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Helium II Film Transport. IV. The Role of Temperature*

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The study of film transport, inaugurated in previous papers in this series, is concluded with a statistical analysis of the temperature dependence of the transport rates, with particular attention to the possibility of detecting systematic variations with substrate, surface finish, and film height. Analysis of 153 rate *vs* temperature characteristic curves reveals that the temperature dependence of the transport is relatively insensitive to changes in the aforementioned parameters. The functional dependence of the *normalized, average* transport rates for all materials, finishes, and film heights discussed in papers I–III, is given by $[1-(T/T_\lambda)^7]$, in the temperature range under investigation ($1.1°\text{K} \leqslant T < T_\lambda$). These results are compared with various theoretical predictions based on the two-fluid model of liquid helium as well as with relevant reports of other observers.

INTRODUCTION

THIS, the fourth and final paper in the current series of studies of helium II film transport,[1–3] is concerned with the clarification of the answer to the question: How do measured transport rates vary with temperature?

A suggestion by Daunt and Mendelssohn[4] that the variation of transport rate with temperature might be a measure of the concentration of superfluid atoms was subsequently bolstered, in a modified form, by a theoretical argument due to London.[5] Although London was then forced to conclude that the experimental evidence was too scant to permit safe generalization and that it would be very desirable to have more data available, only one previous study[6] of transport rates includes data (for glass, platinum and nickel) which have been analyzed with respect to the actual functional form of the temperature dependence. Furthermore, unusual rate *vs* temperature characteristic curves (observed for Lucite)[7,8] have been the source of suggested interpretations of the role of surface finish.[8] A recent theoretical prediction by Dash[9] has also emphasized the need for more experimental evidence.

The present objective has therefore been to study the functional dependence of the transport rate on temperature, with particular emphasis on possible systematic variations with material, surface finish, and film height.

EXPERIMENTAL RESULTS[10]

In analyzing the functional dependence on temperature, it has been found convenient to normalize the rates to unity at 1.1°K; although measurements have been made at lower temperatures, this represents the lowest temperature at which data were obtained with satisfactory consistency. The characteristic curves are automatically coincident at the high temperature extreme (lambda point). Further normalization anywhere other than at the low temperature extreme would then yield a deceptively exaggerated similarity of plotted curves, since little opportunity for departure of the curves from one another would be available within the resultant short range of all points from the normalization temperature. Using data and notation cited in previous papers,[1–3] division of each value of $R(T)$ by the corresponding value of $R(1.1°)$ accomplishes the desired normalization. The similarity of the results so obtained for all heights and finishes (see papers I–III) is striking, and may be observed for metals from the average values and average absolute deviations from these mean values as indicated in Table I. In order to insure that the deviations represent appropriately conservative estimates, all values of deviation less than 0.01 have arbitrarily been listed as 0.01. For comparison of the over-all average obtained from 132 curves for metals, the results of similar calculations for all glass and quartz specimens[1–3] (including those studied with other than "4-section" depth gauges)[1] are also given. The latter two sets of averages are plotted in Fig. 1 where the calculated average absolute deviations from the mean are in all cases no greater than the size of the plotted points. The horizontal bars showing the extreme values encountered at each temperature, as well as the histograms portraying the frequency distribution of observations at 1.7°K, where the distance from both normalization temperatures accentuates the dispersion, serve to complete the statistical picture. The calculated curves, based on the indicated seventh power dependence on temperature, were drawn after suitable logarithmic

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[1] B. Smith and H. A. Boorse, Phys. Rev. **98**, 328 (1955). (I. The Role of Substrate, designated as I in the present text.)

[2] B. Smith and H. A. Boorse, this issue [Phys. Rev. **99**, 346 (1955)]. (II. The Role of Surface Finish, designated as II herein.)

[3] B. Smith and H. A. Boorse, this issue [Phys. Rev. **99**, 358 (1955)]. (III. The Role of Film Height, designated as III herein.)

