presented to the film. The present estimates of the difference between geometric and microscopic perimeters give quantitative support to the rejection by some authors of a purely perimetric explanation of anomalously high flow rates which have sometimes been reported for metals. The inapplicability of the perimetric explanation to such results suggests the desirability of re-examining the accepted perimetric explanation of the very high rates which result from contamination (in the sense of Bowers and Mendelssohn), with specific attention to the probable values of the ratio (δ/λ) as employed in Eq. (1).

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Helium II Film Transport. III. The Role of Film Height*

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Transport rates have been measured for film heights up to 5.3 cm. A variation with film height, whose average behavior is similar to that which has been reported for the film thickness, was observed. This height dependence of the flow rates was found to be temperature independent in the range under investigation $(1°\text{K} \lesssim T < T_\lambda)$. The results are also relatively insensitive to changes in substrate and microfinish. Consideration of this height dependence was essential to the analysis of the role of substrate and microfinish presented in the previous papers of this series.

INTRODUCTION

IN proceeding with the study of film transport rates inaugurated in previous papers,[1,2] the present investigation is concerned with the clarification of the answer to the question: How do measured transport rates vary with the distance of the liquid source from the rim of the beaker?

In the earliest relevant investigation, it was found that the rate decreased "only by 20 percent when the level had dropped from within 0.5 mm of the rim to some 20 mm from the rim."[3] It was concluded that "the rate of transfer depends only on the temperature and is practically independent of the difference in height between the two levels (except if the higher level is very near—1.5 cm or less—the top of the barrier)."[4] In a recent review, Daunt and Smith have given the following more quantitative description of these early results: "Daunt and Mendelssohn observed a small variation of the rate of transfer with height when observations were made over height differences as large as 6 cm. The variation in the rate was approximately 2 percent per cm change in height."[5] It was concluded that "the change in gravitational potential does influence the transfer but that this influence plays only the role of a correction of higher order."[6] Consequently, "to avoid the small error which might have been introduced by the slight influence of gravity, the determinations were all carried out in the same range of the beaker."[7] Later, on the basis of these experiments, the following summary was offered: "The most striking feature of the film transfer is its independence of the pressure head and of the length of the path over which it takes place. A beaker filled with liquid helium will empty itself at a steady rate · · · which does not depend on the position of the level or on the height of the intervening wall. These facts indicate that the film transport must be free of friction, since dissipation of the kinetic energy of flow would introduce a dependence of the flow rate on the pressure head and the length of path."[8]

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[1] B. Smith and H. A. Boorse, Phys. Rev. **98**, 328 (1955), designated as I in the present text.
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Later some observers,[9,10] using glass beakers, reported very high rates characterized by a severe height dependence and a variability which was excessive even by the standards previously enumerated for such results in paper I.[1] Bowers and Mendelssohn,[11,12] as a result of impressive detective work, were able to show that such high transfer rates are produced by the contamination of glass surfaces with solidified gases. This conclusion was promptly confirmed in other laboratories.[13–15] (The data previously alluded to in the present summary as well as in I[1] and II,[2] have been limited to results generally considered to be unaffected by such contamination. This restriction will also apply to subsequent references unless otherwise specified.)

The fact remains that most observers have agreed that, in the absence of such contaminating influences, proximity of the liquid level source to the rim is associated with increased transport rates. The magnitude of this variation, as well as the distance along the beaker over which the variation is pronounced, is however not clear. For example, in addition to those already quoted, the following observations have been reported: "Although the stainless steel beaker has mono-layers of barium stearate on the outside, it apparently behaves as a 'clean beaker' in the sense of Bowers and Mendelssohn, since the creep rate is practically independent of the height of the inner liquid level except for the first few millimeters, where, as usual, the creep rate is considerably greater."[16] In another paper it was reported that the motion of the liquid surface "was invariably a linear function except in the initial stages of an emptying process when the liquid · · · was very near the rim of the tube."[17] The routine acceptance of this effect is indicated by statements such as "the dependence of the transport rate on the position of the liquid level below the container rim was found to correspond qualitatively with the results of other investigators."[18] Other authors have reported that "the increased transfer near the rim still persisted after baking out and can therefore hardly be ascribed to adsorbed water or air. · · · Thus while it is tempting to see in the observed variation of R a true, if slight, dependence on the height of the film above the liquid level, the effect is so small that we hesitate to regard it at present as being of fundamental significance."[19] Consideration of these data has often led to the conclusion that "the flow rate height dependence, if it exists at all, cannot be large."[20]

On the other hand, it has also been observed that even "when adequate precautions were taken to prevent the formation of solid deposits on the surface holding the film, this characteristic rate of flow · · · varied with the height of the film · · · .This type of behaviour was observed in all the experiments. · · · In their original experiments, Daunt and Mendelssohn observed an increase in the rate of flow when one level was near the rim of their beaker, but they suggested that it was due to surface-tension effects. In actual fact the surface-tension rise of a liquid helium meniscus near a wall is about 0.7 mm, whereas · · · there is a significant variation of the rate of flow at film heights of several centimeters, and this must be regarded as a genuine film effect."[13] However, on reproducing an apparent flow rate height dependence, Brown and Mendelssohn have been "inclined to ascribe the small variation which was actually observed not so much to a real change in the flow rate but to a slight variation of the diameter of the beaker over its length. This is the more probable since in the following experiment the same variation was observed."[21]

With few exceptions, it has been common practice to exploit the diminution of this ill-defined effect with increased distance from the rim by calculating flow rates from data taken with the liquid source far removed from the rim (see papers I and II)[1,2] while according only cursory attention to the extent of the observed variation. This practice makes it rather difficult to analyze and compare much of the existing data on the magnitudes of the rates for the various substrates, since the aforementioned precaution[7] of comparing only those determinations carried out in the same range of the beaker has fallen into disuse. (This problem was also noted in I, where the data reported in Table III and Fig. 8 were therefore acquired at identical film heights.)[1]

Thus, in order for the companion studies of the role of substrate[1] and surface finish[2] to be meaningful, it has likewise been necessary to chart the variation of rate with film height.

