Decay of Spin-Zero Mesons into Two Leptons*

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The following general theorem is formulated: The matrix element for the decay of spin-zero mesons with mass m into two leptons with masses μ_1 and μ_2 respectively contains only terms proportional to μ_1/m and μ_2/m, if in the open polygonal arc of lepton lines the number of matrices γ_μ plus the number of internal lines (S_F-functions) is odd. There can also be terms proportional to μ_i/m if virtual leptons with $\mu_i \neq \mu_1, \mu_2$ appear in the arc. Application of this theorem to the reaction $\pi^0\rightarrow e^+ + e^-$ leads to a ratio of the one pair to the two pair decay of the order $(\mu_e/m_{\pi^0})^2 \sim 10^{-5}$. *A priori* one would expect this ratio to be of the order one. Furthermore, the theorem provides a more general basis for the discussion of the relative probability of the reactions $\pi\rightarrow\mu+\nu$ and $\pi\rightarrow e+\nu$.

THE recent development of bubble chambers makes it possible to detect higher order decays of pions into leptons. Therefore it might be of interest to study processes like

$$\pi^0\rightarrow e^+ + e^-,\quad \pi^0\rightarrow e^+ + e^- + e^+ + e^-,\quad \pi^+\rightarrow e^+ + \nu,\quad \text{etc.}$$

It is the purpose of this note to show, on the basis of general principles, that in many cases the transition probability for the decay of spin-zero mesons into two leptons is reduced by a factor of the order $(\mu/m)^2$, where μ is the mass of the heaviest lepton emitted and m is the meson mass. The matrix element for a process of this kind can be written in the general form[1]

$$M(p,q)=\bar{\psi}_1(p)F(p,q)\psi_2(q)\phi(q-p). \tag{1}$$

Here $\phi(q-p)$ is the wave function of the decaying spin zero meson having four-momentum $q-p$, and $\bar{\psi}_1(p)$ and $\psi_2(q)$ are the corresponding wave functions of the emitted leptons. They obey the Dirac equations

$$i\bar{\psi}_1(p)\mathbf{p}=-\mu_1\bar{\psi}_1(p),\quad i\mathbf{q}\psi_2(q)=-\mu_2\psi_2(q),$$

where we use the notation $\mathbf{A}\equiv\gamma_\mu A_\mu=-i\beta\alpha_\kappa A_\kappa+i\beta A_0$ and $\bar{\psi}\equiv\psi^\dagger\gamma_4$; μ_1 and μ_2 are the masses of the emitted leptons. $F(p,q)$ is a spinor matrix which transforms as a scalar or pseudoscalar function depending on the relative parity of $\bar{\psi}_1(p)\psi_2(q)$ and $\phi(q-p)$. If a, b, etc., are scalar functions of $p^2=-\mu_1^2$, $q^2=-\mu_2^2$, and $pq=\frac{1}{2}(m^2-\mu_1^2-\mu_2^2)$, we can write the 4×4 spinor matrix $F(p,q)$ for reasons of covariance in the form

$$F(p,q)=\begin{cases} a+bm^{-1}\mathbf{p}+cm^{-1}\mathbf{q}+dm^{-2}(\mathbf{qp}-\mathbf{pq}) \\ \gamma_5\{a'+b'm^{-1}\mathbf{p}+c'm^{-1}\mathbf{q}+d'm^{-2}(\mathbf{qp}-\mathbf{pq})\}. \end{cases} \tag{2}$$

The first row is for scalar F and the second for pseudoscalar F. Introducing this into Eq. (1) we obtain by use of the Dirac equation

$$M=\begin{cases} \left[a+ib\dfrac{\mu_1}{m}+ic\dfrac{\mu_2}{m}+2d\dfrac{\mu_1\mu_2+pq}{m^2}\right] \\ \qquad\qquad\times\bar{\psi}_1(p)\psi_2(q)\phi(q-p) \\ \left[a'-ib'\dfrac{\mu_1}{m}+ic'\dfrac{\mu_2}{m}-2d'\dfrac{\mu_1\mu_2+pq}{m^2}\right] \\ \qquad\qquad\times\bar{\psi}_1(p)\gamma_5\psi(q)\phi(q-p). \end{cases} \tag{3}$$

