Associated Photons in K-Particle Decays Observed with a Multiplate Cloud Chamber*

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The existence of photons among the decay products of K-mesons is confirmed [see Bridge, Courant, DeStaebler, and Rossi, Phys. Rev. **91**, 1024 (1953)]. If these photons arise through a two-body decay process, it is not possible to assume that they are produced directly as the neutral product; however, they can be accounted for quite naturally by assuming that the neutral product is a π^0-meson. The frequency with which the photons are observed is low and for this reason it is not possible to interpret all the decay events observed in the cloud chamber in terms of a single two-body decay process in which the neutral particle is a π^0-meson. The statistical arguments leading to this conclusion are given.

I. INTRODUCTION

RECENT observations of K-particles by means of multiplate cloud chambers have been reported by the Ecole Polytechnique[1] and the M. I. T. groups.[2,3] The general features of the events and a bibliography of previous work will be found in these two publications. In this paper we consider K-particles which stop in the cloud chamber and so decay at rest. This type of decay is classified phenomenologically as an S-event. To identify such an event we have required either (a) that the charged secondary product traverse at least 10 g cm^{-2} of lead without multiplication, or (b) that the neutral decay product be detected.

In all of the analysis which follows, we assume in each case a two-body decay scheme (however, not necessarily the same decay scheme for all events). The main justification of this assumption is that the range spectrum of the charged secondaries of the decay events does not contain many short-range cases. These would be expected if the decay process were a three-body one.

The experimental data consist of 33 S-events, all of which were obtained using a plate assembly of 11 lead plates, each 1.1 radiation lengths thick. Among these 33 examples, we have found 9 in which there is an electron cascade close to the point of decay. The present paper presents a study of this particular phenomenon. Additional details concerning the events as well as some more recent data obtained with a different plate assembly have been reported previously.[3]

II. ELECTRON CASCADES

Figures 1, 2, and 3 are pictures of S-particle decay events in which a related electron cascade is observed. In Fig. 1 (25178), the decay occurs in the fourth plate from the top, and the electron cascade manifests itself as a spray of five electrons emerging directly from the plate of decay in a general direction opposite to that of the charged decay product.

In Fig. 2 (32531), the decay occurs in the sixth plate, and the electron cascade is seen to emerge from the seventh plate. The nonionizing link between the point of decay and the plate where materialization occurs is strong evidence that the cascade is initiated by a photon. In this case the direction of the photon is

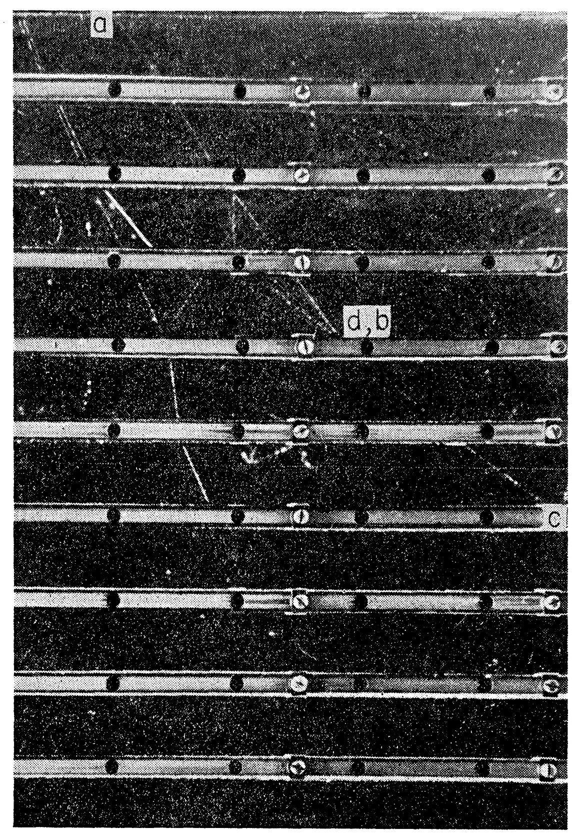

FIG. 1. Example of associated S-event and electron cascade. a is the point at which the K-meson enters the chamber; b is the point of decay; c is the point at which the charged secondary leaves the chamber; and d is the point at which the electron cascade is initiated.