Furthermore, the quantitative data on this subject have hitherto been acquired only for transport over glass surfaces, and to the best of our knowledge had been restricted to the plotting of four isotherms.[13,21] Further work of this nature,[22] also restricted to glass surfaces, was brought to our attention after the completion of the work described herein. The paucity of such data for glass, and its nonexistence for metals, serve to emphasize the need for a systematic study of this effect. Interest in this variation is further stimulated by various

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[21] J. B. Brown and K. Mendelssohn, Proc. Phys. Soc. (London) **A63**, 1312 (1950).
[22] B. N. Eselson and B. G. Lazarev, J. Exptl. Theoret. Phys. (U.S.S.R.) **23**, 552 (1952). We are grateful to Dr. B. S. Chandrasekhar for supplying us, in April, 1954, with a translation of this paper prepared at the University of Illinois.

TABLE I. R_1: average transport rate between 0.45 cm and 1.33 cm from beaker rim (midpoint at 0.89 cm).

Specimen	Run No.	(Rate in cm³/cm-sec) ×10⁵ 1.1°	1.3°	1.5°	1.7°	1.9°	2.1°
Al-I (B)	38, 39, 48	16.0	15.7	14.9	13.1	9.7	3.9
Al-II (B)	55	12.5	12.3	11.6	10.4	7.8	3.5
Cu	59	17.3	16.8	16.0	13.5	10.2	4.7
Cu	66	15.5	15.2	14.3	12.6	9.3	3.7
Ni-Ag-I (B)	33, 34, 37	15.6	15.4	14.8	13.1	9.6	4.0
Ni-Ag-I (B)	35	···	18.9	17.9	16.1	12.0	···
Ni-Ag-II (B)	45, 46, 49	13.4	13.2	12.6	11.4	8.4	3.5
Ni-II (R)	53	16.0	15.8	15.2	13.9	10.4	4.4
Ag	58	14.3	14.1	13.3	12.0	9.5	3.9
S.St. (B)	41–44, 47	14.9	14.7	14.0	12.6	9.4	4.3
Al-II (ES)	62	15.5	15.5	15.4	14.4	10.9	4.4
Al-II (ES)	65	12.1	12.0	11.6	10.4	7.9	3.5
Ni-Ag-II (ES)	52	10.8	10.7	10.5	9.4	7.4	3.1
Ni-II (ES)	60	11.2	11.1	10.9	9.9	7.7	3.8
S.St. (ES)	50, 51	11.1	10.8	10.3	9.2	7.3	3.7
Al-II (IS)	68	17.0	16.7	15.3	13.1	9.9	4.0
Ni-Ag-II (IS)	64	11.2	11.1	10.7	9.7	7.3	3.3
Ni-II (IS)	67, 69, 70	11.1	10.9	10.3	9.1	6.9	2.8
S.St. (IS)	61, 63	9.0	9.0	8.9	8.4	6.6	3.0
G-III	56	16.2	15.9	15.0	13.0	9.1	3.6
G-IV (FG)	54	18.4	18.2	17.8	16.2	12.8	6.1
G-V (CG)	57	15.5	15.3	14.7	13.3	10.2	4.2

TABLE II. R_2: average transport rate between 1.78 cm and 2.67 cm from beaker rim (midpoint at 2.22 cm).

Specimen	Run No.	(Rate in cm³/cm-sec) ×10⁵ 1.1°	1.3°	1.5°	1.7°	1.9°	2.1°
Al-I (B)	38, 39	13.1	13.0	12.6	11.3	8.2	3.3
Al-I (B)	48	11.9	11.7	10.9	9.5	6.9	2.9
Al-II (B)	55	9.7	9.5	9.0	8.0	6.2	2.7
Cu	59, 66	11.5	11.4	11.3	10.2	7.4	3.2
Ni-Ag-I (B)	33–35, 37	12.8	12.7	12.3	10.3	7.1	2.6
Ni-Ag-II (B)	45, 46, 49	11.5	11.4	10.9	9.8	7.1	2.7
Ni-II (R)	53	12.2	12.1	11.7	10.0	7.1	2.9
Ag	58	11.4	11.3	10.9	10.0	7.4	2.6
S.St. (B)	41–44, 47	11.4	11.2	11.0	9.5	6.8	2.8
Al-II (ES)	62, 65	11.7	11.5	11.0	9.6	6.7	2.4
Ni-Ag-II (ES)	52	9.4	9.3	8.8	7.8	6.0	2.8
Ni-II (ES)	60	8.2	8.1	7.8	7.2	5.7	2.3
S.St. (ES)	50, 51	9.0	8.9	8.5	7.8	5.9	2.6
Al-II (IS)	68	11.9	11.5	10.7	9.2	6.8	2.5
Ni-Ag-II (IS)	64	9.2	9.0	8.6	7.7	5.5	2.2
Ni-II (IS)	67, 69, 70	9.8	9.5	8.9	7.8	6.0	2.4
S.St. (IS)	61, 63	9.5	9.4	9.3	8.6	6.4	2.5
G-III	56	13.3	12.8	11.7	10.0	7.4	3.3
G-IV (FG)	54	14.2	14.1	13.9	12.6	9.1	3.6
G-V (CG)	57	13.2	13.0	12.3	11.0	8.2	3.1

theoretical correlations[5,13,20 22,23] between the height dependence of the film thickness and the transport rates.

Consequently, the objective of the present investigation has been the study of the variation of film transport rates with the height of the film, with particular emphasis on a possible systematic variation with material[1] and surface finish.[2] The temperature independence of this effect, hitherto tacitly assumed when not completely ignored, has also received careful attention.

EXPERIMENTAL RESULTS[24]

The data to be reported were acquired for transport from "4-section" beakers of the design shown in paper I (Fig. 2 therein).