[4] J. G. Daunt and K. Mendelssohn, Nature **150**, 604 (1942).

[5] F. London, Revs. Modern Phys. **17**, 310 (1945).

[6] K. Mendelssohn and G. K. White, Proc. Phys. Soc. (London) **A63**, 1328 (1950).

[7] J. G. Dash and H. A. Boorse, Phys. Rev. **82**, 851 (1951).

[8] B. S. Chandrasekhar, Phys. Rev. **86**, 414 (1952).

[9] J. G. Dash, Phys. Rev. **94**, 825 (1954). We are grateful to Dr. Dash for providing us with these results prior to publication.

[10] Preliminary accounts of some of these results have already been presented: B. Smith and H. A. Boorse, Phys. Rev. **94**, 772 (1954); B. Smith, Bull. Am. Phys. Soc. **30**, No. 1, 57 (1955).

TABLE I. Summary of average normalized temperature dependence of the transport rates. Entries represent average values of the quantity $[R(T)/R(1.1°\mathrm{K})]$ appropriate to the temperatures indicated in column headings. Average absolute deviations from the mean are given in columns headed "A.d." (where values less than 0.01 have been listed as 0.01 as noted in the text). For a description of heights pertaining to beaker sections 1–4, finishing procedures, specimen composition and other experimental details, see papers I–III.

Beaker section (n)	1.3°K Av	1.3°K A.d.	1.5°K Av	1.5°K A.d.	1.7°K Av	1.7°K A.d.	1.9°K Av	1.9°K A.d.	2.1°K Av	2.1°K A.d.
Machined metals: Data for 8 specimens, averaged over 20 runs.										
1	0.98	0.01	0.94	0.01	0.84	0.01	0.62	0.01	0.29	0.03
2	0.99	0.01	0.96	0.01	0.84	0.02	0.60	0.02	0.24	0.02
3	0.99	0.01	0.96	0.02	0.85	0.03	0.67	0.04	0.25	0.04
4	0.99	0.01	0.95	0.02	0.84	0.02	0.62	0.02	0.25	0.03
Externally superfinished metals: Data for 4 specimens, averaged over 6 runs.										
1	0.99	0.01	0.96	0.02	0.87	0.03	0.67	0.02	0.32	0.02
2	0.99	0.01	0.94	0.01	0.85	0.03	0.63	0.04	0.26	0.04
3	0.99	0.01	0.96	0.03	0.85	0.04	0.64	0.04	0.26	0.02
4	0.99	0.01	0.96	0.01	0.85	0.02	0.62	0.05	0.25	0.05
Internally superfinished metals: Data for 4 specimens, averaged over 7 runs.										
1	0.99	0.01	0.95	0.03	0.85	0.05	0.65	0.05	0.28	0.04
2	0.97	0.01	0.91	0.01	0.80	0.01	0.60	0.01	0.24	0.01
3	0.99	0.01	0.94	0.02	0.83	0.03	0.63	0.01	0.27	0.01
4	0.99	0.01	0.95	0.01	0.84	0.04	0.61	0.02	0.24	0.01
Over-all average[a] from above 132 $R(T)$ curves for metals (all heights and finishes), plotted in Fig. 1(b).										
All	0.99		0.95		0.84		0.63		0.26	
Over-all average[a] from 21 $R(T)$ curves for glass and quartz (all heights and finishes), plotted in Fig. 1(a).										
All	0.98		0.94		0.84		0.62		0.25	

[a] Average absolute deviations calculated for these over-all averages are in all cases less than 0.01 and are therefore unlisted.

plots yielded straight lines of slope seven. Comparison of the plotted points with the calculated curves and other values indicated in Fig. 1 is reserved for the discussion to follow.