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[1] Gregory, Lagarrigue, Leprince-Ringuet, Muller, and Peyrou, Nuovo cimento **11**, 292 (1954).
[2] Bridge, Peyrou, Rossi, and Safford, Phys. Rev. **90**, 921 (1953).
[3] Bridge, Courant, Dayton, DeStaebler, Rossi, Safford, and Willard, Nuovo cimento **12**, 81 (1954).

$180°\pm5°$ from the line of flight of the charged decay product.

Figure 3 (2877), is similar to Fig. 2 in that there is a nonionizing link between the point of decay and the point where materialization occurs. In this case, however, the angle between the trajectory of the photon and that of the charged decay product is $155°\pm5°$.

Before discussing the significance of the observation of the electron cascades, it is necessary to establish unambiguously that the observed cascades are related to the S-particle decays. To do this we must evaluate the probability of chance association, i.e., the probability of finding an unrelated electron cascade that has the characteristics of direction, proximity, and general behavior which might lead us to identify it erroneously as a manifestation of the neutral decay product.

For this purpose we have examined the pictures exhibiting S-particle decay and counted the number of cascades observed anywhere in the chamber. Each of these electron cascades shows a general direction of propagation, and we assume that the origin of the initiating photon must be somewhere within a cone of 15° half-angle from this inferred line of propagation.

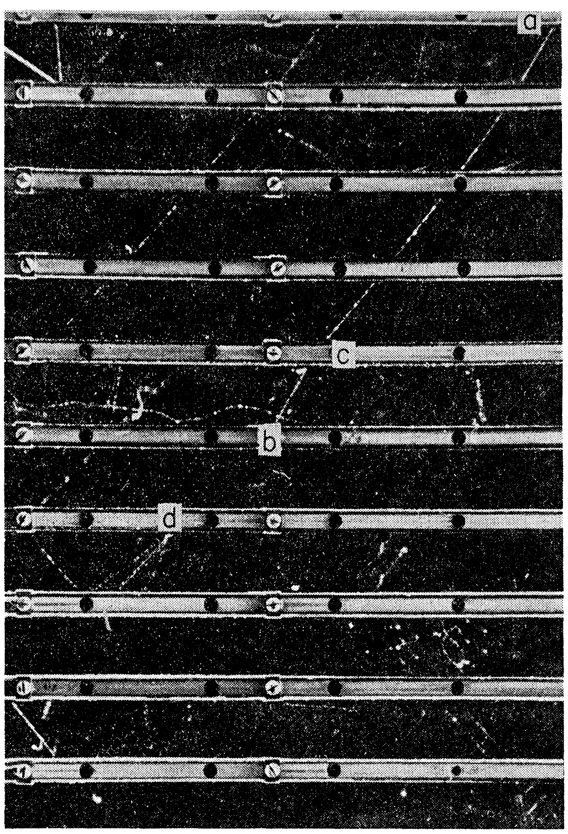

FIG. 2. Example of associated S-event and electron cascade. a is the point at which the K-meson enters the chamber; b is the point of decay; c is the point at which the charged secondary leaves the chamber; and d is the point at which the electron cascade is initiated.

Furthermore, we assume that the origin of the initiating photon must be within two radiation lengths (2 plates of our chamber) from the point where the electron cascade is observed. In this manner we define a volume of material in the cloud chamber in which a fortuitous S-event would erroneously be associated with the electron cascade.

To obtain the probability for a chance association between an electron cascade and an S-particle decay we take the ratio of the volume of material available for making such an association and that available for the S-event to be observed at all. This ratio has a value of 0.0013, and it is a measure of the probability that there be a chance association between a particular S-event and an electron cascade. This figure is based on 31 suitable examples of S-decay in which related electron cascades might have been observed if they had occurred. The probability that all nine of our cases of cascades associated with S-decay are due to chance is

$$P=(0.0013)^9=10^{-26}.$$

This is the probability that none of the observed cascades are causally related to the phenomenon of S-decay. The probability that some of our nine cases are not genuine is

$$P=1-(1-0.0013)^{31}=0.04.$$

It therefore seems likely that most of our events which have been identified as exhibiting electron cascades in association with S-particle decay have been correctly interpreted on this point.

III. NEUTRAL PARTICLE AS π^0-MESON

In this section, we investigate the possibility that the observed electron cascades originate from the decay photons of π^0-mesons under the assumption that the π^0-meson is produced in a two-body decay process. To do this we compare the observed angular distribution of the photons with that expected under the above assumptions.

