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Letters to the Editor

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Possible Mathematical Formulation of the Gell-Mann Model for New Particles

BERNARD D'ESPAGNAT AND JACQUES PRENTKI
CERN, Geneva, Switzerland

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GELL-MANN and Pais[1] have recently proposed several schemes for describing the existence and interactions of hyperons and heavy mesons. One of these is essentially a refined version of a hypothesis originally proposed by Gell-Mann[2] and by Nakano and Nishijima.[3] As has been pointed out by the authors,[1] this particular model does not appear at first sight to be the most attractive for theorists, mainly because it involves an apparent arbitrariness in the choice of the relation Q *versus* I_3 for the various particles. On the other hand, it is perhaps the model best fitted for the interpretation of present experimental evidence.

In view of these facts, we wish to point out in this note that an equivalent but more axiomatic formulation of the Gell-Mann rules is possible. The theory will be developed in a three-dimensional isotopic spin space. As is well known,[4] in such a space it is possible to define two kinds of spinors which differ by their symmetry properties with respect to an oriented plane. If ξ, ξ' denote spinors of the first kind and η, η' spinors of the second kind, the true scalars that can be formed with these quantities are $\xi'^{*T}\xi$, $\eta'^{*T}\eta$, $\eta'^{T}\tau_2\xi$, $\xi'^{T}\tau_2\eta$, where the superscripts $*$ and ^{T} mean "imaginary conjugate" and "transposed", respectively. Families of particles are now defined by their transformation properties in Lorentz and isotopic spin space. In Table I different fields are so defined, the list being obviously not limitative. We postulate only that family A contains the nucleons and family δ the π mesons.

TABLE I. Several types of fields.

	Lorentz space	Isotopic space	
ψ_A	spinor	spinor of first kind	$\mathfrak{N}$
ψ_B	spinor	spinor of second kind	Ξ
ψ_a	spinor	no spin (scalar)	Λ
ψ_b	spinor	spin 1 (pseudo-vector)	Σ
φ_Δ	no spin	spinor of first kind	θ
φ_δ	no spin	spin 1 (pseudo-vector)	π

The other correspondences given in column 4 are not postulated; they will follow [with respect to interactions, charges Q, and relation $Q(I_3)$] from the mathematical formulation.

Along with Lorentz invariance, the invariance with respect to rotations *and reflections* in isotopic space will be postulated for strong interactions. The most general Lagrangian that follows from these assumptions is easily constructed. For the interaction part, terms like

$$\bar{\psi}_a\varphi_\Delta{}^{*T}\psi_A,\quad \bar{\psi}_b\varphi_\Delta{}^{*T}\tau\psi_A,\quad \bar{\psi}_a\psi_B{}^T\tau_2\varphi_\Delta,\cdots$$

appear, together with others of the same general form which for the sake of brevity will not be written.

Let us define the operator

$$U=\int dv[\bar{\psi}_A{}^T\beta\psi_A-\bar{\psi}_B{}^T\beta\psi_B - i(\pi_\Delta{}^T\varphi_\Delta-\pi_\Delta{}^{*T}\varphi_\Delta{}^*)]. \quad (1)$$

It can be verified that U commutes with the total Hamiltonian containing the fields of Table I.[5] U is therefore a constant of the motion. On the other hand the conservation of fermions is respected and, of course, I^2 and I_3 are constants. The latter can be written

$$\begin{aligned}2I_3=n(A)-n(\bar{A})-n(A')+n(\bar{A}')+n(B)\\ -n(\bar{B})-n(B')+n(\bar{B}')+2[n(b)-n(\bar{b})-n(b')\\ +n(\bar{b}')]+n(\Delta)-n(\bar{\Delta})-n(\Delta')+n(\bar{\Delta}')\\ +2[n(\delta)-n(\delta')],\end{aligned} \quad (2)$$

where A and A' refer to different charge states of the nucleon, etc., and the bars refer to the antiparticles; while

$$\begin{aligned}U=n(A)-n(\bar{A})+n(A')-n(\bar{A}')-n(B)+n(\bar{B})\\ -n(B')+n(\bar{B}')+n(\Delta)-n(\bar{\Delta})+n(\Delta')-n(\bar{\Delta}').\end{aligned}$$

One sees that the constant $I_3+\frac{1}{2}U$ is, for known particles (nucleons and π mesons), equal to the charge Q and can thus be considered as a general definition of this quantity.[6] The eigenvalues of U give therefore the desired relation between charge and I_3. In particular, $Q=I_3+\frac{1}{2}$ for one quantum of the ψ_A or φ_Δ fields, while $Q=I_3$ for ψ_a, ψ_b, and φ_δ; and finally $Q=I_3-\frac{1}{2}$ for ψ_B field quanta. The correspondence given in Table I, column 4 between the field families ψ_A, ψ_B, etc., and the observed particles $\mathfrak{N}$, Ξ, etc., is therefore justified as far as charge is concerned. On the other hand, it can be verified that the postulate of complete invariance leads to an interaction Lagrangian which allows only the virtual processes given by the Gell-Mann rules. The equivalence is therefore complete.

