show that, in the laboratory angle interval of interest, the primary effect is to suppress the cross section and not to change the angular distribution. Consequently we ignore the effect of such scattering in calculating a minimum a/b.

(3) The isotropic part of the free proton angular distribution will, if anything, be larger than the isotropic part of the elastic deuteron angular distribution (before modification by the deuteron form factor). Again we take the extreme case of no difference in the two distributions in calculating a lower limit for a/b.

On the basis of these arguments, and using our measured cross-section ratio $R(168°/124°)=0.41\pm0.12$, we calculate $a/b\geq 0.35^{+0.25}_{-0.06}$.

If we take the impulse approximation at its face value, use Chew and Lewis[2] values for $I^2(\theta)$, and make no scattering corrections, we calculate $a/b=0.80^{+0.70}_{-0.20}$.

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Energy Distribution of γ Rays from π^0 Decay*

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It is shown that the γ-ray energy distribution resulting from the decay of π^0 mesons produced in a target bombarded by a high-energy particle beam is related in a simple manner to the differential π^0 production cross section, for sufficiently high energies of the γ's ($\gtrsim$500 Mev). An expression is obtained for the π^0 production cross section in terms of the γ-ray energy distribution. This result is extended to the case of an arbitrary two-body decay, for which an expression is obtained for the production cross section of the primaries in terms of the energy distribution of the secondaries emitted in the decay.

I. INTRODUCTION

INFORMATION about the π^0 meson production in a target bombarded by a high-energy particle beam can be obtained from a measurement of the energy distribution of the γ rays from the π^0 decay at various angles to the beam. At incident energies in the range of 200–400 Mev,[1] the interpretation of the γ-ray spectrum is very complicated, because at each angle of observation, a wide range of angles of the π^0's is involved. However, with increasing energy of the incident particles and of the resulting γ rays from π^0 production, the maximum possible angle between the observed γ and the decaying π^0 becomes very small, and it can be assumed that the π^0 differential production cross section remains approximately constant over the small range of π^0 angles involved. It will be shown that in this high-energy region (γ energy$\gtrsim$500 Mev), the π^0 cross section can be expressed in a simple manner in terms of the γ-ray energy spectrum. A similar expression will also be obtained for an arbitrary two-body decay for the production cross section of the primaries in terms of the energy distribution of the secondaries which are emitted in the decay.

II. RELATION BETWEEN π^0 PRODUCTION CROSS SECTION AND γ-RAY ENERGY SPECTRUM

The velocity v_π of the π^0 in the laboratory system is related as follows[2] to the laboratory angle ψ between the observed γ and the π^0:

$$\bar{k}=\gamma_\pi k(1-v_\pi\cos\psi), \tag{1}$$

where k is the energy of the γ-ray in the laboratory system, $\bar{k}$ is its energy in the π^0 rest system, and $\gamma_\pi=(1-v_\pi^2)^{-\frac{1}{2}}$. Upon squaring Eq. (1) and solving for v_π, one obtains

$$v_\pi=\frac{k^2\cos\psi\pm\bar{k}(\bar{k}^2-k^2\sin^2\psi)^{\frac{1}{2}}}{k^2\cos^2\psi+\bar{k}^2}. \tag{2}$$

The total energy E_π of π^0 is given by

$$E_\pi=\frac{m_\pi(k^2\cos^2\psi+\bar{k}^2)}{k[\bar{k}\pm\cos\psi(\bar{k}^2-k^2\sin^2\psi)^{\frac{1}{2}}]}, \tag{3}$$

where m_π=mass of π^0. It is seen that for a given ψ, there are in general two values of E_π. Moreover, since the expression under the radical must be positive, ψ is

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[1] See, for example, A. Silverman and M. Stearns, Phys. Rev. **88**, 1225 (1952); G. Cocconi and A. Silverman, Phys. Rev. **88**, 1230 (1952); Goldschmidt-Clermont, Osborne, and Scott, Phys. Rev. **89**, 329 (1953); Phys. Rev. **97**, 188 (1955); Walker, Oakley, and Tollestrup, Phys. Rev. **89**, 1301 (1953); Marshall, Marshall, Nedzel, and Warshaw, Phys. Rev. **88**, 632 (1952); R. H. Hildebrand, Phys. Rev. **89**, 1090 (1953).