We define φ as the angle between the trajectory of the charged decay product and that of the observed photon. Let ϑ^* be the angle between the direction of the photon and the direction of flight of the π^0-meson as viewed in the frame of reference in which the π^0-meson is at rest. The angles φ and ϑ^* are related by the expression

$$-\gamma_0 \tan\varphi=\sin\vartheta^*/(\cos\vartheta^*+\beta_0),$$

where β_0 is the velocity of the π^0-meson in the laboratory system, and $\gamma_0=1(1-\beta_0^2)^{\frac{1}{2}}$.

It is evident from the above relation that the photon which in the frame of reference of the π^0-meson is emitted in a forward direction ($0°\leqslant\vartheta^*\leqslant90°$) will appear in the laboratory within a cone of semi-apex angle $\vartheta_0=\tan^{-1}[1/(\gamma_0\beta_0)]$. For the purpose of graphically presenting the angular probability distribution of

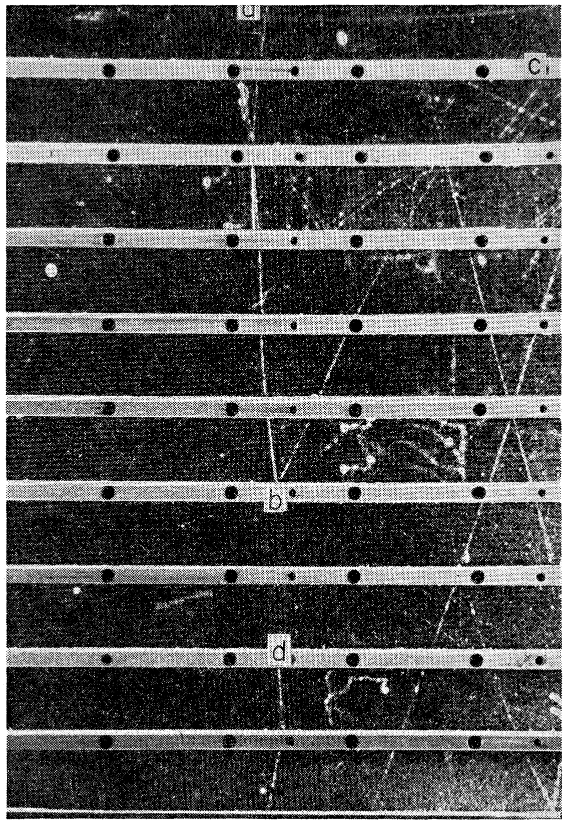

FIG. 3. Example of associated S-event and electron cascade. a is the point at which the K-meson enters the chamber; b is the point of decay; c is the point at which the charged secondary leaves the chamber; and d is the point at which the electron cascade is initiated.

π^0-meson decay photons as observed in the laboratory, we define the variable x as follows: $x=\gamma_0 \tan\vartheta$. Let us construct a cone of semi-apex angle ϑ with the trajectory of the neutral decay particle as its axis. Consider the circle defined by the intersection of such a cone of height γ_0 with a plane normal to its axis. The value of the variable $x(\vartheta)$ then is equal to the radius of this circle. We define $W(x)dx$ as the probability that the variable x lie in dx at x. $W(x)dx$ can be considered as the probability that a photon traverse the above-mentioned circle in a ring of width dx at the radius x. Figure 4 shows the differential probability distribution $W(x)$ of the forward photon ($0° \leqslant \vartheta^* \leqslant 90°$) from the decay of a π^0-meson of momentum $p_{\pi^0}=200$ Mev/c. (In this case, our definition gives $x=1.78 \tan\vartheta$.)

Although the cone containing all photons emitted in the forward direction has a semiapex angle of 34° (for $p_{\pi^0}=200$ Mev/c), graphical integration of $W(x)$ leads to the result that half of these photons are contained in a coaxial cone of semi-apex angle 20°.

In the analysis which follows, only the first seven of the nine observed decay photons are employed. This is due to the fact that the last two cases were added to our collection after the statistical analysis had been performed, and that the determination of the angle φ in one of them is not very precise.

In the frame of reference where the π^0-meson is at rest, the differential angular probability distribution for the forward-going decay photon is

$$W^*(\vartheta^*)d\vartheta^*=\sin\vartheta^* d\vartheta^*.$$

Since $W^*(\vartheta^*)d\vartheta^*=d(-\cos\vartheta^*)$, we introduce the variable z according to the definition $z=\cos\vartheta^*$. The resulting distribution is the rectangular distribution, i.e., it has a constant value in the range $0\leqslant z\leqslant 1$, and the value zero elsewhere.

