The mobility, v, can be estimated theoretically[6] as about 100 cm/sec per volt/cm for MgO. Similarly the value of δ is approximately 4×10^5 cm^{-1}. According to Fig. 1, $md/V_c=1.2\times10^{-3}$ cm volt^{-1}. Hence τ, the lifetime of the carriers, is roughly 3×10^{-11} sec. This is consistent with the fact that, at room temperature, no time effects compatible with a lifetime as long as 10^{-6} sec have been observed in oscillographic display of the pulses.

In accordance with Eq. (1), the bombardment-induced current follows Ohm's law up to the highest applied fields ($\sim$200 kv/cm). Furthermore, for peak primary currents ranging from 2.5 μa to 2.5 ma, the induced current is proportional to the primary current.

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[6] N. F. Mott and R. W. Gurney, in *Electronic Processes in Ionic Crystals* (Clarendon Press, Oxford, 1950), p. 107; H. Fröhlich and N. F. Mott, Proc. Roy. Soc. (London) **171**, 496 (1939).

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Effect of Pressure on the Electrical Conductivity of InSb

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The electrical conductivity of InSb has been measured as a function of temperature from −78°C to +300°C and pressures up to 12 000 kg/cm^2. It is found that the activation energy increases at a rate 15.5×10^{-6} ev/(kg/cm^2), that the electron mobility is approximately inversely proportional to the activation energy, and that the hole mobility is independent of pressure. On the basis of these mobility effects it is concluded that the k of the valence band energy extremum is not zero. An examination of the effects of pressure and temperature above 200°C suggests that there are three or four equivalent energy minima in the conduction band.

1. INTRODUCTION

The unusual properties of the semiconducting compound InSb, in particular the extremely high electron mobility and the partial degeneracy of the electron gas,[1–3] have recently attracted much interest. The significant variations in the energy gaps of semiconductors[4,5] which are produced by hydrostatic pressure suggested that some light might be cast on these unusual properties by a study of their variation with pressure and inspired the work reported here. For certain quantities characteristic of InSb which are not evaluated here reference has been made to the extended investigations of Madelung and Weiss[1] and Austin and McClymont.[2] In some cases constants of the conduction band have been estimated, in the absence of any more acceptable theory, from the perturbed free electron model of the electronic states.[6] The small values of the energy gap and effective mass in InSb lend some plausibility to this approximation. An experiment similar to the one reported here has been performed by Long and Miller[7] over a considerably more restricted range of conditions.

2. MATERIAL

n- and p-type specimens of InSb were generously provided by Dr. S. W. Kurnick of the Chicago Midway Laboratories. The n-type material was of high purity, having in the exhaustion region a low-field Hall constant of −6000 cm^3/coul, indicating a donor concentration of 10^{15} cm^{-3}, and a mobility ($\mathcal{R}\sigma$) of 80 000 cm^2/volt sec at 77°K. The Hall constant of the p-type material at low temperatures was 70 cm^3/coul and the acceptor concentration thus was 10^{17} cm^{-3}.

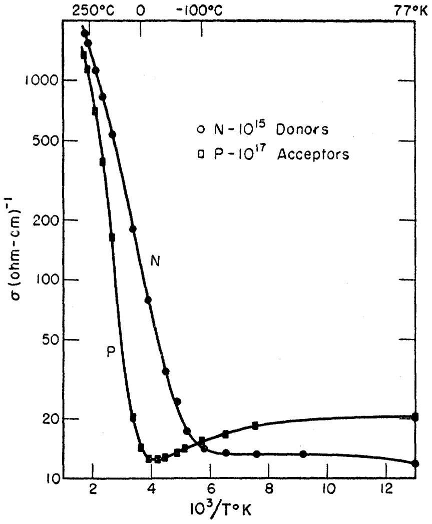

FIG. 1. The temperature dependence of the conductivity at atmospheric pressure of the materials used for this experiment.

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[1] O. Madelung and H. Weiss, Z. Naturforsch. **9a**, 527 (1954).

[2] I. G. Austin and D. R. McClymont, Physica **20**, 1077 (1955).

[3] E. Burstein, Phys. Rev. **93**, 632 (1954).

[4] J. Bardeen, Phys. Rev. **75**, 1777 (1949).

[5] R. W. Keyes, Phys. Rev. **92**, 580 (1953).

