Multiple Scattering of Low-Energy Alpha Particles in a Gas

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Alpha particles from a polonium source were slowed down and allowed to enter a cloud chamber containing a mixture of hydrogen, water, and ethyl alcohol at pressures of 12.25 cm, 1.50 cm, and 2.25 cm, respectively, during the sensitive time. Eighty-four sharp tracks of particles which stopped in the illuminated part of the chamber and which did not show any sharp bends due to single scattering were selected for study. Starting at the termini of the projected tracks, the angles between successive chords of three millimeters length were measured, the energies at the first three intersections of the chords being approximately 18, 45, and 85 kev. The magnitudes of the angles at which the tails of the distributions could be distinguished from the Gaussian parts were determined to be 10°, 6° to 8°, and 4°, respectively. The total standard deviations (which included the experimental error of 2.2°) for the Gaussian part of the distributions at these energies were 8.0°, 4.3°, and 3.0°, respectively, so that the standard deviations due to multiple scattering were, respectively, 7.7°, 3.7°, and 2.0°.

INTRODUCTION

In recent years considerable progress has been made in the experimental study and theoretical interpretation of the multiple scattering of charged particles of medium energies in a gas, and the agreement between experiment and theory is moderately good. In the case of low-energy nuclear particles, however, there is still much to be done in this field because, on the one hand, there is a scarcity of experimental work; on the other hand, no satisfactory theory of scattering has been developed in the absence of an adequate treatment of such effects as charge exchange which become important in this energy range. The present communication is devoted to an account of the results of some cloud-chamber experiments on the multiple scattering of alpha particles with energies below one-hundred kev in a mixture of hydrogen and alcohol and water vapor.

In studying the multiple scattering by cloud-chamber techniques it is sufficient to make use of photographs giving the projection of the paths of the individual particles on a single plane, since from measurements of such photographs the multiple-scattering law can be specified just as completely as by measurements of the actual path in three dimension. In the present work the expression "projected path" will be used to mean the projection of the path of a particle on the plane of the photograph of its track. As applied to such projected paths, the results of the theories of multiple scattering are conventionally expressed in terms of the angle φ between two tangents to the projected path at points a given distance l apart. Although differing in details, different theories all give the result that for φ not too large, the number of particles $dF(\varphi)$ for which this angle lies between φ and $\varphi+d\varphi$ is represented by an expression of the form:

$$dF(\varphi)=(2\pi\sigma^2)^{-\frac{1}{2}}\exp(-\varphi^2/2\sigma^2)d\varphi+dF'(\varphi). \quad (1)$$

The first term represents the true multiple-scattering distribution, i.e., the effect of a large number of individual collisions each leading to a small deflection, while the second term, $dF'(\varphi)$, gives the contribution of the single and plural scattering, i.e., of individual deflections through relatively large angles which occur relatively infrequently. If l is large enough there will exist an angle $\varphi_c \gg \sigma$ such that the distribution is essentially Gaussian for $\varphi<\varphi_c$ and only for angles $\varphi>\varphi_c$ does $dF'(\varphi)$ amount to an appreciable fraction of $dF(\varphi)$. The quantity which characterizes the multiple scattering at a given energy and which is to be compared with experiment is then σ. In carrying out measurements on cloud-chamber tracks, however, greater accuracy can be achieved by measuring the angles ω between adjacent chords of a given length ($\approx l$) along a projected path than by measuring the angles between tangents to the curve at these points. It can be shown[1] that for angles $\varphi<\varphi_c$ for which the Gaussian part of Eq. (1) is dominant, the distribution function for ω is Gaussian in form with σ replaced by $\sigma_\omega=(2\sigma^2/3)^{\frac{1}{2}}$. It follows that the angles ω are distributed according to

$$dG(\omega)=(2\pi\sigma_\omega^2)^{-\frac{1}{2}}\exp(-\omega^2/2\sigma_\omega^2)d\omega+dG'(\omega), \quad (2)$$

and that there exists an angle ω_c such that to a good approximation dG' may be neglected for $\omega<\omega_c$. In the case of multiple scattering the experimental problem is then to determine for a given energy and path length the variance σ_ω^2 of the Gaussian part of the distribution in ω and if possible to give an estimate for ω_c. It is impossible to ascertain in advance whether this can be done with any accuracy for a given set of conditions since before analyzing the data one does not know whether the angle at which the distribution starts to deviate from a Gaussian distribution is large compared to the variance σ_ω^2 or not. In the present experiment we obtained the distribution of the angles ω for low-energy alpha particles for the case in which the plane of projection was essentially parallel to the direction of incidence. The distribution of the measured angles ω will have a form like that of Eq. (2) but will not be