[2] It is assumed that the units are such that $c=1$.

$\leq\psi_{max}$, where ψ_{max} is given by

$$\psi_{max}\equiv\sin^{-1}(\bar{k}/k). \quad (4)$$

ψ_{max} becomes very small at high energies k; thus $\psi_{max}=7.8°$ for $k=500$ Mev, and 3.9°, for $k=1$ Bev. Hence the spectrum of γ rays above ~500 Mev depends only on the production cross-section for π^0 at about the same angle as the angle of observation of the γ rays. This result is not inconsistent with the fact that π^0 mesons of all energies emit some γ rays at large angles to their direction of motion, and even backwards, because these large angle γ's have low energy in the laboratory system ($\lesssim$100 Mev). Equation (4) corresponds to the case in which the γ ray is emitted at right angles to the direction of π^0 in the π^0 rest system. The transverse momentum in the laboratory system is then $\bar{k}$, so that: $\sin\psi=\bar{k}/k$.

Figure 1 shows E_π *vs* ψ for photon energies $k=200$, 400, 600, and 800 Mev. The lower branch of the curves corresponds to the + sign in Eq. (3); the upper branch corresponds to the − sign. The smallest E_π is obtained for $\psi=0°$ and is given by

$$E_{\pi,\min}=k(1+\bar{k}^2/k^2), \quad (5)$$

which approaches k as k is increased. For $\psi=\psi_{max}$, we have $E_\pi=2k$. The largest E_π is $E_{\pi,\max}=\infty$, and is obtained for $\psi=0°$ on the upper branch of the curves.

We now obtain an expression for $P(k,\theta_\gamma)$, the number of γ rays per unit energy interval dk and per unit solid angle at an angle θ_γ to the incident beam, in terms of $f(E_\pi,\theta_\pi)$, the differential π^0 production cross section per unit energy interval dE_π and per unit solid angle at an angle θ_π to the beam (see Fig. 2). The number of γ rays in a small energy interval dk can be written

$$P(k,\theta_\gamma)dk=nN\sum\int d\theta_\pi\int d\varphi_\pi f(E_\pi,\theta_\pi) \times\sin\theta_\pi(2/4\pi)JdE_\pi, \quad (6)$$

where n = number of incident particles, N = number of

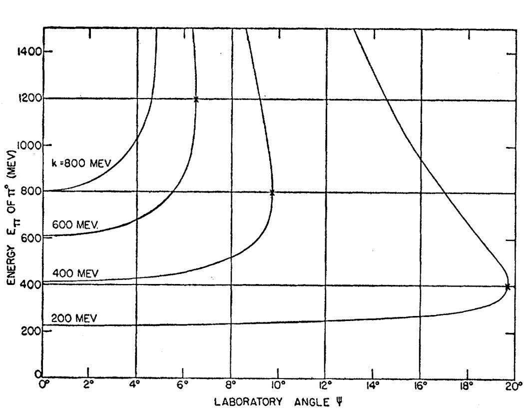

FIG. 1. Total energy E_π of π^0 as a function of the angle ψ between π^0 and γ for various γ energies k. The crosses correspond to the maximum angle ψ_{max}.

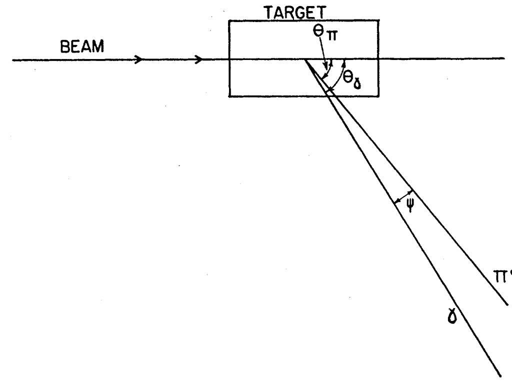

FIG. 2. Relationship between the angles involved in Eqs. (6) and (7). The lines marked π^0 and γ indicate the directions of motion of the π^0 meson and the γ ray.