We observe that the terms in F which are linear in γ_μ lead in the matrix element M to contributions proportional to the lepton masses. These contributions vanish for $\mu_1\rightarrow0$, $\mu_2\rightarrow0$, provided the coefficients do not become infinite in this limit. A behavior like that would be very unusual and can be excluded in all applications discussed below. If, for a special decay process, the function $F(p,q)$ contains only terms involving products of odd numbers of matrices γ_μ besides those which are *a priori* proportional to μ_1 or μ_2, it can be reduced to

$$F(p,q)=\begin{cases} bm^{-1}\mathbf{p}+cm^{-1}\mathbf{q}+f_1m^{-1}\mu_1+f_2m^{-1}\mu_2 \\ \gamma_5(b'm^{-1}\mathbf{p}+c'm^{-1}\mathbf{q}+f_1'm^{-1}\mu_1+f_2'm^{-1}\mu_2). \end{cases} \tag{4}$$

The corresponding matrix element therefore contains only terms proportional to the lepton masses μ_1 and μ_2. Here we have made the assumption that the open polygonal arc of lepton lines contains no internal lines corresponding to spin one half particles with masses different from μ_1 and μ_2. Thus every internal lepton line is associated with a propagation function

$$S_F(k)=(i\mathbf{k}-\mu)/(k^2+\mu^2),$$

where μ is either μ_1 or μ_2. After having performed the integrations over intermediate momenta, the four vector k_μ must be of the form $k_\mu=x(m,\mu_1,\mu_2)p_\mu+y(m,\mu_1,\mu_2)q_\mu$. We see that every S_F-function introduces a term proportional to γ_μ and a term which is *a priori* proportional to μ_1 or μ_2. For the applications it is only important that the masses of the virtual leptons in the open polygonal are smaller or of the same order of magnitude than those of the final particles.

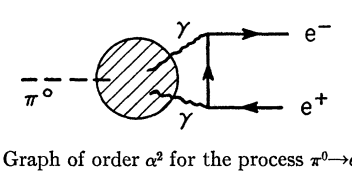

FIG. 1. Graph of order α^2 for the process $\pi^0\rightarrow e^+ + e^-$.

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[1] R. Oehme, Z. Naturforsch. 7a, 55 (1952).

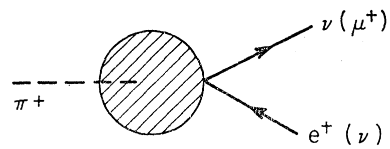

FIG. 2. Graph for the reactions $\pi^+\rightarrow e^+ + \nu$ and $\pi^+\rightarrow\mu^+ + \nu$.

On the basis of these arguments we can formulate the theorem: "The matrix element for the decay of spin-zero mesons with mass m into two leptons with masses μ_1 and μ_2 respectively contains only terms proportional to μ_1/m and μ_2/m, if in the open polygonal arc of lepton lines the number of matrices γ_μ at vertices plus the number of internal lines (S_F-functions) is odd. Should the lepton arc contain internal lines corresponding to virtual spin-one-half particles with masses μ_i being unequal to μ_1 and μ_2, then the matrix element contains also terms proportional to μ_i/m."

As a first application of this theorem we consider the reaction $\pi^0\rightarrow e^+ + e^-$. We assume only that the π^0 meson has spin zero and that the interaction occurs via the electromagnetic field, i.e., that the pair is produced by virtual photons. No specific assumption about the meson-photon interaction is necessary. Under these conditions the matrix element of first order in the fine structure constant α vanishes identically.[2] The process involving two virtual photons is in principle possible, but we have $N(\gamma_\mu)+N(S_F)=\text{odd}$. (See Fig. 1.) Therefore, according to the theorem formulated above, the matrix element is proportional to μ_e/m_{π^0} and the transition probability contains the factor

$$(\mu_e/m_{\pi^0})^2\alpha^4.$$

One can easily see that $N(\gamma_\mu)+N(S_F)=\text{odd}$ is independent of the number of virtual photons involved. Thus the contributions to the matrix element which are of higher order in α must be also proportional to μ_e/m_{π^0}. We conclude that the reaction $\pi^0\rightarrow e^+ + e^-$ is less probable by a factor of the order 10^{-5} than the decay into two pairs, for which Kroll and Wada[3] find

$$(\pi^0\rightarrow e^+ + e^- + e^+ + e^-)/(\pi^0\rightarrow\gamma+\gamma)\approx 3.5\times 10^{-5}.$$

For the ratio of one pair to double γ decay we obtain

$$r\equiv(\pi^0\rightarrow e^+ + e^-)/(\pi^0\rightarrow\gamma+\gamma)\sim 10^{-9}.$$

The measurements of Lindenfeld, Sachs, and Steinberger[4] lead to an upper limit of 5×10^{-4} for this ratio.

