93**, 768 (1954).
11 Davies, Bethe, and Maximon, Phys. Rev. **93**, 788 (1954).
12 A. I. Berman and K. L. Brown, Phys. Rev. **96**, 83 (1954).
13 Barber, George, and Reagan, Phys. Rev. **98**, 73 (1955).

TABLE I. Results of experiment, showing major corrections.

Row	Radiator	$(E_0-\mu)=24$ Mev Cu	Ta	Pb	U	$(E_0-\mu)=34$ Mev Cu	Ta	Pb	U
1	Basic counting data	76.23 ± 0.2	83.54 ± 0.2	78.32 ± 0.3	81.41 ± 0.3	103.34 ± 0.3	110.27 ± 0.4	102.35 ± 0.3	108.36 ± 0.5
2	Integrator correction (percent)	2.40 ± 0.2	1.50 ± 0.2	1.34 ± 0.2	1.38 ± 0.2	2.14 ± 0.2	1.45 ± 0.2	1.10 ± 0.2	1.15 ± 0.2
3	Degradation corrections (percent)	6.66 ± 1.3	5.15 ± 1.0	4.73 ± 0.9	4.75 ± 1.0	2.67 ± 0.5	2.20 ± 0.4	2.10 ± 0.4	2.18 ± 0.4
4	Data after corrections	83.14 ± 1.2	89.10 ± 0.9	83.08 ± 0.9	86.40 ± 0.9	108.31 ± 0.7	114.30 ± 0.7	105.63 ± 0.6	111.97 ± 0.8
5	Effect of first foil	9.34 ± 0.1	9.34 ± 0.1	9.34 ± 0.1	9.34 ± 0.1	12.18 ± 0.1	12.18 ± 0.1	12.18 ± 0.1	12.18 ± 0.1
6	Difference Row 4 −Row 5 = effect of radiator	73.80 ± 1.2	79.76 ± 0.9	73.74 ± 0.9	77.06 ± 0.9	96.13 ± 0.7	102.12 ± 0.7	93.45 ± 0.6	99.79 ± 0.8
7	Radiator thickness (mg/cm²)	310.09 ± 2.2	167.95 ± 1.2	145.20 ± 1.0	147.58 ± 1.5	310.09 ± 2.2	167.95 ± 1.2	145.20 ± 1.0	147.58 ± 1.5
8	Yield per atom of radiator $Y_{\mathrm{exp}}(E_0,Z)$	15.12 ± 0.27	85.89 ± 1.2	105.22 ± 1.4	124.30 ± 1.9	19.70 ± 0.20	109.98 ± 1.1	133.37 ± 1.3	160.98 ± 1.7
9	Yield per radiation length	3044 ± 55	3012 ± 42	2959 ± 38	2809 ± 42	3966 ± 40	3857 ± 39	3750 ± 38	3638 ± 40
10	Theoretical yield per atom $Y_T(E_0,Z)$ unnormalized	5.179	31.15	39.02	48.80	6.783	39.87	49.83	62.17
11	$R(E_0,Z)=C\dfrac{Y_{\mathrm{exp}}(E_0,Z)}{Y_T(E_0,Z)}$	1.003 ± 0.018	0.947 ± 0.013	0.926 ± 0.012	0.875 ± 0.013	0.997 ± 0.010	0.947 ± 0.009	0.919 ± 0.009	0.889 ± 0.010

technique was to pass the external electron beam of the Mark II (38-Mev peak energy) linear accelerator through a stack of foils into a Faraday-cup electron integrator. The foil stack consisted of an 0.002-in. copper foil followed by a radiator foil of approximately 0.025 radiation length and a second 0.002-in. copper foil. To prevent errors due to variations in detector-foil thickness the same matched pairs of foils were used with the various radiators. The Cu^{62} activity (9.73-minute half-life) induced in the front and rear foils was measured with a 4π scintillation counter.