Each complete emptying of these capacitor-beakers yields four transport rates which will be designated by R_n, where $n=1, 2, 3, 4$, corresponding to the similarly numbered sensitive regions of the capacitor core shown in I, Fig. 2. Thus, four rate *vs* temperature characteristic curves result from each day's run. A complete summary of the data for beaker sections 1–3 inclusive is provided in Tables I–III, where numerical values, abstracted from smooth curves drawn through the experimental points, are reported at conveniently spaced temperatures. The corresponding data for R_4 have already been tabulated elsewhere[1,2] and are consequently omitted here. Since the notation of the present paper is identical with that introduced in the preceding papers,[1,2] correlation of symbols, specimen composition, surface condition, run chronology, etc., may be established with the aid of tables in papers I and II. Since each row in Tables I–III of the present paper represents a separate rate *vs* temperature curve, it is clear that space does not permit the graphical exhibition of all the experimental points, although as has previously been noted,[1,2] a rather complete display of the data is vital in a field whose history is so filled with conflicting reports. However, the original data for R_4 have already been pre-

TABLE III. R_3: average transport rate between 3.11 cm and 4.00 cm from beaker rim (midpoint at 3.56 cm).

Specimen	Run No.	(Rate in cm³/cm-sec) ×10⁵ 1.1°	1.3°	1.5°	1.7°	1.9°	2.1°
Al-I (B)	38, 39, 48	10.9	10.7	10.0	8.7	6.3	2.8
Al-II (B)	55	9.2	9.1	8.7	7.8	5.8	2.2
Cu	59	11.0	10.9	10.7	9.8	7.3	2.7
Cu	66	9.5	9.4	9.1	8.2	5.9	2.2
Ni-Ag-I (B)	33–35, 37	12.3	12.2	11.9	10.4	7.0	2.1
Ni-Ag-II (B)	45, 46, 49	10.5	10.4	10.0	9.0	7.1	3.4
Ni-II (R)	53	13.4	13.0	12.1	10.4	7.1	2.6
Ag	58	9.1	9.1	9.0	8.5	6.9	2.9
S.St. (B)	41–44, 47	9.8	9.7	9.5	8.6	6.3	2.6
Al-II (ES)	62	12.2	11.8	10.9	9.5	6.7	2.3
Al-II (ES)	65	9.3	9.3	8.9	7.4	5.2	2.4
Ni-Ag-II (ES)	52	8.8	8.8	8.7	8.1	5.9	2.4
Ni-II (ES)	60	8.4	8.3	8.2	7.6	5.9	2.3
S.St. (ES)	50, 51	8.0	7.9	7.8	6.8	5.1	2.2
Al-II (IS)	68	10.8	10.5	9.7	8.4	6.3	2.6
Ni-Ag-II (IS)	64	8.7	8.6	8.2	7.3	5.6	2.5
Ni-II (IS)	67, 69, 70	9.4	9.3	8.8	7.6	5.9	2.5
S.St. (IS)	61, 63	8.6	8.5	8.4	7.5	5.5	2.3
G-III	56	12.3	11.9	11.1	9.3	6.6	2.7
G-IV (FG)	54	11.7	11.5	11.1	10.1	8.1	3.7
G-V (CG)	57	13.3	13.1	12.9	11.6	8.6	3.7

[23] J. G. Dash, Phys. Rev. **94**, 1091 (1954). We are indebted to Dr. Dash for acquainting us with these results prior to publication.

[24] Preliminary accounts of some of these results have already been presented: B. Smith and H. A. Boorse, Proceedings of the Third International Conference on Low Temperature Physics and Chemistry (Rice Institute, Houston, 1953); B. Smith and H. A. Boorse, Phys. Rev. **94**, 772 (1954); B. Smith, Bull. Am. Phys. Soc. **30**, No. 1, 57 (1955).