[6] C. Kittel, *Introduction to Solid State Physics* (John Wiley and Sons, Inc., New York, 1953).

[7] D. Long and P. H. Miller, Jr., Bull. Am. Phys. Soc. **30**, No. 1, 53 (1955).

The conductivity of both materials at atmospheric pressure is shown in Fig. 1 throughout the temperature range of interest.

3. EXPERIMENTAL

The high-pressure system used was of the usual Bridgman type.[8] The temperature was controlled by immersing the high-pressure vessel containing the sample in a heated or cooled liquid bath. The pressure generating and measuring apparatus was separated from the sample container by a connecting pipe as described by Kurnick.[9] From −80°C to +100°C isopentane was used as the pressure transmitting fluid; from +100 to +300°C a silicone oil was used. Pressure was measured in the conventional Bridgman way with a manganin wire resistance gauge calibrated at the freezing point of mercury.

For the determination of the conductivity, samples 1 mm×2 mm×1 cm were cut from the original ingots. Spring contacts of Nichrome wire were used as electrodes. The conductance of the samples was determined by the direct current-potential probe method; a Leeds and Northrup Type *K* potentiometer was used to measure the potential drop in the sample and the potential across a standard resistor in series with the sample. Readings were taken for both directions of sample current.

4. RESULTS AND DISCUSSION

The principle data are presented in Figs. 2 and 3, which show the conductivity as a function of pressure for several temperatures between −80°C and +100°C.

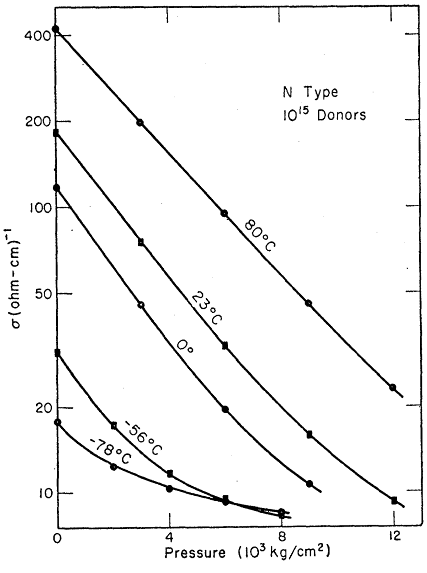

FIG. 2. The conductivity of the *n*-type InSb as a function of pressure at several temperatures.

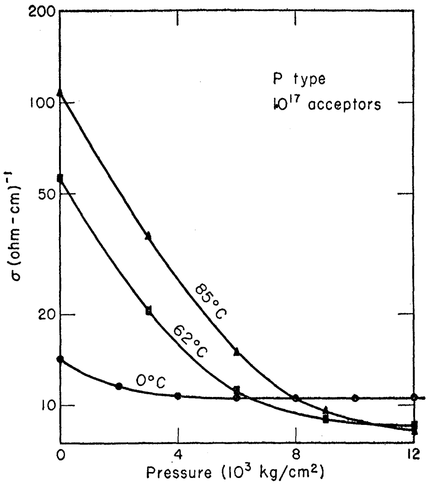

FIG. 3. The conductivity of the *p*-type InSb as a function of pressure at several temperatures.

The qualitative features of the data are immediately apparent; the initial rapid decrease in the conductivities is indicative of an activation energy which increases with pressure. At all temperatures for which curves are given, an appreciable number of "intrinsic" carriers, or carriers due to electrons excited across the gap from the valence to the conduction band, are present at atmospheric pressure. As the activation energy increases, the number of intrinsic carriers decreases, and the sample approaches the "exhaustion" region of conductivity, in which the number of carriers is constant and is determined by the impurities in the sample. This accounts for the leveling of the curves at high pressure. The *P* sample at 0°C is in the exhaustion region above 6000 kg/cm^2, and the constancy of the conductivity from 6000 to 12 000 kg/cm^2 shows that the hole mobility is independent of pressure

The phenomena in the *n*-type sample are slightly more complex. Use of the value to be derived for the rate of change of activation energy with pressure shows that at −78°C the conductivity due to intrinsic carriers is negligible above 4000 kg/cm^2. The continuing decrease of conductivity above this pressure is therefore due to a decreasing electron mobility. It can be seen from Fig. 1 that the donor impurities are just beginning to deionize at 100°K, and a very large change in donor ionization energy would be required if we attribute the decrease in conductivity above 4000 kg/cm^2 to a change in the number of current carriers. In addition, the constancy of the conductivity from −78°C to −56°C at 8000 kg/cm^2 reinforces the belief that the donors are completely ionized.