The quantity of interest is that part of the induced activity in the second foil which is due only to the photons produced in the 0.025-radiation-length radiator. The activity induced by the virtual quanta associated with the electromagnetic field of the moving electrons, by neutrons or stray gamma radiation entering with the electrons, is the same in the front as in the rear foil (except for corrections due to electron energy losses in passing through the foils). Therefore, except for minor corrections, the activity induced in the rear foil minus the activity induced in the front foil, normalized to the integrated charge, yields a quantity proportional to the number of photons produced in the front foil plus radiator. The fraction of this effect that is due to the front foil may be subtracted by reducing the data for the copper radiator first. The quantity finally determined, $Y(E_0,Z)$, is the induced activity per incident electron due to the radiator; then

$$Y_{\mathrm{exp}}(E_0,Z)=N_rN\int_0^{E_0-\mu}\varphi_{\mathrm{exp}}(k,E_0,Z)\sigma_{\gamma,n}(k)dk,\quad (1)$$

where k is the quantum energy, E_0 is the electron energy, N_r is the number of radiator nuclei per cm², and N is the number of Cu^{63} nuclei per cm² in the detector foil, $\sigma_{\gamma,n}(k)$ is the $Cu^{63}(\gamma,n)Cu^{62}$ cross section *vs* energy k, and $\varphi_{\mathrm{exp}}(k,E_0,Z)$ is the experimental 'thin-target' integral bremsstrahlung cross section for the element Z.

From theory, and a knowledge of the shape of $\sigma_{\gamma,n}(k)$ for copper, it is possible to calculate, by numerical integration, the quantity

$$Y_T(E_0,Z)=N_rN\int_0^{E_0-\mu}\varphi_T(k,E_0,Z)\sigma_{\gamma,n}(k)dk,\quad (2)$$

where $\varphi_T(k,E_0,Z)$ is the Bethe-Heitler bremsstrahlung cross section including screening and modified by the factor $[(Z+\delta)/Z]$ to take into account the contribution of atomic electrons in producing radiation. Values of δ varying from 1.08 in the case of uranium at 24 Mev to 1.12 for copper at 34 Mev were calculated from the equations given by Wheeler and Lamb.[14] To compare experiment and theory, the ratio

$$R(E_0,Z)=CY_{\mathrm{exp}}(E_0,Z)/Y_T(E_0,Z)\quad (3)$$

was calculated for each Z and both values of E_0. The value of the constant C was chosen arbitrarily to make the average value of $R(E_0,29)$ equal to unity.

RESULTS

The experimental results, the important experimental corrections, and the comparison with theory are shown in Table I. The quantity listed under the heading "Basic Data" (Row 1 of Table I) represents the average value of the second-foil Cu^{62} activity (corrected for multiple-scattering effects) minus the first-foil Cu^{62} activity, all divided by the observed integrated charge collected by the Faraday cup. The probable errors listed were determined from the internal consistency of fifteen bombardments made with each set of conditions. The technique of measuring the radioactivity in the foils and of measuring the integrated charge are discussed in detail by Berman and Brown.[12] The integrator correction is an experimentally determined quantity and represents the change in the observed integrated charge

[14] J. A. Wheeler and W. E. Lamb, Phys. Rev. **55**, 858 (1939).

when the foils and radiators were placed in the path of the electron beam. This is due to the secondary electrons produced in the stacked foils which outweigh the loss of charge due to scattering. The degradation corrections are applied in order to compensate for both the energy loss of the electrons and the loss of photons in passing through the foils. After these corrections are applied, the resulting quantity is the "thin-target" yield due to the first foil plus radiator. The data obtained with the copper radiator are then used to determine the effect produced by the first foil (Row 5, Table I). The net effect due to the radiator alone (Row 6) is divided by the foil thickness in mg/cm^2 (Row 7) and multiplied by the atomic weight to give the relative yield per atom of radiator (Row 8). Estimated errors due to foil nonuniformity of 0.7 percent in the cases of Cu, Ta, and Pb, and one percent in the case of U, are added statistically at this point. The relative yield per radiation length is tabulated in Row 9 where the radiation length was computed from the formula

$$X_0=\left[\frac{4NZ(Z+1)}{137}\left(\frac{e^2}{mc^2}\right)^2\ln 183Z^{-\frac{1}{3}}\right]^{-1}. \tag{4}$$

The theoretical yield defined in Eq. (2) is listed in Row 10. The ratio Eq. (3) giving the comparison of experiment and theory is in Row 11.