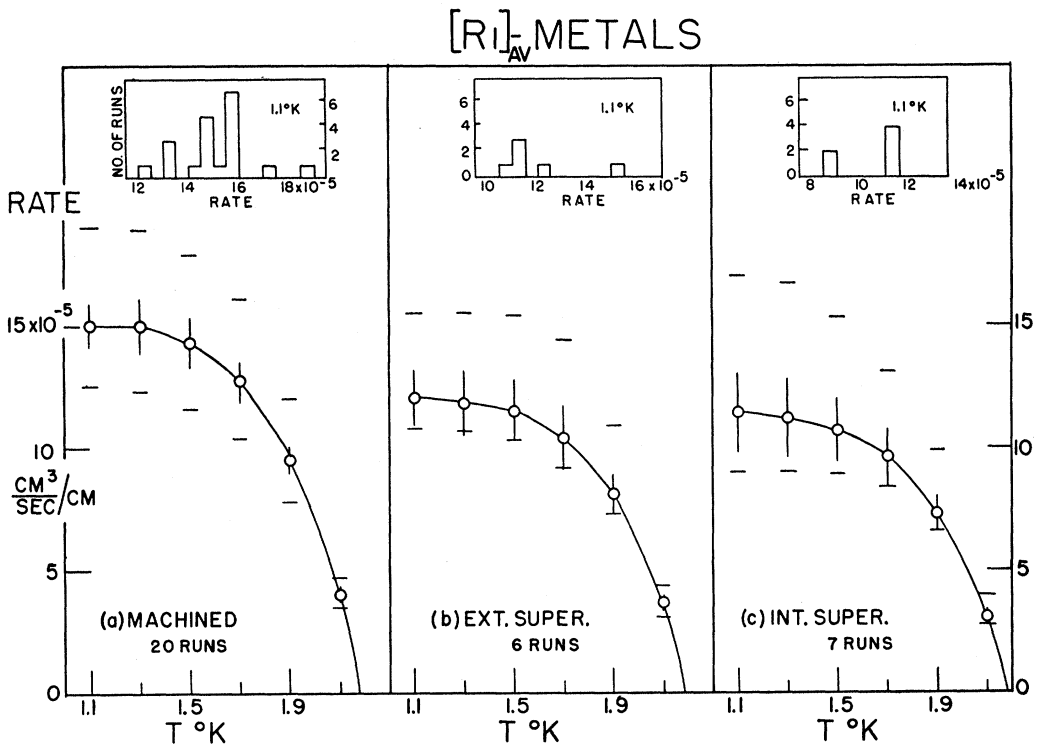

FIG. 1. Average values of transport rates calculated from those listed in Table I for various metal surfaces (R_1: 0.45 cm to 1.33 cm from beaker rim). The plotted points are calculated arithmetic mean values; the vertical lines are drawn to extend above and below the points by an amount equal to the average absolute deviation from the mean and the horizontal bars represent the highest and lowest values contributing to each average. The histograms show the frequency distributions of the observed rates at 1.1°K.

sented (see I: Fig. 8 and II: Figs. 2–6); this should prove adequate since there exist one to one correspondences between the plotted experimental values of R_4 and those for R_1, R_2, and R_3, which will not be shown.

As previously noted,[2] the striking similarity of the data for all materials and finishes at each height suggests an analysis based on the statistical comparison of average behavior illustrated in Figs. 1–3 of the present paper and Fig. 7 of paper II. Indeed, these results have already been invoked in the previous analysis of the role of substrate[1] and surface finish.[2]

The variation of transport rate with height for each complete isothermal emptying of the capacitor may be obtained by plotting R_n against abscissas corresponding to the distance of the midpoint of the nth section from the rim. Thus, each emptying yields a height dependence isotherm similar to those plotted by Eselson and Lazarev[22] and Atkins.[13] The prohibitive number of such graphs (one for every experimental point shown in the graphs of R_4 *vs* T in papers I and II) may greatly be reduced by making use of the fact that this effect was found to be temperature independent over the range of temperatures which was investigated. This may conveniently be observed by plotting a separate graph of the dimensionless ratio (R_n/R_4) against temperature for each value of n; a complete isothermal emptying provides a point on each such graph. Representative results are portrayed in Fig. 4 in plots of (R_3/R_4) *vs* T obtained for a number of materials and surface finishes. Since such graphs failed to reveal any dependence on temperature, an arithmetic mean of values obtained over all temperatures was then calculated for each ratio. A complete summary of these average values appears in Table IV. Individual values for the three glass runs are also plotted in Fig. 5(d). The rows in Table IV reflect the observed temperature independent average height variation in a conveniently normalized fashion since the value $(R_4/R_4)=1$ (omitted in the table) is common to each row. This normalization may further be exploited by averaging items in each column of Table IV which correspond to groups of similarly prepared metal beakers. This operation yields the graphs shown in Fig. 5(a–c), which thus represent a distillation of the information available in hundreds of height dependence isotherms. The points, vertical lines, and horizontal bars in Fig. 5 were drawn using the conventions previously described in reference to Figs. 1–3. Histograms are included in the insets of Fig. 5 to exhibit the frequency distribution of actual values in the range observed for one particular ratio. For reasons which will later become apparent, it is of interest to investigate the possibility of representing the data in Fig. 5 by algebraic expressions of the form

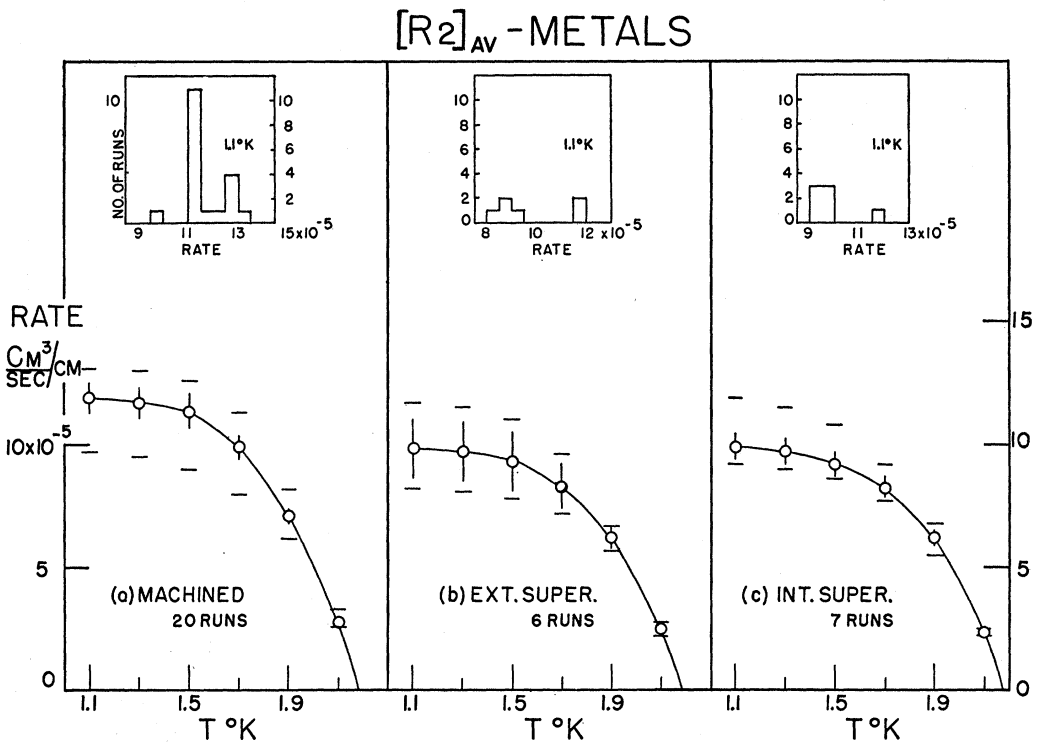

FIG. 2. Average values of transport rates calculated from those listed in Table II for various metal surfaces (R_2: 1.78 cm to 2.67 cm from the rim). Symbols have the same significance as in Fig. 1.