The analysis of the results in terms of a few parameters will be described in some detail, since its understanding is necessary in evaluating the significance of these parameters. The conductivity is given by the formula

$$\sigma = (n\mu_n + p\mu_p)e, \tag{1}$$

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[8] P. W. Bridgman, *Physics of High Pressure* (G. Bell and Sons, London, 1939).

[9] S. W. Kurnick, J. Chem. Phys. **20**, 218 (1952).

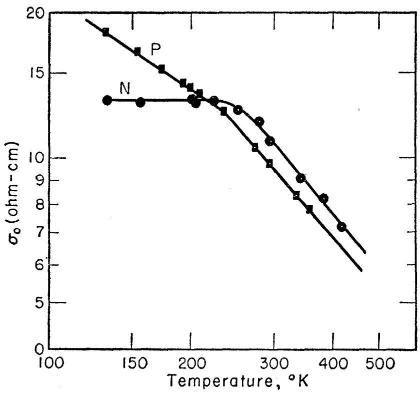

FIG. 4. The quantities $N_D\mu_n e$ and $N_A\mu_p e$ as functions of temperature. The electron mobility referred to is Hall mobility, while the hole mobility is conductivity mobility.

where the carrier densities are determined by

$$np = n_0^2 e^{-W/kT} \tag{2}$$

and

$$n + N_A = p + N_D. \tag{3}$$

In the p-type sample, we have set $N_D = 0$ and in the n-type sample $N_A = 0$, and in addition neglected the conductivity due to the holes in the n-type sample. The impurity concentrations were determined by measurement of the Hall constant in the exhaustion range. The temperature variation of the electron mobility was found by measuring the Hall constant from 77°K to 144°C. The temperature variation of the hole mobility was found from the impurity conductivity, which was in turn determined by fitting the σ *vs* P curves at the high pressure end. These mobility variations are shown by Fig. 4, in which $N_A\mu_p e$ and $N_D\mu_n e$ are plotted against temperature. Data from the low-temperature exhaustion range of the p-type sample are also included.

To take account of pressure variation in Eq. (2), it was rewritten

$$np = n_0^2 e^{-(W_0+Pv)/kT} = [n_1(p)]^2 e^{-Pv/kT}. \tag{4}$$

The pressure dependence of n_1 is due to the fact that it includes a factor $(m_e^* m_h^*)^{\frac{3}{4}}$ which may depend on the energy gap. Because the hole mobility is independent of pressure we assume that m_h^* is also indepent of pressure. We estimate the pressure variation of m_e^* from the perturbed free electron calculation of energy levels in a periodic potential,[6] according to which $m^* \sim W$. At $P = 0$, $n_1(0)$ is calculated from Eqs. (1), (3), and (4), and then $n_1(P)$ by the formula $n_1(P) = (1 + Pv/W_0)^{\frac{3}{4}} n_1(0)$. In order to represent the mobility variation in terms of a physically significant parameter we write $\mu_n \sim W^{-s}$. A value of W_0 is not obtained in the present experiment and for the purpose of these calculations a value $W_0 = 0.25$ ev has been used throughout, in agreement with thermal energy gap and infrared absorption measurements at low temperatures.

The quantities to be determined are now v and s. At each temperature values of these quantities which gave a reasonable representation of the observed curves were found by trial. The curves drawn in Figs. 2 and 3 were calculated from the selected values of s and v. It was found that a single value of v, namely $v = 15.5 \times 10^{-6}$ ev/(kg/cm^2) $= 15.3$ cm^3/mole, was satisfactory for all the data. The necessary values of s were all of the order of one and are shown in Fig. 5.

It should be pointed out that the effect of pressure on the conductivity over most of the region studied is primarily due to the variation of the activation energy and that the calculated value of v is very insensitive to the assumptions made about the variation of mobilities and density of states. These details were presented in order to explain what is meant by s. Values of s are determined most accurately by the data at -78°C and -56°C, where the exhaustion range is attained at high pressure.