The values of $R(E_0,Z)$ are plotted as functions of Z^2 in Fig. 1, and within experimental error they fall on straight lines. The lines plotted were obtained by least-square fitting of the four points at each value of E_0. From the straight lines it is found that $R(E_0,Z) = R_0(E_0)(1-KZ^2)$ where $K=(1.54\pm0.2)\times10^{-5}$ for $E_0=24$ Mev, and $(1.38\pm0.14)\times10^{-5}$ for $E_0=34$ Mev.

It should be noted that if the curves φ_{exp} and φ_T as functions of k were parallel over the range of k where $\sigma_{\gamma,n}(k)$ is finite, the ratio $R(E_0,Z)$ would be proportional to $\varphi_{\text{exp}}/\varphi_T$. Since the curves are expected to be nearly parallel and since $\sigma_{\gamma,n}(k)$ is sharply peaked near $k=17.5$ Mev, the ratio $R(E_0,Z)$ is very nearly proportional to $\varphi_{\text{exp}}(17.5,E_0,Z)/\varphi_T(17.5,E_0,Z)$.

From the definition of R, it follows then that the average value [weighted by $\sigma_{\gamma,n}(k)$] of φ_{exp} deviates from a similarly weighted φ_T and, to close approximation,

$$\frac{\varphi_T(17.5,E_0,Z)-\varphi_{\text{exp}}(17.5,E_0,Z)}{\varphi_T(17.5,E_0,Z)}=KZ^2. \tag{5}$$

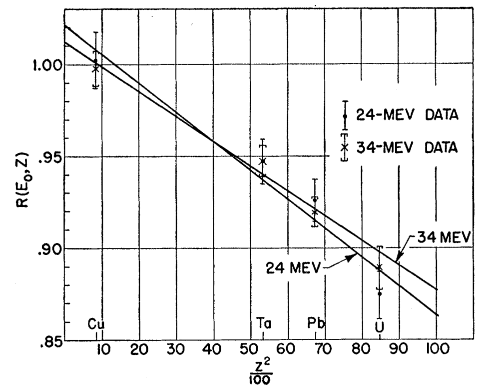

FIG. 1. The ratio $R(E_0,Z)$ of the experimental to theoretical photon yield is plotted as a function of $Z^2/100$. The results have been normalized to make the average of $R(34, 29)$ and $R(24, 29)$ equal to unity. The straight lines are least-square representations of the data.

DISCUSSION

The results of the experiment are appreciably affected by the corrections due to finite radiator thickness (Rows 2 and 3 of Table I). However, it should be emphasized that even before the corrections are applied the conclusion can be drawn that the experimental cross section relative to the Bethe-Heitler theoretical cross section decreases as a function of Z. Because of the higher ionization losses in the radiators of low Z, the effect of the thickness corrections is to increase this deviation.

It is interesting to note that the experimental deviation from the Bethe-Heitler theory is nearly the same as is observed in the case of pair production by photons of 15 to 50 Mev.[6–8] For example, Berman[6] working with photons of 19.5 Mev, expressed results in a form similar to Eq. (5) as

$$(\sigma_{\text{theor}}-\sigma_{\text{exp}})/\sigma_{\text{theor}}=1.55\times10^{-5}Z^2,$$

for pair production. This similarity between bremsstrahlung and pair production at intermediate energies is consistent with the theories of Bethe *et al.*[10,11]

It is desirable to extend the bremsstrahlung experiments to higher initial electron energies where the screening is nearly complete. In this case, the recent theories[10,11] predict that the bremsstrahlung cross section is correctly given by the Born approximation whereas the pair-production cross section is not.

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