We now define the quantity K_n as follows:

$$K_n=\sum_{i=1}^{n} z_i.$$

Since z_i can assume any value between 0 and 1, it is clear that K_n may have values between 0 and n.

In what follows, we will need the differential distribution W_n of K_n. It can be shown that the probability $W_n(K_n)dK_n$ of finding K_n in dK_n is given by[4,5]

$$W_n(K_n)dK_n=\frac{dK_n}{(n-1)!}\sum_{r=0}^{m}(-1)^r\binom{n}{r}(K_n-r)^{n-1},$$

for values of K_n between m and $m+1$, where

$$\binom{n}{r}=n!/(n-r)!r!$$

is the rth binomial coefficient of the power n. The probability function $W_7(K_7)$ is shown in Fig. 5. One sees that it has a maximum at $K_7=3.5$, which is thus

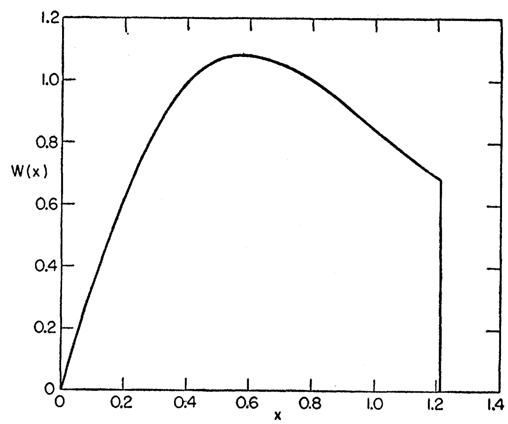

FIG. 4. The differential probability $W(x)$ that in the laboratory the more energetic photon emitted in the decay of a π^0-meson of momentum 200 Mev/c has a particular value of x. The variable x is related to ϑ, the laboratory angle between the line of flight of the photon and the π^0-meson, by the equation $x=1.78 \tan\vartheta$.

[4] M. G. Kendall, *The Advanced Theory of Statistics* (Charles Griffin and Company Ltd., London, 1952), Vol. 1, p. 240 ff.

[5] H. Cramer, *Mathematical Methods of Statistics* (Princeton University Press, Princeton, 1951), p. 244 ff.

TABLE I. The experimentally observed values of K_7 and the corresponding values of $P(\Delta K_7)$ computed as described in the text for several values of π^0-momentum, p_{π^0} (Mev/c). $P(\Delta K_7)$ represents the *a priori* probability that K_7 deviates from the most probable value by at least the amount observed.

S-particle with photon	Space angle φ (in lab system)	$p_{\pi^0}=150$	$p_{\pi^0}=200$	$p_{\pi^0}=250$	$z_i=\cos\vartheta_i^*$ $p_{\pi^0}=300$	$p_{\pi^0}=350$	$p_{\pi^0}=400$	$p_{\pi^0}=450$
1	180°	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	178°	0.999	0.998	0.997	0.995	0.994	0.993	0.991
3	178°	0.999	0.998	0.997	0.995	0.994	0.993	0.991
4	157°	0.567	0.392	0.210	0.000	0.000	0.000	0.000
5	140°	0.000	0.170	0.340	0.456	0.566	0.607	0.677
6	155°	0.510	0.318	0.120	0.000	0.120	0.170	0.240
7	162°	0.708	0.588	0.470	0.340	0.210	0.120	0.000
	$K_7=\sum_1^7 z_i$	4.78	4.46	4.13	3.79	3.88	3.88	3.89
	$P(\Delta K_7)$	0.1	0.2	0.4	0.6	0.6	0.6	0.6

the most likely value of K_7. We can now compute the function

$$P(\Delta K_7)=\int^{\bar{K}_7-\Delta K_7} W(K_7')dK_7' +\int_{\bar{K}_7+\Delta K_7}^{7} W(K_7')dK_7'.$$

This function represents the total probability of finding for $\sum_1^7 z_i$ a value that deviates from the most probable value, $\bar{K}_7$, ($\bar{K}_7=3.5$) by more than a given amount, ΔK_7.