5. DISCUSSION OF THE MOBILITY

The large change of electron mobility with pressure suggests that the band separation at the k value characteristic of the electrons has changed by an appreciable fraction, which implies that the separation of the order of the thermal activation energy. On the other hand, the negligible change of hole mobility means that the band separation at the K of the holes is large. Since there is evidence that the electrons have k near zero,[10,11] this means that the holes are not at $k = 0$. Thus the $E(k)$ curves must have roughly the form of Fig. 6, in which the band separation increases rapidly as k goes away from zero because of the small effective mass of the electrons ($m^* = 0.013 m_0$).[10]

It is of some interest to compare the result $\mu \sim W^{-1}$ obtained in the preceding section with theoretical interpretations of mobility. Explicit formulas have been derived for three mechanisms: (1) ionized impurity

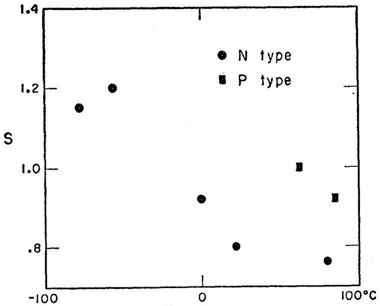

FIG. 5. Values of s, the exponent in the expression $\mu \sim W^{-s}$, obtained by fitting the data of Figs. 2 and 3 in the way described in the text.

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[10] Dresselhaus, Kip, Kittel, and Wagoner, Phys. Rev. 98, 556 (1955).

[11] G. L. Pearson and M. Tannenbaum, Phys. Rev. 90, 153 (1953).

scattering, (2) neutral impurity scattering,[12] and (3) deformation potential scattering.[13] Dependence of the mobility on the energy gap enters these formulas through the effective mass and the dielectric constant. For the former we assume as previously $m^* \sim W$, for the latter we assume the Moss relationship, $\kappa^2 W = \text{constant}$,[14] where κ is the effective dielectric constant.

(1) The well-known Conwell-Weisskopf formula is

$$\mu = \frac{2^{7/2}\kappa^2(kT)^{3/2}}{N\pi^{3/2}e^3m^{*1/2}}\left\{\log\left[1+\left(\frac{6\kappa dkT}{e^2}\right)^2\right]\right\}^{-1}, \qquad (5)$$

and thus, with our assumptions, $\mu \sim W^{-\frac{3}{2}}$. The exponent $-\frac{3}{2}$ cannot be regarded as inconsistent with the data presented above. Figures 1 and 4 show however that the temperature dependence of the mobility is not correctly described by this formula, and in addition the numerical result for the mobility at 200°K using $N=10^{15}$ cm^{-3} is $\mu = 3\times10^6$ cm^2/volt sec, much larger than the observed value in the n-type sample.

(2) The constancy of the electron mobility over a large range of temperature would be expected according to the formula of Erginsoy,

$$\mu = e\tau/m^* = 8\pi^3 m^* e^3/20\kappa N h^3.$$

The pressure dependence of μ according to this formula is $\mu \sim W^{\frac{3}{2}}$, a result not in agreement with the data.

(3) The deformation potential formula is

$$\mu T^{3/2} = \left(\frac{(8\pi)^{1/2} e h^4 C}{m^{*5/2} k^{3/2} E_1{}^2}\right), \qquad (6)$$

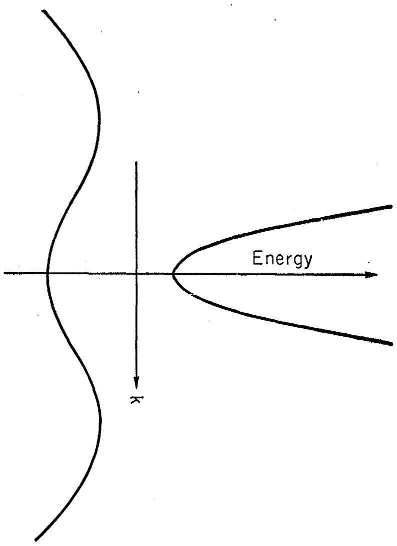

FIG. 6. A dependence of energy on k which explains the variation of the mobilities with pressure and is consistent with the magnetoresistance and cyclotron resonance experiments.