DISCUSSION

It may be observed from Table I and Fig. 1 that the temperature dependence of the transport rates was so insensitive to variations in beaker material, microfinish, and height of the film, that observed differences were unsystematic and could not definitely be ascribed to changes in these parameters. The difficulty of exactly reproducing beaker surface conditions[1–3] must again be considered in any attempt to evaluate the significance of small differences between the results obtained on separate runs. Although enough tabular information has already been presented[1–3] to permit the analysis of the temperature dependence of many individual runs, the current status of theory and experiment does not seem to warrant the extension of the present discussion beyond the consideration of the average values shown in Table I and Fig. 1. The shape of the histogram in Fig. 1(b) also encourages reliance on the statistical picture.

Before discussing this average behavior, it is well to recall that departures from the pattern exhibited in Fig. 1 have received some attention in the literature. According to Chandrasekhar,[8] the maxima he observed in the R *vs* T curves for transport over methyl methacrylate polymers, "seem to indicate that the observed flow on Lucite and Perspex can be regarded as being composed of two parts: (1) pure film flow giving a characteristic similar to the one observed on glass; (2) a pressure dependent flow of bulk helium, which takes place when the surface irregularities are of a shape and size favorable to this type of syphon flow. The maxima in the flow characteristics would require this type of flow to decrease with decreasing temperature." Support of this conclusion is adduced[8] from the observation that the most pronounced maximum was obtained for a Perspex surface which showed a rough streaky structure with ridges and valleys about 10^{-3} cm wide.

It has already been noted[1,11,12] that the results obtained for Lucite in the current study of the role of

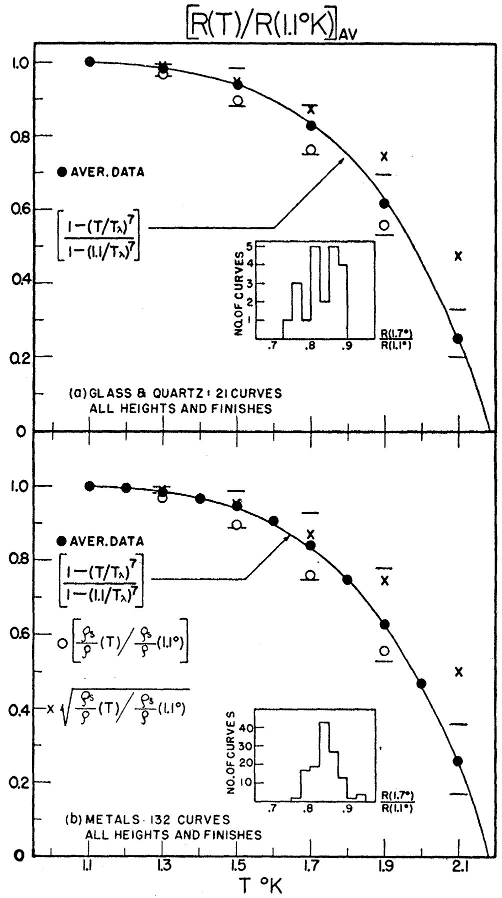

FIG. 1. Average, normalized temperature dependence of the transport rates at all heights (see Table I), (a) averaged over 21 curves for glass and quartz, and (b) averaged over 132 curves for metals. The apparent absence of (ρ_s/ρ) values at some temperatures stems from the identity of these values with the average data close to the normalization temperatures. Details of specimen composition, beaker identification, machining and superfinishing procedures, film heights appropriate to various beaker sections, and other relevant experimental details may be determined by reference to the previous papers in this series (I–III).

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[11] B. Smith and H. A. Boorse, Phys. Rev. **90**, 156 (1953).

[12] J. G. Daunt and R. S. Smith, Revs. Modern Phys. **26**, 172 (1954).

substrate do not exhibit such a maximum (see I: Fig. 7). Furthermore, the 10^{-3} cm wide ridges do not seem very significant in view of the conventional results obtained for the channels about $5(10)^{-3}$ cm by $2.5(10)^{-3}$ cm in size which covered the surface of a broached nickel beaker in work previously described.[2] A recent search for anomalous flow in glass capillaries has also yielded negative results.[13] Transport rates, for the comparatively rough specimens of glass and metal used in control experiments on the role of microfinish,[1,2] did not display maxima in R *vs* T curves.