In Table I, we have tabulated the experimental values of K_7 and the corresponding calculated values of $P(\Delta K_7)$ for six different assumed values of p_{π^0}. As explained above, the values of $P(\Delta K_7)$ given in Table I represent the *a priori* probability of finding a value of K_7 which deviates from the most probable value as much or more than does the experimental value. If $P(\Delta K_7)$ is sufficiently small we may conclude that the experimentally observed angular distribution is not related to that on which the probability calculations are based. It is seen that the angular distribution of the photons observed in the cloud chamber is consistent with that expected from π^0-mesons of momentum between 150 and 400 Mev/c.

We should point out that when we transform the observed angles into the rest system of the π^0-meson some of the photons were emitted with $\vartheta^*>90°$, i.e., in the backward cone. The number of these cases varies with the assumed value of p_{π^0}. In these events the value of z_i for the forward photon was used in the analysis. The estimated errors in the measurement of the space angles would allow the quantity $P(\Delta K_7)$ to vary by as much as ± 0.3 for $p_{\pi^0}=150$ Mev/c. This does not alter the conclusions of the analysis.

If the electron cascades are interpreted in terms of a π^0-meson, one might expect to detect both photons occasionally. However, in the nine cases of S-decay where related photons have been detected we have not seen evidence for a second photon which could possibly be the other photon from the decay of the π^0-meson. We have examined each of these nine cases to determine the trajectory of this second photon (assuming a momentum of 200 Mev/c for the π^0-meson) and then calculated the probability for this photon to be detected. We find that because of the geometry of our nine events the likelihood of detecting the second photon is essentially zero in all cases. Thus the fact that we have not observed the second photon from the decay of a π^0-meson in any of our examples is entirely consistent with the assumption that the observed photons originate in the decay of π^0-mesons created in the S-events.

We should emphasize here that the treatment of this section is merely a test to see whether our data would fit the hypothesis of a π^0-meson as a decay product in two-body decay. Our data may in fact fit some other hypothesis equally well. For example, a three-body decay in which one of the neutral particles is a photon may well be possible. However, the two-body decay scheme which has a single photon as the neutral product is certainly ruled out.

IV. PROBABILITY OF DETECTING π^0-MESON AS DECAY PRODUCT

In this section, we now examine the hypothesis that a π^0-meson is created in each of our 33 examples of S-decay.

We shall approach this problem from the point of view that the number of cases in which we have detected a photon is small compared to the number of cases we would expect to see under the assumption that a π^0-meson is created in each decay event.

Thus, our treatment of the data is to be an attempt to account for the small number of photons observed in our collection of 33 examples. We shall consistently use approximations which underestimate the probability of detecting photons, and which exaggerate the number of observed photons which should be included in the analysis. If this biased approach yields the result that our data are not compatible with the hypothesis that a

TABLE II. Numerical values of b_m and q_m for 33 S-events. b is the detection probability for an electron cascade and $q_m=1-b_m$.[a]

m	b_m	q_m	m	b_m	q_m	m	b_m	q_m
1*	0.00	1.00	12	0.38	0.62	23	0.67	0.33
2	0.00	1.00	13*	0.39	0.61	24*	0.69	0.31
3	0.00	1.00	14	0.39	0.61	25	0.69	0.31
4	0.06	0.94	15*	0.48	0.52	26	0.74	0.26
5	0.10	0.90	16	0.48	0.52	27*	0.75	0.25
6*	0.19	0.81	17	0.51	0.49	28	0.75	0.25
7	0.20	0.80	18	0.51	0.49	29	0.76	0.24
8	0.29	0.71	19*	0.52	0.48	30	0.82	0.18
9	0.29	0.71	20	0.55	0.45	31*	0.87	0.13
10*	0.38	0.62	21	0.63	0.37	32	0.87	0.13
11	0.38	0.62	22	0.66	0.34	33	0.95	0.05

[a] The numbers indicated by a star (*) correspond to examples in which associated electron cascades were found.

π^0-meson is created in each decay, we may certainly conclude that the premise is incorrect.

In proceeding with the analysis of the individual events, this over-all approach has the following direct consequences:

(a) We shall accept as *bona fide* examples of associated electron cascades those cases in which the charged secondary from the decay is not under observation in the cloud chamber for a range sufficient to identify the event independently as an S-event (the charged secondary should be observed to penetrate at least 10 g cm^{-2} of lead within the illuminated region of the chamber[1]). Among our examples one case falls into this category.