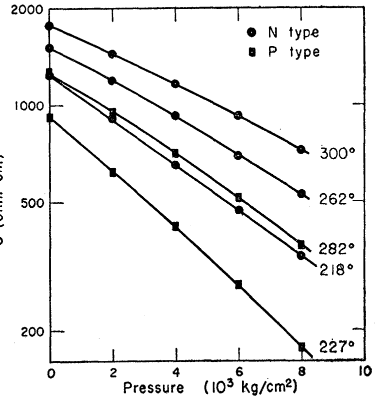

FIG. 7. The conductivity of n- and p-type InSb as a function of pressure at temperatures between 200°C and 300°C.

where C is the elastic constant for longitudinal sound waves and $E_1 = V(dW/dV)$. In the perturbed free electron approximation W is a Fourier coefficient of the potential in which the electrons move. Since this potential is not expected to exhibit any unusual features when W is small the pressure variation of E_1 should not be marked. Consequently $\mu \sim W^{-5/2}$, suggesting a somewhat stronger pressure dependence of mobility than that which is actually found.

In order to calculate μ numerically, the elastic constants must be known. An estimation of these by using a Lindemann relation in the form $CV/T_m = \text{constant}$ with $C =$ elastic constant, $V =$ molar volume, $T_m =$ melting temperature to compare InSb with germanium suggests that the elastic constants of InSb are about half those for germanium. Thus, attributing the entire gap variation to the conduction band, we use $E_1 = 6$ ev and $C = 0.75\times10^{12}$ dynes/cm^2 and find at 300°K that $\mu = 10^7$ cm^2/volt sec. The main reason for this extremely high value is the small effective mass. Dilation may, of course, shift both band extrema in the same direction, and if $E_{1c} = 60$ ev the calculated mobility is of the correct magnitude. This possibility cannot be definitely rejected.

6. HIGH-TEMPERATURE MEASUREMENTS

Some data obtained up to 8000 kg/cm^2 in the temperature range 200°C–300°C are presented in Fig. 7. Comparison of these curves with those of Figs. 2 and 3 shows that the ressure dependence of the conductivity is distinctly smaller. The reason for this can be seen from the work of Madelung and Weiss,[1] who have concluded that at high temperatures the Fermi level approaches the bottom of the conduction band and the Boltzmann approximation to the Fermi distribution function is not adequate, or, in other words, the electron gas becomes degenerate. In order to confirm and amplify the picture of the high-temperature conductivity formulated by Madelung and Weiss the high-temperature data of Fig. 7 have been examined

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[12] C. Erginsoy, Phys. Rev. **79**, 1013 (1950).

[13] J. Bardeen and W. Shockley, Phys. Rev. **80**, 72 (1950).

[14] T. S. Moss, *Photoconductivity in the Elements* (Academic Press, Inc., New York, 1952).

closely and an attempt has been made to derive values of some parameters characteristic of InSb which allow a consistent representation of this data.

At high temperatures the equation which determines the Fermi level is

$$N_c g_v \alpha_v^{-\frac{3}{2}} F_{\frac{1}{2}}(-w-\xi) = (\tfrac{1}{2}\sqrt{\pi}) N_A + N_c g_c \alpha_c^{-\frac{3}{2}} F_{\frac{1}{2}}(\xi). \quad (7)$$

(The donor concentration in our samples is entirely negligible at the temperatures in question and has been set equal to zero.) In this equation

$$N_c = 2(2\pi m_c kT/h^2)^{\frac{3}{2}},$$

g_v and g_c are the degeneracies of the extrema of the of the valence band and the conduction band, respectively, and the α's are the effective mass ratios, m_0/m^*, in the case of spherical energy surfaces and more complicated functions of the effective mass tensor which give the density of states in the case of ellipsoidal surfaces. Energies are measured from the bottom of the conduction band; the magnitude of the gap is wkT and the energy of the top of the valence band is $-wkT$. The Fermi level is at energy ξkT. The acceptors are assumed completely ionized. $F_{\frac{1}{2}}(\xi)$ is the Fermi integral, tabulated by McDougall and Stoner.[15] By using the Boltzmann approximation for the holes in the valence band and defining $\gamma = (g_c/g_v)(\alpha_v/\alpha_c)^{\frac{3}{2}}$ and $f = (\sqrt{\pi}) N_A \alpha_c^{\frac{3}{2}}/2N_c g_c$, we transform Eq. (7) to

$$e^{-w} = \gamma e^{\xi}[F_{\frac{1}{2}}(\xi)+f]. \quad (8)$$

If an energy-independent mean free path for the electrons is assumed and the hole mobility is neglected, the conductivity is related to the Fermi level[1] by $\sigma = \bar{\sigma} \log(1+e^{\xi})$ with $\bar{\sigma} = N_c \alpha_c^{-\frac{3}{2}} g_c \mu_n$.