It is also noteworthy that transport rate characteristic curves with and without such maxima have now been reported not only for plastics, but also for platinum[6] and glass,[14] while Mendelssohn and White[6] remark that this "peculiar fall of the rate with lower temperatures · · · was also observed on glass beakers by Daunt and Mendelssohn." The evanescent nature of such behavior was vividly portrayed in some of our early experiments,[11] in which capacitor design and reproducibility from run to run were being investigated. As is evident from Fig. 2, a maximum was observed for transport over a machined nickel beaker (run 5) which, except for outgassing,[1] was left undisturbed in the cryostat between prior and subsequent runs which did not reveal such behavior (runs 4 and 6). Thus the size and shape of surface irregularities were unchanged while the rate-temperature characteristics varied. (Since later work[3] indicated that the transport rates vary considerably with height in the range used here (between 0.5 cm and 3.5 cm from the rim), the absolute values of transport rate in Fig. 2 can be interpreted only in connection with the discussion of height dependence presented in paper III. When so interpreted, these data are consistent with the other results obtained for nickel.)[1–3]

It thus seems appropriate to add a note of caution about the interpretation of data from which the existence of transport rate maxima has been inferred. On perusal of the graphs illustrating this effect,[6–8] one is immediately struck by the comparatively small number of experimental points which serve to justify the curvature ascribed to the rate-temperature characteristics. This is also true in Fig. 2, where run 5 ended before more data could be secured at the lower temperatures. Furthermore, on some occasions, it appears from plotting experimental values in chronological order in the present experiments that, had some runs ended earlier, a somewhat distorted view of the final rate-temperature characteristic would have been obtained. Consequently, it would appear that only repeated and painstaking traversals of the entire range of accessible temperatures may be used to gauge the true scatter of the data with sufficient accuracy to establish the shape of a particular curve.

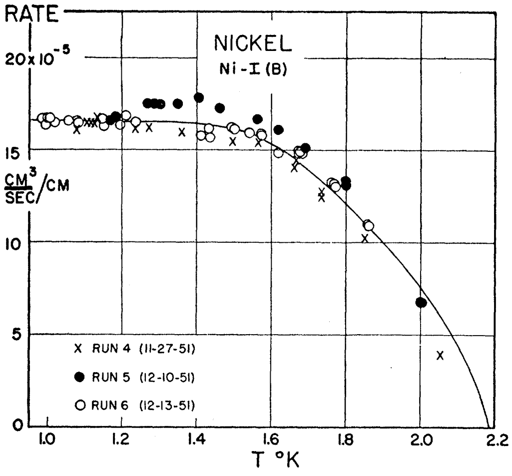

FIG. 2. Transport rates over nickel specimen Ni-(B) (see papers I and II for beaker identification), averaged over region between 0.5 cm and 3.5 cm from the rim. Interest in these results centers in the variation of curve shape from run to run, although the specimen remained in the cryostat and was merely outgassed between runs. The magnitudes may be compared with data obtained in other experiments only if the previous discussion of height dependence (reference 3) is used to interpret the results; such a comparison indicates that these data are consistent with the results obtained for this and other specimens on other occasions (see papers I–III).

In view of these considerations, it will henceforth be assumed that the occurrence of maxima in R *vs* T curves is not a fundamental property of the transport.