The only direct coupling between pseudoscalar π^0-mesons and electrons for which the above theorem is not applicable is: $i\bar{\psi}_e\gamma_5\psi_e\phi_{\pi^0}$. If we assume this hypothetical interaction to be about as strong as the Fermi coupling for β decay, it would lead to a ratio r of the order 10^{-6}.

[2] See reference 1; for vector mesons this is the most probable decay process.

[3] U. M. Kroll and W. Wada, Phys. Rev. **98**, 1355 (1955).

[4] Lindenfeld, Sachs, and Steinberger, Phys. Rev. **89**, 531 (1953).

Let us further consider the decay of a charged π meson into a μ meson and a neutrino and into an electron (positron) and a neutrino. Assuming that both processes are caused by the same interaction (see Fig. 2), one would expect from the difference of phase spaces a ratio

$$R\equiv(\pi\rightarrow e+\nu)/(\pi\rightarrow\mu+\nu)\sim 3.3,$$

whereas experimentally one finds that $R\lesssim 10^{-4}$.[5]

We can write the matrix element for both reactions in the form (1). If the coupling to the lepton field is such that the number of γ_μ matrices contained in F is odd, we find, according to our theorem,

$$R\sim 3.3(\mu_e/\mu_\mu)^2\sim 10^{-4}.$$

This result does not depend on whether the interaction is direct or occurs via nucleon pairs or other intermediate fields. We assume only that the charged π meson has spin zero and that the μ meson and the neutrino have spin one-half. Terms proportional to μ_ν/μ_μ have been neglected, because μ_ν is zero or at least small compared with μ_e. We see that vector and pseudovector lepton couplings lead to a ratio R which is compatible with the experimental limit. All other lepton couplings give in general $R\gtrsim 1$. Of course there are other selection rules which forbid both reactions under certain assumptions about the intermediate fields and couplings.[6] Application of the generalized Furry theorem for intermediate closed nucleon loops leads in some cases to a simultaneous reduction of both decay processes by a factor of the order α^2.[7]

A ratio R of the order 10^{-4} for pseudovector lepton couplings has been found by several authors.[8] They have calculated the matrix elements for both decay processes in first order of the meson nucleon coupling, assuming that the interaction occurs via an intermediate nucleon field. According to our theorem this result holds in a more general sense. We finally remark that for some of the lepton couplings which lead to a ratio $R\sim 10^{-4}$ the reaction $\pi\rightarrow e+\nu+\gamma$ should be more probable by a factor of the order $(m_\pi/\mu_e)^2\alpha\sim 10^2$ than the decay $\pi\rightarrow e+\nu$. For the process $\pi\rightarrow e+\nu+\gamma$ our theorem is not applicable.

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[5] S. Lokanathan and J. Steinberger (private communication via J. Steinberger); H. Friedman and J. Rainwater, Phys. Rev. **81**, 644 (1951).

[6] See, for example, C. N. Yang and J. Tiomno, Phys. Rev. **79**, 485 (1950).

[7] C. B. van Wyck, Phys. Rev. **80**, 487 (1950); A. Pais and R. Jost, Phys. Rev. **87**, 871 (1952). R. Oehme, in *Kosmische Strahlung*, edited by W. Heisenberg (Springer-Verlag, Berlin, 1953), p. 557.

[8] For a survey of the literature and for references see L. Michel, in *Progress in Cosmic Ray Physics*, edited by G. J. Wilson (Interscience Publishers, Inc., New York, 1952), pp. 176–181. More recently, M. Gell-Mann and V. Telegdi have made calculations on this subject (unpublished; private communication from V. Telegdi).