Since no temperature dependence of v appeared in the low temperature data, the value derived there, $v = 15.5\times10^{-6}$ ev/kg/cm^2, is used here. In view of the scatter of the points of Fig. 5 and the uncertainty of the value of s to be expected theoretically, the introduction of additional parameters to describe the mobility variation would not be meaningful, and the expression $\mu \sim (W_0+Pv)^{-s}$ is used in this section in the identical form as before, with $W_0 = 0.25$ ev and $s = 1.0$, the arithmetic mean of the values given in Fig. 5.

By using the m^* of reference 10 and an electron mobility $\mu_n = 3\times10^4$ cm^2/volt sec at 500°K,[1] we calculate $\bar{\sigma}$ to be $400 g_c$ (ohm cm^{-1}). If $g_c = 1$, the observed conductivity requires that $\xi = +4$, and reconciliation of this value with the infrared energy gap requires $\gamma = 2\times10^{-4}$. When the Fermi level is this high the dependence of the conductivity on ξ is very weak, and the conductivity calculated as a function of temperature or pressure varies much too slowly. We therefore conclude that $g_c > 1$ and use $g_c = 3$ in trying to reproduce the pressure effect, since, although higher values are not excluded by the data, very high multiplicity is not to be expected for spherically symmetrical energy surfaces. Another possibility is that the effective mass measured at 4°K is not the correct one to use at high temperatures.

Knowing g_c and N_A allows us to calculate f for the p-type material, while for the n-type material, $f = 0$. Then Eq. (8) can be solved for ξ and the conductivity obtained as a function of γ and w. The rate at which σ changes with pressure depends on γ and on the value of w at $P = 0$. The energy gaps at $P = 0$ which best reproduce the data of Fig. 7 are shown for two choices of γ in Fig. 8. It is seen that $\gamma = 0.02$ gives fair agreement between the gap derived from the N sample and the extrapolated infrared absorption edge of Austin and McClymont,[4] while $\gamma = 0.01$ results in better consistency of the data from the n- and p-type samples. Because the mobility effects discussed previously and the cyclotron resonance experiment[10] suggest considerable complexity in the structure of the valence band extrema, no attempt has been made to draw inferences from the values of α_v and g_v suggested by these values of γ.

7. CONCLUSIONS

The compound InSb is somewhat similar to germanium in the magnitude of the dependence of energy gap on dilation, but the very small effective mass leads to a deformation potential scattering 100 or 1000 times smaller for InSb. However, none of the conventional scattering mechanisms appear to provide sufficient scattering to limit the electron mobilities to the observed values. Adams[16] has pointed out that electron-

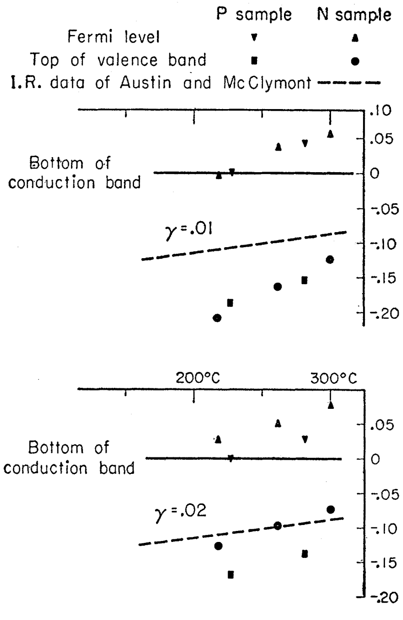

FIG. 8. The values of the energy gap at $P=0$ and the corresponding Fermi levels which give the best fit to the pressure dependence of the conductivity illustrated in Fig. 7.

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[15] J. McDougall and E. C. Stoner, Trans. Roy. Soc. (London) **A237**, 67 (1938).

[16] E. N. Adams (private communication).

hole collisions[17] provide sufficient scattering to limit the mobility to the observed values at high temperatures and may be the dominant mechanism there. Using the estimate of E_1 previously given and the thermal expansion coefficient $\alpha = 14.4\times10^{-6}$ deg^{-1} of Mokrowski and Regel,[18] the change of activation energy with temperature due to thermal expansion of the lattice can be calculated. It is 1.0×10^{-4} ev/deg, which is to be compared with the value 2.6×10^{-4} ev/deg derived from the shift of the infrared absorption edge by Austin and McClymont.[2] Even allowing for possible error in the compressibility, a considerable portion of the temperature variation of the gap must be attributed to direct electron-phonon interactions.