$h^{-1/z}$. Consequently, the average data have been replotted logarithmically in Fig. 6, where successful representation by the expression $h^{-1/z}$ should produce linearity with slope $(-1/z)$. Except for the average data for bored specimens, it is clear that the departures from linearity are rather pronounced. A rough measure of this departure may be inferred from the line segments which have been drawn connecting adjacent points on the two lower curves. Values of z appropriate to each segment are indicated on the graphs. The curves shown in Fig. 5 were drawn for integral values of z suggested by the range of slopes in Fig. 6; each curve has been scaled to correspond to the observed data at a distance of 1 cm from the rim. These curves are included only for reference in the discussion to follow and are not intended to convey an adequate representation of the experimental results.

Direct verification of the consistency of earlier observations (shown in I: Figs. 4–7 and II: Fig. 1) with those just described is rendered difficult by the different heights appropriate to the earlier data. However, graphs of R *vs* h similar to Fig. 5 but differing in the omission of normalization, serve to verify the complete consistency of all the data which have previously been presented.[1,2] Thus, although the uniform geometry characterizing specimens in the final series of control experiments[1,2] provides a fertile source of easy and direct comparisons, thereby encouraging almost exclusive reference to these results in subsequent discussion, the conclusions drawn will be equally applicable to the other data.

DISCUSSION

As mentioned in the Introduction, the present study of the height dependence of the transport rate was undertaken as a precautionary measure intended to minimize ambiguity in the analysis of the dependence on substrate[1] and microfinish.[2] An undeniable variation of transport rate with film height, regardless of its origin, has indeed been observed. The value of the precaution of comparing only those "determinations carried out in the same range of the beaker"[7] has therefore amply been demonstrated.

The arguments most frequently cited for judging previously observed height dependences to be spurious do not seem to apply here. For example, the results cannot be ascribed to uncertainties in beaker geometry, because the effect was observed for so many beakers whose inner diameters had been accurately measured (see I for discussion of bore measurement). The results cannot simply be ascribed to surface roughness since the height dependence persisted even for (IS) beakers.[2] There is no reason to believe that contamination in the sense discussed by Bowers and Mendelssohn[11,12] is responsible for the observations, in view of the leak tightness and outgassing precautions which were adopted.[1] Furthermore, the observed rates not only fail to exhibit the anomalously high values characteristic of

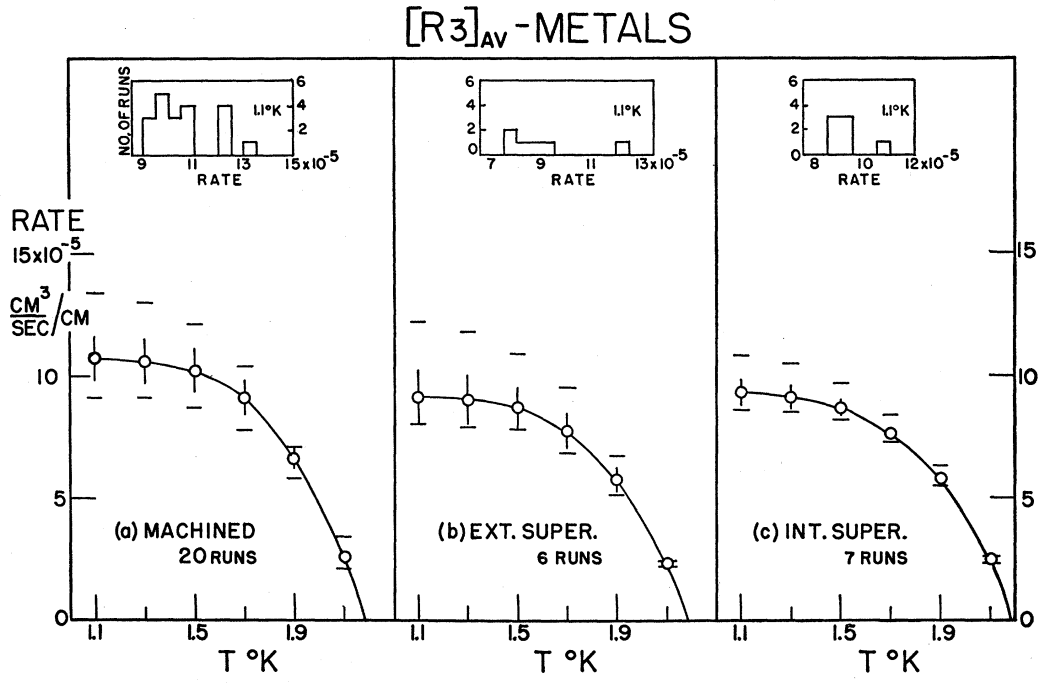

FIG. 3. Average values of transport rates calculated from those listed in Table III for various metal surfaces (R_3: 3.11 cm to 4.00 cm from rim). Symbols have the same significance as in Fig. 1.

contaminated surfaces, but are even lower than most previous results for "clean metal beakers."[1] Finally, as will be discussed in a subsequent paper,[25] the temperature dependence of the flow was observed to be the same (on the average) at all heights.

The present observations are similar to those described by Atkins[13] and Eselson and Lazarev,[22] both with respect to order of magnitude and also with respect to their conclusion that the film height (i.e., the distance of the level of the liquid source from the beaker rim) is the significant parameter with which the observations may be correlated. Since detailed evidence for this conclusion is available elsewhere,[13,22] the present discussion will be limited to the observation that the effect of film height is most vividly observed by comparison of filling rates, measured at effectively constant film height, with those obtained on emptying which takes place at continuously varying film height.