(b) Although the analysis is based on the detection only of the forward photon (which is more energetic), we shall accept as examples of associated electron cascades those cases in which we have detected the backward-going photon. Among our examples two cases fall into this category.

(c) The evaluation of the probability of detecting the forward photon in each of our 33 cases is based on an assumed momentum of 200 Mev/c of the π^0-meson. This is the lowest value that one can ascribe to the momentum of the π^0-mesons if one assumes that the sample is homogeneous. For in one case, the charged secondary penetrates 85 g cm^{-2} of lead before it leaves the chamber, and if it is a π-meson, its momentum exceeds 200 Mev/c. A decrease in this momentum increases the apex angle of the cone in which the decay photons must lie, and, since the portion of the cone that is found in the illuminated region of the cloud chamber usually decreases with increasing apex angle, the probability of finding the decay photon in the cloud chamber decreases with decreasing energy of the π^0-meson.

To determine the probability that we should see n associated electron cascades among the 33 examples of S-decay in our collection, we must first calculate the probability for detecting the photon in each case. This calculation must take into account the following: (1) the particular geometry of the event; (2) the probability that a photon will undergo materialization; and (3) the probability that at least one of the created electrons will emerge from the plate where materialization occurs. We shall outline the method used for this calculation by considering the above points in order.

(1) *Geometry considerations.*—In each case we have estimated the fraction of decay photons which were available for materialization in the illuminated region of the cloud chamber. This fraction in general has a new value for each lead plate which is to be considered.

(2) *Materialization probability.*—The materialization probability of the photon was evaluated for each lead plate on the basis of the estimated average angle of incidence of the photon to the plate. The energy of the photon was taken as 100 Mev. The materialization probability in lead for this photon is 0.6 per radiation length.

(3) *Probability of observing at least one of the electrons.*—This part of the calculation takes into account the probability of catastrophic (radiative) energy loss by the electrons of the first generation in the cascade.[6] In most cases this results in a value of the electron detection probability of about 0.95.

The evaluation of the photon-detection probability as outlined above is consistent with the general approach detailed at the beginning of this section. In part 1 of this outline, we have made the assumption that the photon probability density is roughly constant within the π^0-meson decay cone. This approximation results in an exaggeration of the probability density along the periphery of the cone in which the photons lie. The consequence of this is to overestimate slightly the probability that the photon leave the illuminated region of the cloud chamber, thus causing the likelihood of detection by materialization to be underestimated. In part 2, we base the materialization probability on that

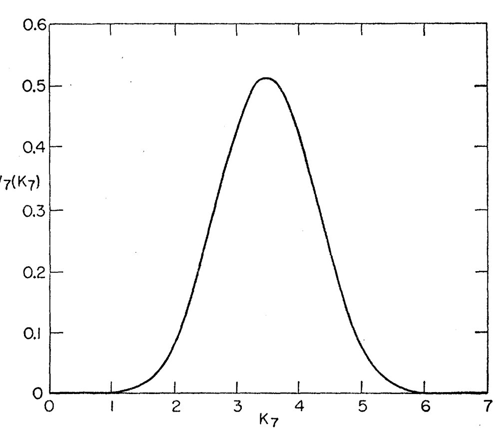

FIG. 5. The differential probability $W_7(K_7)$ plotted *versus* K_7.

[6] This calculation was performed by H. C. DeStaebler and has been adjusted to give agreement with R. R. Wilson's Monte Carlo histories of photon-initiated electron cascades in lead. Professor Wilson kindly lent us the original histories; see H. C. DeStaebler, Jr., Massachusetts Institute of Technology thesis, 1954 (unpublished).

for a photon of 100-Mev energy. The energy of the decay-photon from a π^0-meson of momentum 200 Mev/c varies from 121 Mev to 221 Mev within the cone. Since the materialization probability increases with increasing energy we thus underestimate the over-all materialization probability.

We now define the quantity b_m as the overall probability to observe an electron cascade in the picture of the mth decay event, and the quantity

$$q_m = 1 - b_m,$$

representing the probability of *not* observing an electron cascade in the mth picture. The quantities b_m and q_m of each of our 33 S-particle decays are tabulated in Table II.