[1] Groetzinger, Berger, and Ribe, Phys. Rev. 77, 584 (1950).

identical with it because these values of ω are affected by the experimental errors. We shall therefore have for the experimental distribution the equation

$$d\bar{G}(\omega)=(2\pi\bar{\sigma}_\omega^2)^{-\frac{1}{2}}\exp(-\omega^2/2\bar{\sigma}_\omega^2)d\omega+d\bar{G}'(\omega), \quad (2a)$$

where the barred quantities have essentially the same significance as their counterparts in Eq. (2). The analog of ω_c, we shall designate $\bar{\omega}_c$.

EXPERIMENTAL PROCEDURE

Measurements were made on 84 cloud-chamber tracks produced by alphas which entered the chamber with energies of about 500 kev, these tracks having been obtained in the course of an investigation[2] concerned with the ranges of low-energy alpha particles. The chamber employed, which had a diameter of 25 cm, was filled with a mixture of hydrogen, water, and ethyl alcohol, and was operated at a temperature of 20.5°C. In a previous investigation[3] of the operation of this chamber under the conditions used, it was verified that the composition of the filling gases remains essentially constant during the period in which sharp tracks are produced (the sensitive time) and it was determined that at this time the partial pressures amount to 12.25 cm for hydrogen, 1.5 cm for water, and 2.25 cm for ethyl alcohol. In accordance with the aforementioned definition of the sensitive time, the tracks analyzed were selected from a considerably larger number available by excluding those which were thick or fuzzy. An image of each suitable track was projected to full size onto a sheet of transparent celluloid using a Recordak microfilm reader. Starting at the end of the track at which the particles stopped successive points with a 3-mm separation were marked along each track with the aid of a pair of dividers. In each case, the angles between the chords defined by these points were then measured with a drafting machine. Both the plotting of the points and the measurement of the angles between the chords were performed independently by two of us (G. A. and R. W.) and the averages of the resulting measurements of the angles were used as the individual scattering angles ω.

The aforementioned choice of the chord length d was governed by the following considerations. First the most important requirement is that d should be considerably larger than the track thickness in order to avoid unduly large errors of measurement. Second, d should ideally be sufficiently great that the condition $\bar{\omega}_c \gg \bar{\sigma}_\omega$ be satisfied in order that it be possible to estimate the latter quantity as accurately as possible. Third, if possible, d should be small enough so that the change of energy over the part of the track spanned by a pair of chords be reasonably small in order that the observed value of $\bar{\sigma}_\omega$ can be properly associated with a reasonably small range of energies. For low-energy alphas the absence of even an approximate theory makes it impractical at present to attempt to apply the second criterion. In any event, however, the tracks obtained are so thick and the energy loss per unit distance is so high in the present case that d is fixed by the first and third conditions alone. On this basis the value of d chosen was 3 mm.

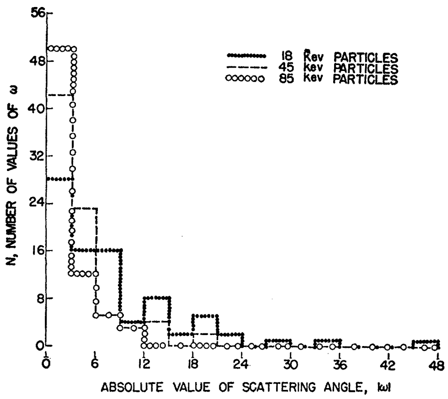

FIG. 1. Frequency distributions of the scattering angles.

The following procedure was used for estimating and correcting the experimental error due to optical distortion and inaccuracy of angular measurement. A straight line and a circle which had a diameter of 3.4 cm were constructed on a sheet of drawing paper. The width of both lines was chosen the same as the average width of a track. The sheet of paper was placed at the proper position in the cloud chamber and photographed. The angles between successive 3-mm segments of the straight line and between successive 3-mm chords of the circle were then measured by the procedure described previously. The standard deviations in the measured values of these angles in these two cases were 2.2°. On the assumption that the multiple scattering angles and the error angles are statistically independent, the variance of the Gaussian part can be corrected by subtracting the error variance.