As a result of the primary interest of the present work in comparing the behavior of beakers of various materials and finishes under identical conditions,[1,2] most of the systematic observations were made on the emptying of beakers which had previously been filled by immersion. Those filling rates which were measured did not appear appreciably different from emptying rates obtained at comparable film heights (see graphs in papers I and II), although other observers have noted measurably higher,[22] as well as lower[19] rates on emptying. The present measurements did not extend to the small level differences which have been the concern of some investigations.[13,22,26]

In the absence of any clear evidence for an alternative explanation, it seems reasonable to identify the observed dependence on height with the similar behavior of the film thickness, as has been done by Atkins.[13] However,

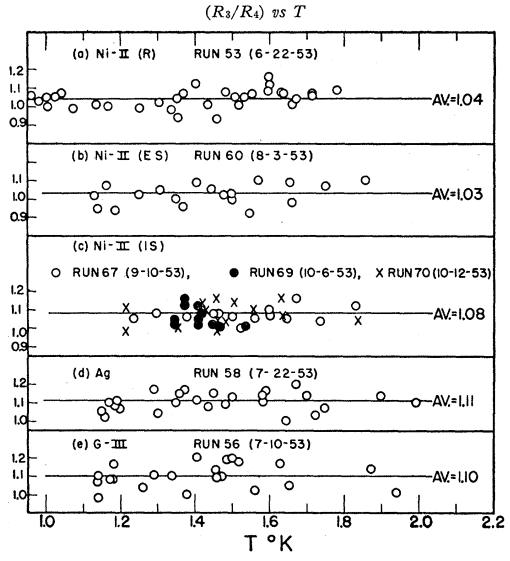

FIG. 4. Typical plots of (R_3/R_4) as a function of temperature for some metal and glass specimens. These and similar graphs reveal the temperature independence of this ratio for all specimens, thereby providing a single average value for each run as shown. Averages so obtained are listed in Table IV.

[25] B. Smith and H. A. Boorse, following paper [Phys. Rev. 99, 367 (1955)], designated as IV in the present text.

[26] G. S. Picus, Phys. Rev. 94, 1459 (1954).

TABLE IV. Normalized average height dependence of the transport rates. These are temperature independent values (obtained from graphs such as those shown in Fig. 4) as explained in the text.

Specimen	Run Nos.	$(R_1/R_4)_{Av}$	$(R_2/R_4)_{Av}$	$(R_3/R_4)_{Av}$
Al-I (B)	38	1.63	1.29	1.02
Al-I (B)	39	1.35	1.19	
Al-I (B)	48	1.65	1.21	
Al-II (B)	55	1.35	1.05	1.01
Cu	59	1.64	1.11	1.12
Cu	66		1.31	1.03
Ni-Ag-I (B)	33	1.35		
Ni-Ag-I (B)	34	1.40	1.06	1.01
Ni-Ag-I (B)	35	1.49		
Ni-Ag-I (B)	37	1.33		
Ni-Ag-II (B)	45	1.38	1.23	1.14
Ni-Ag-II (B)	46	1.33	1.19	1.04
Ni-Ag-II (B)	49	1.28	1.03	1.01
Ni-II (R)	53	1.29	0.96	1.04
Ag	58	1.68	1.34	1.11
S.St. (B)	41–44, 47	1.54	1.33	1.11
Over-all average for machined [i.e., (B) and (R)] metals [See Fig. 5(a)]		1.47	1.19	1.06
Average absolute deviation from the mean		0.12	0.11	0.04
Al-II (ES)	62	1.57	1.21	1.13
Al-II (ES)	65	1.27		0.95
Ni-Ag-II (ES)	52	1.19	0.96	0.92
Ni-II (ES)	60	1.41	1.01	1.03
S.St. (ES)	50	1.49	1.24	1.09
S.St. (ES)	51		1.19	
Over-all average for externally superfinished (ES) metals [See Fig. 5(b)]		1.39	1.14	1.04
Average absolute deviation from the mean		0.13	0.10	0.07
Al-II (IS)	68	1.57	1.14	1.01
Ni-Ag-II (IS)	64	1.34	1.09	1.02
Ni-II (IS)	67, 69, 70	1.27	1.10	1.08
S.St. (IS)	61, 63	1.09	1.06	0.97
Over-all average for internally superfinished (IS) metals [See Fig. 5(c)]		1.27	1.09	1.03
Average absolute deviation from the mean		0.13	0.02	0.04
G-III	56	1.49	1.18	1.10
G-IV (FG)	54	1.58	1.24	1.00
G-V (CG)	57	1.23	1.02	1.06

Eselson and Lazarev,[22] upon noting a lack of instantaneous adjustment to a new equilibrium flow rate when the liquid level progressed from the wide upper part to the narrow lower part of a specially designed beaker,[27] question the validity of a simple correlation with film thickness. They suggest that only the initial high rate of emptying a full beaker should be ascribed to the large film thickness arising from proximity to the rim. The remainder of the height dependence, which manifests itself at considerable distances from the rim, is then depicted as the result of a tendency of the rate to resist changes from the initial high value. This inertial effect is said to be more pronounced under conditions of poor thermal contact between the bath and the helium in the beaker. Even this picture relies on the height dependent film thickness to furnish the initial tendency for the rate to be high, while the exact nature of the special effects, as well as the details of the inertial hypothesis, remain obscure. The present discussion will therefore be limited to a comparison of the known properties of the film thickness with the present flow rates, on the assumption that "the rate of transfer is probably a rather sensitive measure of the thickness."[28]

The calculation of Schiff[29] which predicts that the stationary film thickness should vary as $h^{-\frac{1}{3}}$ has already been discussed in connection with the size of the helium-substrate interaction [see I: Eq. (2)]. Although somewhat similar considerations also led Frenkel[30] to predict the same functional dependence of the thickness on h, a variation with $h^{-\frac{1}{2}}$ was derived from other assumptions by Bijl, deBoer, and Michels.[31] Consequently, experimental results are traditionally compared with functions of the form $h^{-1/z}$ to provide numerical estimates of the appropriate values of z. Temperley[28] incorporated features of both these theories in a quantum-mechanical