The product

$$(b_1+q_1)(b_2+q_2)\cdots(b_{32}+q_{32})(b_{33}+q_{33})$$

is equal to one, since each term (b_m+q_m) is equal to one. If we express this product as a polynomial we may

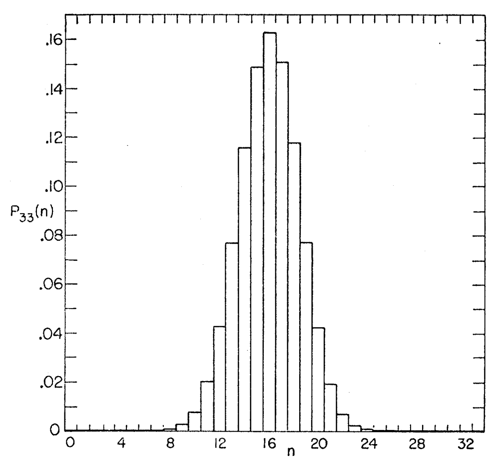

FIG. 6. The probability of observing exactly n electron cascades in the experimental data as a function of n.

interpret the sum of all terms having dimensions $b^n q^{33-n}$ as the probability that we should find n examples of associated electron cascades among our 33 events. Thus by arranging the terms appropriately we may find the numerical value of $P_{33}(n)$, which we define as the

TABLE III. Numerical values of the quantity $P_{33}(n)$, the probability of observing exactly n electron cascades in the sample of 33 S-events.

n	$P_{33}(n)$	n	$P_{33}(n)$	n	$P_{33}(n)$
1	6.44×10^{-11}	12	4.31×10^{-2}	23	2.24×10^{-3}
2	2.01×10^{-9}	13	7.70×10^{-2}	24	5.47×10^{-4}
3	3.71×10^{-8}	14	1.16×10^{-1}	25	1.04×10^{-4}
4	4.76×10^{-7}	15	1.45×10^{-1}	26	1.50×10^{-5}
5	4.40×10^{-6}	16	1.63×10^{-1}	27	1.58×10^{-6}
6	3.09×10^{-5}	17	1.51×10^{-1}	28	1.10×10^{-7}
7	1.70×10^{-4}	18	1.18×10^{-1}	29	4.56×10^{-9}
8	7.52×10^{-4}	19	7.73×10^{-2}	30	8.23×10^{-12}
9	2.72×10^{-3}	20	4.26×10^{-2}	31	~0
10	8.15×10^{-3}	21	1.94×10^{-2}	32	~0
11	2.05×10^{-2}	22	7.32×10^{-3}	33	~0

probability of detecting n associated photons among our 33 S-particles.

Table III gives the numerical values obtained for $P_{33}(n)$, and Fig. 6 shows these values plotted. By inspection of Fig. 6 or Table III, it is apparent that we should have seen about 16 cascades among our 33 S-decay events. Instead, we only found 9 cases of associated cascades, a number which deviates by 7 units from the expected value. The total probability of a deviation equal or larger than 7 is

$$\sum_{1}^{9} P_{33}(n) + \sum_{23}^{33} P_{33}(n) = 3.3\times10^{-3}.$$

Thus the hypothesis that each of our events has given rise to a π^0-meson as the neutral decay product is ruled out.

V. CONCLUSIONS

We have presented conclusive evidence that electron cascades are related to the S-decays in the events in which they were observed (Sec. II). We have shown that the observed cascades *may* be due to the creation of a π^0-meson as the neutral decay product (Sec. III). We have shown that our data are *not* consistent with the assumption that a π^0-meson is created in each decay (Sec. IV).

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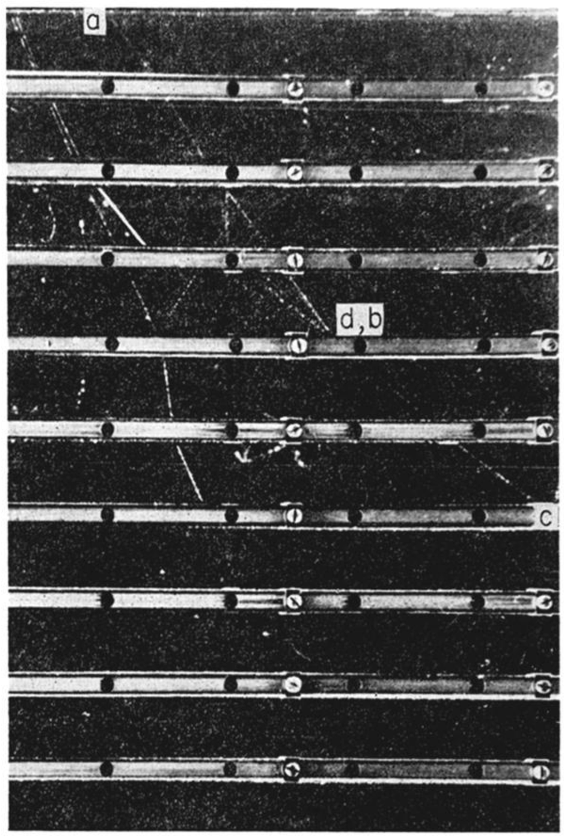