RESULTS

The energies E_1, E_2, and E_3 at each intersection of adjacent chords was estimated by using the range energy curves for alpha particles in air given by Cook *et al.*[4] together with the value of the air equivalent of the cloud-chamber mixture estimated in a previous paper.[2] These energies were found in this way to be approximately 18, 45, and 85 kev, respectively. The frequency distributions, in intervals of 6°, for the measured values of the corresponding angles are shown in Fig. 1.

In order to determine the value of the quantity $\bar{\sigma}_\omega$ from these results it is, as remarked earlier, necessary to separate the contribution of the Gaussian part of $d\bar{G}$ from that of the tail $d\bar{G}'$ in the total distribution.

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[2] Barile, Webeler, and Allen, Phys. Rev. **96**, 673 (1954).
[3] S. Barile and R. Webeler, Rev. Sci. Instr. **25**, 389 (1954).
[4] Cook, Jones, and Jorgensen, Phys. Rev. **91**, 1417 (1953).

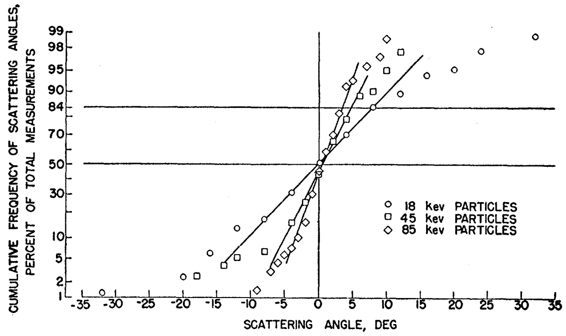

FIG. 2. Cumulative frequency distributions of the scattering angles.

To indicate the method used, it is convenient first to define the quantity $\bar{G}(\omega)$ as equal to $\int_{-\infty}^{\omega} d\bar{G}$. It can then be shown[5] that there exists a function $\Gamma(\bar{G})$ depending on $\bar{\sigma}$ only through $\bar{G}$ such that for $\bar{G}$ Gaussian, the relation

$$\Gamma(\bar{G})-\Gamma(0.5)=[\Gamma(0.8413)-\Gamma(0.5)](\omega-\bar{\mu})/\bar{\sigma}_\omega \quad (3)$$

holds so that $\Gamma(\bar{G})$ varies linearly with ω. In Eq. (3) $\bar{\mu}$ is the average of the measured values of ω and will be close to zero. In order to determine the Gaussian part of the distribution, the experimental points were represented graphically in Fig. 2 with Γ as ordinate and ω as abscissa. It can be seen that in each case these points lie on a straight line over a considerable range centering about $\Gamma(0.5)$. We estimate in each case $\bar{\sigma}_\omega$ from the slope of this line by use of Eq. (3)[6] and took $\bar{\omega}_c$ as the value of ω at which the points began to deviate appreciably from the line. The resulting values are shown in Table I which also gives the values of σ_ω obtained by correcting $\bar{\sigma}_\omega$ for the experimental error. These do not suffice to determine a curve of σ *versus* energy, but it does follow that σ increases much faster than $1/E$ with decreasing energy whereas the usual theories applying to high-energy charged particles predict a $1/E$ dependence.[7] The enhanced low-energy scattering is to be expected, of course, because of the increasing importance of nuclear collisions in this energy range.

TABLE I. Summary of results of the multiple scattering experiment. E is the kinetic energy of the alpha particle; $|\bar{\omega}_c|$ is the maximum magnitude of the angles for which the distribution remained Gaussian; $\bar{\sigma}_\omega$ is the total measured standard deviation; σ_ω is the standard deviation due to multiple scattering.

E, kev	$\lvert\bar{\omega}_c\rvert$, deg	$\bar{\sigma}_\omega$, deg	σ_ω, deg
18	10	8.0	7.7
45	6–8	4.3	3.7
85	4	3.0	2.0

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[5] R. S. Burington and D. C. May, Jr., *Handbook of Probability and Statistics with Tables* (Handbook Publishers, Inc., Sandusky, Ohio, 1953), p. 109.

[6] A χ^2 test was made of the conclusion that the points within the limits $\bar{\omega}_c$ in each case belonged to a Gaussian distribution with the indicated experimental parameters. The test showed that such a conclusion was not statistically inconsistent with the experimental distribution (reference 5, p. 141).

[7] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Clarendon Press, Oxford, 1949), p. 278.