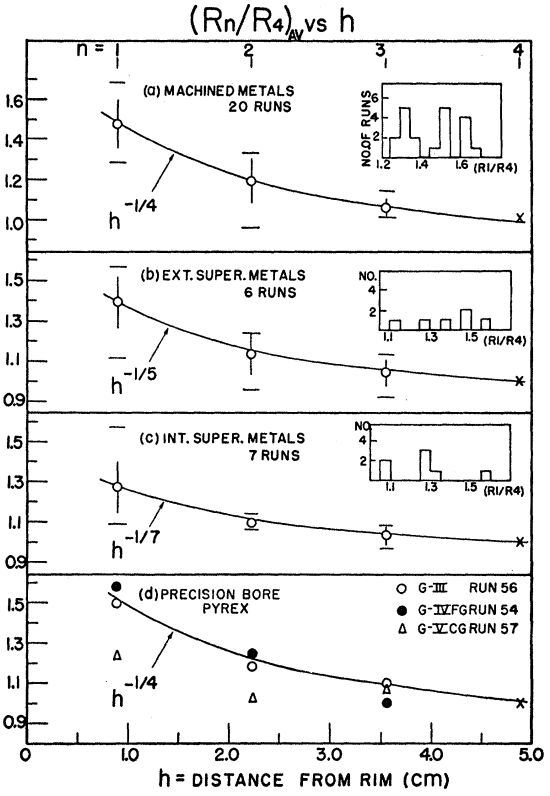

FIG. 5. Average normalized dependence of transport rates on film height for various materials and surface finishes.

[27] Although not specifically discussed in their paper, it appears that Daunt and Mendelssohn observed a similar effect in their original experiments (see reference 6, Fig. 7).

[28] H. N. V. Temperley, Proc. Roy. Soc. (London) **A198**, 438 (1949).

[29] L. I. Schiff, Phys. Rev. **59**, 839 (1941).

[30] J. Frenkel, J. Phys. (U.S.S.R.) **2**, 365 (1940).

[31] Bijl, deBoer, and Michels, Physica **8**, 655 (1941).

treatment which leads to a qualitatively similar result which cannot be reduced to such a simple algebraic relation between thickness and height. Atkins[32] has also deduced a more complicated result which yields a value of z between 2 and 3, if an approximation of the form $h^{-1/z}$ is sought. A discussion of the merits and limitations of the various theories will be omitted here in view of the availability of two excellent review articles which have recently appeared.[5,33]

It must first be noted that the variation with height of the present transport rates has been found to be temperature independent over the range which was investigated (see Fig. 4). This compares favorably with a similar temperature independence exhibited by the film thickness in both optical[34] and gravimetric[35] studies.

Only Temperley[28] has made a serious attempt to propose a theory of the variation of film thickness with temperature. His work leads to three possible results depending on three different assumptions which can be made in comparing the energy attributed to the lowest state of the liquid with the energies of the film states. One assumption leads to a constant film thickness below the lambda point, another leads to a vanishing of the film at absolute zero, and the third implies that the film thickness should exhibit a maximum below the lambda point. Temperley notes that "it is not possible to predict *a priori* which of these three assumptions should be made," although he suggests "a little evidence" in favor of the third possibility. This consists, in part, of the fact that the observed proportionality of transport rates to the minimum perimeter presented by surfaces can be correlated with the third assumption in a fairly obvious manner. Supplementary evidence is based on experimental data available at the time (1949), from which it was inferred that "the rate of transfer seems to pass through a maximum at 1.6°K to 1.8°K," thereby suggesting that "the film thickness passes through a maximum here also." Since later measurements have shown that neither the thickness[34,35] nor the transport rates[1,2,25] normally exhibit such maxima, it might be profitable to develop the consequences of the previously ignored first assumption in a more quantitative fashion. The fact that Temperley's calculation does not predict a maximum on an *a priori* basis seems to have received inadequate attention in the literature.

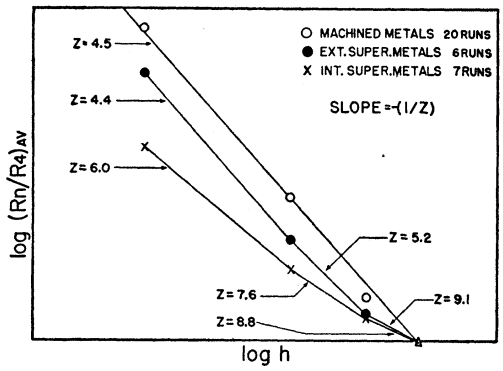

FIG. 6. Logarithmic replot of the average data of Fig. 5 to determine the applicability of the functions shown therein to the data.

TABLE V. Comparison of values of z obtained by fitting measured variation of film thickness with height[a] to the functional dependence $h^{-1/z}$.

Substrate	Range of film height (cm)	T°K	z	Method	Reference
Glass	0.5–5.0	1.47	7	Oscillations	36
Glass	0.3–1.0	1.47	~2	Oscillations	36
Glass	1.0–4.5	1.47	~10	Oscillations	36
Glass		1.3–1.9	1–2	Oscillations	26
Stainless steel covered by barium stearate	0.25–1.2	1.5	2.9–3.1	Optical, static film	34[b]
	0.15–1.5	1.1	3.5	Optical, static	34[b]
	0.15–1.5	2.1	2.5	Optical, static	34[b]
	0.4–1.6	2.05	2.3	Optical, static	40
	0.25	?	2.5	Optical, mobile	34[b]
	1	?	8–12	Optical, mobile	34[b]
Aluminum	0.3–7.9	1.25–2.15	2±0.3	Gravimetric	35

[a] In a paper which appeared after the preparation of this manuscript, L. Meyer [Phys. Rev. **97**, 22 (1955)] offers an interesting discussion and interpretation of "the great variety of the experimental evidence" to be found in reports of the thickness of the static helium film as a function of height.