A fact which is perhaps of some significance is that, in such a straightforward generalization of the ordinary formalism, the Ξ field appears as a spinor of the second kind in isotopic space when the nucleon field is taken as a spinor of the first kind. As a practical rule, one may point out that instead of the conservation of Q and I_3 one may of course use the conservation of Q and U. Possible reactions (including electromagnetic) will therefore be obtained by requiring (a) the usual conservation of fermions, (b) the conservation of charge, (c) the use of the following rule: denote by "isofermions" the nucleons, the θ, and the anti-Ξ, and by "anti-isofermions" the antinucleons, the anti-θ (charge conjugated), and the Ξ; then *the number of isofermions minus the number of anti-isofermions must be conserved.*

[1] M. Gell-Mann and A. Pais. Proceedings of the Glasgow Conference on Nuclear and Meson Physics, 1954 (to be published), p. 342. See also A. Pais, Phys. Rev. **86**, 663 (1952); Physica **19**, 869 (1953); Progr. Theoret. Phys. (Japan) **10**, 457 (1953); Proc. Natl. Acad. Sci. U. S. **40**, 484, 835 (1954).

[2] M. Gell-Mann, Phys. Rev. **92**, 833 (1953).

[3] T. Nakano and K. Nishijima, Prog. Theoret. Phys. (Japan) **10**, 581 (1953).

[4] E. Cartan, *Leçons sur la théorie des spineurs* (Hermann & Cie, Paris, 1938), Vol. I, especially Chap. III.

[5] Other choices of fields also leading to a constant U are possible; they give the same physical results. (1) is obtained from the invariance of the total Lagrangian with respect to the transformation $\psi_A \rightarrow \psi_A e^{i\alpha}$, $\psi_B \rightarrow \psi_B e^{-i\alpha}$, $\varphi_\Delta \rightarrow \varphi_\Delta e^{i\alpha}$.

[6] Some considerations on the total Lagrangian show that this assumption is indeed the most natural one.

Bevatron *K*-Mesons*

Robert W. Birge, Roy P. Haddock, Leroy T. Kerth,
James R. Peterson, Jack Sandweiss,† Donald H. Stork,
and Marian N. Whitehead
Radiation Laboratory, University of California, Berkeley, California

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To facilitate the search for K-mesons from the Bevatron, two of us (L.T.K. and D.H.S.) have suggested the use of a strong-focusing spectrometer (Fig. 1),[1] consisting of a magnetic quadrupole focusing lens[2] followed by an analyzing magnet. Particles of any desired momentum can be brought to a focus, forming an image of the target at a point behind the analyzing magnet. Emulsion stacks are placed at this point. With this arrangement we have found examples of four types of heavy mesons first established in cosmic-ray work.[3] Particles of different mass can be separated according to their ranges in emulsion. For particles of momentum 360 Mev/c, the range of K's is 4.6 times the range of the protons, and pions pass through the emulsion stack at minimum ionization.

A stack of 107 Ilford G.5600-μ pellicles,[4] 3.5 in. by 3.5 in., has been exposed so that 114-Mev K-particles stopped in the center of the stack. The proper time of flight for such particles from the target to the emulsion is about 10^{-8} sec.

This stack has been scanned in a swath across the direction of the meson flux for tracks lying in the plane of the emulsion whose ionization is visibly greater than minimum. Particles stopping in the stack (beyond the position of the swath) have masses less than 1200 m_e. Particles that go all the way through the stack have masses less than 800 m_e.

The tracks are followed until they stop and the endings are examined for decays. To date 300 decays have been observed. Twenty of these are τ^+ mesons whose unique decay into three charged pions is readily identifiable. Among the others, all of which decay into one lightly ionizing secondary, only those with a secondary that is flat or with an ionization obviously higher than minimum have been categorized. Three examples have been found of what is assumed to be the alternate decay of the τ into one charged and two neutral pions, with the pion stopping in the emulsion stack. Two events decay into low-energy muons (less than 55 Mev) and are presumably examples of the κ or $K_{\mu 3}$.

To establish the existence of the $K_{\pi 2}(\theta^+,\chi^+)$ or of the $K_{\mu 2}$ mesons, either very large emulsions are needed to stop the long-range secondary or else very accurate measurements of the multiple scattering and the ionization must be made. Measurements on four fortuitously flat secondaries at a distance of 5 cm from the decay point revealed that three of the primaries were $K_{\pi 2}$'s and one presumably a $K_{\mu 2}$, as determined by the tentative identification of the secondary as a high-energy muon. Excellent calibration on grain count is available from the π mesons of known energy traversing the same region of the emulsions. From the number of K-mesons found here compared to the number found at about 25 cm from the target,[5] it is unlikely that the mean life of any of the K's seen is less than 3×10^{-9} sec.

In the initial exposure, the momentum resolution as determined from the proton ranges allows a mass determination to $\pm 40\ m_e$ on each K-meson. With a few exceptions, all particles with lightly ionizing secondaries fall within a distribution of this width centered about 20 m_e below the average for τ mesons plotted separately (Fig. 2). In a subsequent exposure the momentum resolution has been improved. The scattered points on the high-mass side of the main distribution may be due to particles that suffered inelastic collisions, or scattered off the channel. A

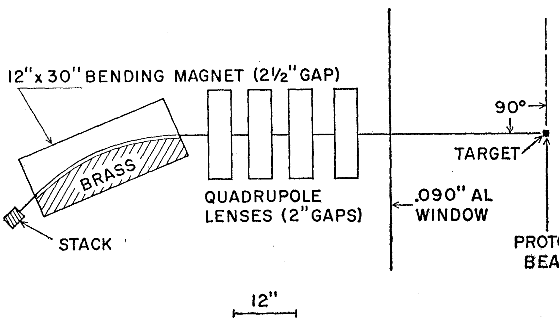

Fig. 1. Strong-focusing spectrometer.