FIG. 1. Example of associated S-event and electron cascade. a is the point at which the K-meson enters the chamber; b is the point of decay; c is the point at which the charged secondary leaves the chamber; and d is the point at which the electron cascade is initiated.

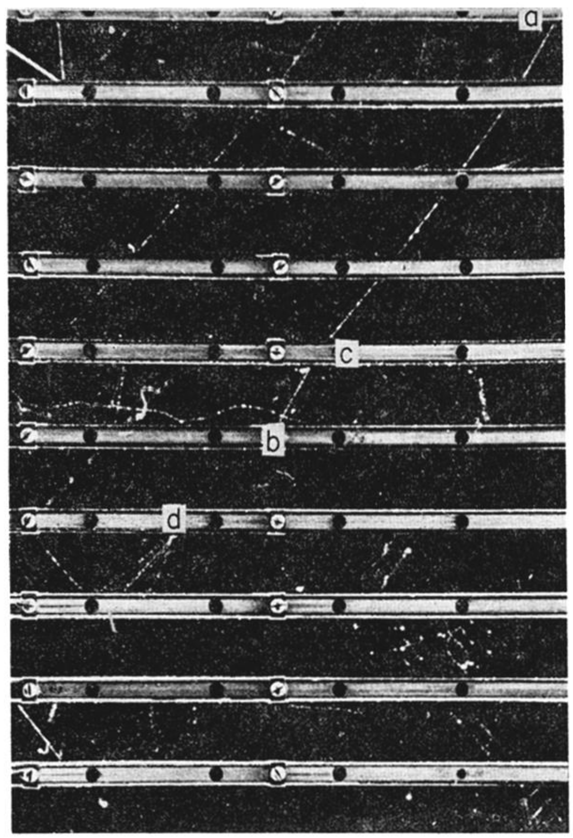

FIG. 2. Example of associated S-event and electron cascade. a is the point at which the K-meson enters the chamber; b is the point of decay; c is the point at which the charged secondary leaves the chamber; and d is the point at which the electron cascade is initiated.

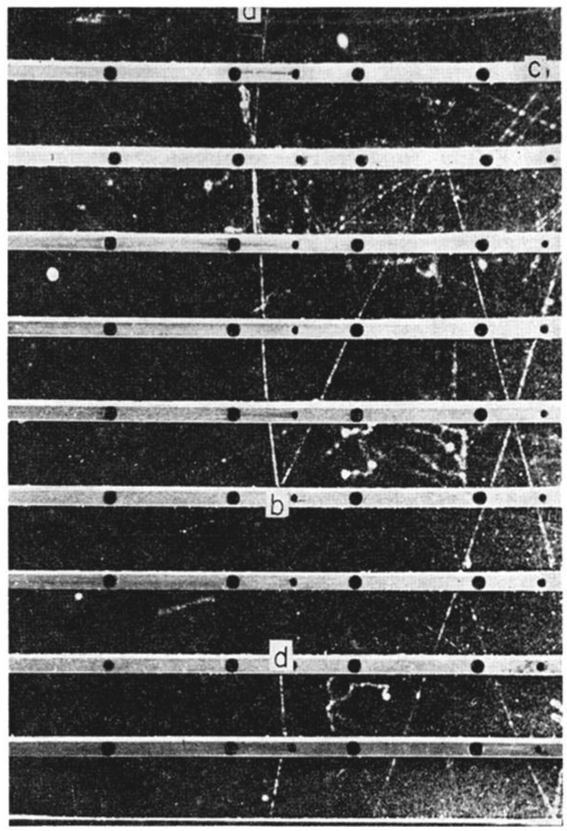

FIG. 3. Example of associated *S*-event and electron cascade. *a* is the point at which the *K*-meson enters the chamber; *b* is the point of decay; *c* is the point at which the charged secondary leaves the chamber; and *d* is the point at which the electron cascade is initiated.