[b] As noted by Daunt and Smith (see reference 5), these values must presumably be corrected in accordance with later observations reported by L. C. Jackson and D. G. Henshaw [Phil. Mag. **44**, 14 (1953)]. See reference 40 for those details of the required modifications which have thus far been reported.

The results for individual runs have been listed in Table IV so as to provide a basis for detailed comparison of the present observations with future theoretical and experimental studies. In view of the current status of both theory and experiment,[5,34,35] it seems reasonable to restrict the present discussion to the average dependence of the transport rate on height which is shown in Fig. 5. In such a discussion it must always be remembered that Table IV and Fig. 5 indicate variations from run to run and specimen to specimen to which the remarks made in connection with the analysis of the role of substrate[1] and microfinish[2] also apply. Here too, the difficulty of exactly duplicating beaker surfaces is worthy of attention. For example, the film thickness data which will be cited for "clean surfaces" may be compared with Atkins' report[36] that the film thickness varied as h^{-1} on contaminated glass surfaces. Atkins has also observed a thickness on soda glass which had been cleaned with hot distilled water, which differed from the thickness he measured for lead glass cleaned in hot chromic acid, although the film "had the same shape in the two cases."[36]

Table V summarizes the available data on the functional dependence of film thickness on height as reflected by values of z in expressions of the form $h^{-1/z}$. Comparison with Fig. 6 indicates that the results are

[32] K. R. Atkins, Can. J. Phys. **32**, 347 (1954).
[33] R. B. Dingle, Phil. Mag. Suppl. (Advances in Physics) **1**, 111 (1952).
[34] E. J. Burge and L. C. Jackson, Proc. Roy. Soc. (London) **A205**, 270 (1951). (Also, see reference 40.)
[35] R. Bowers, Phil. Mag. **44**, 1309 (1953); Phys. Rev. **91**, 1016 (1953).
[36] K. R. Atkins, Proc. Roy. Soc. (London) **A203**, 119 (1950).

indeed similar; although the function appears too simple for an adequate description of the logarithmic plots of Fig. 6, this inadequacy is virtually masked in the curves shown in Fig. 5. These curves were drawn solely for the purpose of demonstrating this observation and do not reflect any attempt to provide the best possible fit to the data. In both thickness and flow rate determinations, the trend is toward larger values of z at greater heights. The scatter of the present height dependence data, as well as the actual transport rate data, was found to be greatest at small film heights. While this effect cannot be dissociated from the increased uncertainty inherent in measurements of the shorter times associated with higher rates,[1] one must not overlook possible correlation with other effects such as the drop formation which has on occasion been observed in association with proximity of the liquid level to the rim.[37] It must also be noted that the validity of the comparison of values in Table V with the present data on flow rates may be limited by the different geometrical conditions of the various experiments. For example, the gravimetric measurements[35] refer to the film thickness on a sheet of aluminum foil, the optical measurements[34] determine the thickness of the film on the outside wall of a beaker, and the oscillation studies[26,36] are performed only at small level differences, whereas the present flow data should be correlated with the thickness of the film on the inner beaker wall during emptying. Comparison should also be confined to measurements of the thickness of mobile rather than stationary films.

It has been suggested that the expression

$$(mv_c)d \sim \hbar, \tag{1}$$

where (mv_c) is the average critical momentum of the superfluid particles and d is the film thickness or channel width, might describe superflow in liquid helium II.[5] If m is then associated with the atomic mass of helium, rather than some effective mass which might depend implicitly on other parameters, it would be expected that any variation of d with height should be counteracted by a variation of v_c tending to maintain the constancy of the product $v_c d$. Consequently the transport rate, which is expected to vary as $v_c d$ on dimensional grounds, would also be height independent.[5] In spite of the interesting features embodied in Eq. (1), its status with respect to the helium II problem is not sufficiently established in theory or experiment to justify the rejection of experimental observations which seem to conflict with its consequences. For example, although Eq. (1) implies $v_c \propto d^{-1}$, Mott[38] has proposed a model in which $v_c \propto d^{-\frac{1}{2}}$, and a recent calculation by Dash[20] also leads to the latter dependence. Experimental evidence for the invariance of the product $v_c d^{\frac{1}{2}}$ is cited by Dash,[20] while Daunt and Smith[5] conclude that the experimental evidence "does not allow a completely clear choice" between the two alternative relations connecting v_c and d. Atkins[39] has called attention to evidence that "$v_c d$ is not a constant but increases with d."

Dash[23] has also suggested that the transport rate might vary with height as the square root of the thickness dependence on height. Such a variation (i.e., $R \propto h^{-\frac{1}{2}z}$, when $d \propto h^{-1/z}$) would also be consistent with the present data if one uses the smaller values of z shown in Table V for the film thickness. A more complete comparison must await further study of the film thickness as a function of material, finish, temperature,[40] etc., in both theory and experiment.

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[37] D. G. Henshaw and L. C. Jackson, National Bureau of Standards, Circular 519 (U. S. Government Printing Office, Washington 25, D. C., 1952) pp. 189–190.

[38] N. F. Mott, Phil. Mag. **40**, 61 (1949).

[39] K. R. Atkins, Phil. Mag. Suppl. (Advances in Physics) **1**, 169 (1952).

[40] A. C. Ham and L. C. Jackson, Phil. Mag. **45**, 1084 (1954). In this paper, it is reported that a thick film persists above the lambda temperature, and that "the thickness of the helium II film was greatly increased by the presence of an extremely thin layer of solid air on the surface of the mirror." These observations are obviously of great interest, not only in the present context, but also in connection with the previous discussion of the roles of substrate (reference 1) and surface finish (reference 2). Further discussion of these findings must however await the publication of the more complete report promised by Ham and Jackson, as well as the extension of these new measurements to mobile films and greater film heights. In this connection, Meyer's recent paper (cited in Table V) is also of interest.