Mendelssohn and White[6] observed that their data on glass, platinum and nickel, when normalized at 1.4°K, were fairly well represented by an expression of the form

$$[1-(T/T_\lambda)^\sigma], \tag{1}$$

with σ equal to 6, except for the lowest rates obtained with an unbaked glass specimen for which $\sigma=8$ seemed preferable. They noted that in spite of the variation in absolute value by up to a factor of approximately 3, the shape of the transfer curves was almost identical. The present data, when averaged over all heights, materials and finishes, follow expression (1) with $\sigma=7$, very closely throughout the entire temperature range under investigation. The extent to which this is really characteristic of all the data is revealed by the separate analysis of the data for glasses and metals (Fig. 1), as well as by the size of the absolute deviations from the mean listed in Table I.

Since it has been suggested that the analogy between superfluidity in helium and superconductivity proposed by Daunt and Mendelssohn[4] implies that "the ratio of the density of the normal constituent to the total density can be evaluated from the curves for the rate of volume flow,"[12] values of the ratio of superfluid to total density (ρ_s/ρ), are included for comparison in Fig. 1. (These have also been normalized to unity at 1.1°K.) Although expression (1), with a value of about 5.5 for σ, is a convenient first approximation[5,9] to the concentration of superfluid atoms in the temperature range of

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[13] Dyba, Lane, and Blakewood, Phys. Rev. **95**, 1365 (1954).

[14] H. A. Fairbank and C. T. Lane, Phys. Rev. **76**, 1209 (1949).

interest, it is known that a careful study of the experimental results for (ρ_s/ρ) does not yield detailed agreement with the result of inserting a unique value of σ in (1).[12] Consequently, the points associated with (ρ_s/ρ) in Fig. 1 were calculated directly from the experimental data obtained in oscillating disk[15,16] and second sound[17] experiments above 1°K.

From Fig. 1 it is evident that although the normalized flow rates show a definite similarity in temperature dependence to the normalized variation of (ρ_s/ρ), the resemblance[18] may be largely superficial in view of the forced agreement of the normalized curves at the lambda point and at 1.1°K. In judging such agreement it must be kept in mind that (1) is so insensitive to the value of σ that, when a family of normalized curves similar to those shown in Fig. 1 is plotted for various values of σ, comparison, at 1.75°K where differences appear most pronounced, shows that the result for $\sigma=6$ differs from those for $\sigma=4$ and $\sigma=8$ by only 20 percent and 12 percent, respectively.

Since the experimental values of (ρ_s/ρ) are regarded as well established, the significance to be attached to the seventh-power dependence of the average flow rates is not clear. The expectation that the transport rate should reflect the variation of (ρ_s/ρ) in a detailed rather than a superficial manner is not sufficiently rooted in an explicit theory for the consequences of the apparent contradiction to be discussed.[12] Preliminary studies[19,20] have indicated that valuable information may be forthcoming from extension of the measurements to very low temperatures.

Dash[9] has recently proposed a model which includes the possibility that the flow rates should vary as the square root of (ρ_s/ρ). Typical normalized values of this quantity, based on the experimental data for (ρ_s/ρ), are also shown in Fig. 1. This function appears to rise too steeply from the lambda point to be consistent with the present data.

ACKNOWLEDGMENTS

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[15] E. L. Andronikashvili, J. Exptl. Theoret. Phys. (U.S.S.R.) (a) **18**, 424 (1948); (b) **18**, 429 (1948).

[16] A. C. Hollis-Hallett, Proc. Roy. Soc. (London) **A210**, 404 (1952).

[17] deKlerk, Hudson, and Pellam, Phys. Rev. **93**, 28 (1954).

[18] In this connection, the recent investigation, by R. T. Swim and H. E. Rorschach [Phys. Rev. **97**, 29 (1955)], of the isothermal, pressure-induced volume flow rates through narrow glass slits, is also of interest. From their data between 1.4°K and the lambda point, normalized to equal (ρ_s/ρ) at 1.81°K, they conclude that "the temperature dependence of the flow is essentially the same as that of the superfluid concentration."

[19] E. Ambler and N. Kurti, Phil. Mag. **43**, 260 (1952).

[20] L. Lesensky and H. A. Boorse, Phys. Rev. **87**, 1135 (1952).