8. ACKNOWLEDGMENTS

The author is indebted to the Chicago Midway Laboratories and Dr. S. W. Kurnick for the InSb used in this experiment and to Dr. C. Kittel and co-workers at the University of California for communication of their cyclotron resonance results prior to publication. He also wishes to thank Dr. E. N. Adams, Dr. R. L. Longini, and Dr. F. Keffer for critical reviews of the manuscript.

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[17] F. J. Morin and J. P. Maita, Phys. Rev. **94**, 1525 (1954).

[18] H. P. Mokrowski and A. R. Regel, J. Tech. Phys. (U.S.S.R.) **22**, 1281 (1952).

PHYSICAL REVIEW VOLUME 99, NUMBER 2 JULY 15, 1955

Distribution of Electrons Scattered by Gold*

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The distribution in energy and angle of 20-kev electrons scattered by gold has been measured. The energy resolution is better than 0.1 percent and the angular resolution better than 10^{-3} radian. Data are presented for gold of different thickness and degree of crystal orientation and for grazing angle reflection.

ALTHOUGH the interactions of electrons with matter have been extensively studied in the past, most of the interest has been directed either to incident energies so high that the most important contribution to the scattering is nuclear, or to energies of at most a few hundred electron volts where secondary emission is of primary interest. With a few notable exceptions,[1] the range of energies from a few kev to one hundred kev has until recently been neglected. Yet this region of the energy spectrum can give information on the imaginary part of the complex dielectric constant used in the semiclassical theory of stopping power, or from another point of view, information on the density effects on the elementary interactions processes. This knowledge is of interest both to the solid state and radiation physics. Interest in this region was revived in Germany just after the war when Mollenstedt[2] with the aid of an improved electrostatic analyzer, began to study the energy losses of electrons in the forward direction (0–10^{-3} rad). This work was extended by the studies of Marton and Leder[3] in America and workers in Japan and Germany.[4] The interest was centered on the characteristic losses of the scatterers which are more or less discrete losses ranging from 1 to 100 electron volts. With a modified Mollenstedt apparatus Leonhard[5] and Watanabe[6] have extended these measurements to include the distribution in angle. The electrostatic analyzer when used in this manner is somewhat unsatisfactory, being nonlinear in energy and intensity and having a poorly defined acceptance angle.

To overcome these limitations we have constructed an apparatus using a double-focusing magnetic analyzer and the geometry shown in Fig. 1. This instrument, which will be described in detail elsewhere, is automatic in operation and gives a record of the energy spectrum of the scattered electrons with an energy resolution of better than 0.1 percent at angular intervals of less than 10^{-3} radian over a range of 300 degrees. The relative value

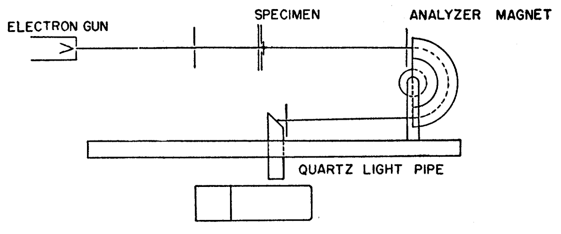

FIG. 1. Electron trajectory showing relative placement of principal elements of the scattering instrument.

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* This work was in part supported by the Office of Naval Research.

[1] G. Ruthemann, Naturwiss. **29**, 648 (1941); Ann. Physik (6) **2**, 113 (1948); W. Lang, Optik **3**, 233 (1948).

[2] G. Mollenstedt, Optik **5**, 499 (1949); **9**, 473 (1952).

[3] L. Marton and L. B. Leder, Phys. Rev. **94**, 203 (1954); L. B. Leder and L. Marton, Phys. Rev. **95**, 1345 (1954).

[4] H. Watanabe, J. Phys. Soc. Japan **9**, 920 (1954); **9**, 1035 (1954); Phys. Rev. **95**, 1684 (1954); W. Klein, Optik **11**, 226 (1954).

[5] F. Leonhard, Z. Naturforsch. **99**, 727 (1954).

[6] H. Watanabe (private communication).