target nuclei per cm², φ_π = azimuthal angle of π^0, $(2/4\pi)$ is the number of γ rays per steradian in the π^0 rest system, and J is the Jacobian for the transformation from the π^0 rest system to the laboratory. In Eq. (6), the integrations extend over the regions of θ_π and φ_π which contribute to the γ intensity at θ_γ (i.e., for which $\psi\leq\psi_{max}$); dE_π is the interval of E_π corresponding to dk at (θ_π,φ_π); the sum refers to the fact that there are, in general, two such intervals for given values of θ_π and φ_π for which $\psi\leq\psi_{max}$. Equation (6) can be evaluated more conveniently by using the direction of the γ as the polar axis. We have $J=\partial\cos\bar{\psi}/\partial\cos\psi$, where $\bar{\psi}$ is the angle between π^0 and γ in the π^0 rest system. One obtains

$$P(k,\theta_\gamma)=nN\sum\int_0^{\psi_{max}} f(E_\pi,\theta_\gamma)\frac{\partial E_\pi}{\partial k}\left(\frac{2}{4\pi}\right)\frac{\partial\cos\bar{\psi}}{\partial\cos\psi} \times(2\pi)\sin\psi d\psi. \quad (7)$$

Here $f(E_\pi,\theta_\gamma)$ has been used instead of $f(E_\pi,\theta_\pi)$ on the assumption that ψ_{max} is small enough so that $f(E_\pi,\theta_\pi)$ does not vary appreciably over the range of θ_π. Thus Eq. (7) is valid for $k\gtrsim500$ Mev (where $\psi_{max}=7.8°$), although it may hold with reasonable accuracy down to $k\sim200$ Mev ($\psi_{max}\sim20°$). An exception may occur near the forward direction[3] if $f(E_\pi,\theta_\pi)$ near $\theta_\pi=0°$ varies rapidly over a region of angles of the order of ψ_{max}.

In order to obtain $\partial\cos\bar{\psi}/\partial\cos\psi$, we note that

$$\cos\bar{\psi}=(\cos\psi-v_\pi)/(1-v_\pi\cos\psi). \quad (8)$$

Differentiation of (8) gives

$$\frac{\partial\cos\bar{\psi}}{\partial\cos\psi}=\frac{1}{\gamma_\pi{}^2(1-v_\pi\cos\psi)^2}=\frac{k^2}{\bar{k}^2}. \quad (9)$$

Since ψ and $\bar{k}^2/k^2$ are small, we set $\cos\psi\approx1$ in Eq. (3) and neglect $\bar{k}^2$ in comparison with k^2, except in the term $\bar{k}^2-k^2\sin^2\psi$. As shown below, the resulting errors

[3] This possibility was pointed out to the author by Dr. M. H. Ross.

are very small above $k=500$ Mev. Equation (3) becomes

$$E_\pi=\frac{m_\pi k}{\bar{k}\pm(\bar{k}^2-k^2\sin^2\psi)^{\frac{1}{2}}}. \tag{10}$$

From Eq. (10), one finds

$$\sin\psi d\psi=4(\bar{k}^2/E_\pi^3)[1-1/(2x)]dE_\pi, \tag{11}$$

$$\frac{\partial E_\pi}{\partial k}=\frac{1}{x(2x-1)}, \tag{12}$$

where $x=k/E_\pi$. Upon inserting (9), (11), and (12) into (7), one obtains

$$P(k,\theta_\gamma)=2nN\int_k^\infty \frac{f(E_\pi,\theta_\gamma)}{E_\pi}dE_\pi. \tag{13}$$

It may be noted that the integral over ψ of Eq. (7) has been transformed into an integral over E_π by means of (11). Upon differentiating both sides of (13), one obtains

$$\frac{\partial P(k,\theta_\gamma)}{\partial k}=-\frac{2nNf(k,\theta_\gamma)}{k}, \tag{14}$$

which gives

$$f(k,\theta_\gamma)=-\frac{k}{2nN}\frac{\partial P(k,\theta_\gamma)}{\partial k}. \tag{15}$$

Equation (15) gives the production cross section in terms of the number of γ rays observed.[4] We note that the argument (k,θ_γ) of f refers to the energy and angle of the π^0, i.e., the expression for f at a given energy and angle involves $k\partial P/\partial k$ evaluated at the same energy and angle. Equation (15) implies that $\partial P/\partial k<0$ in the region of validity of this equation. This also follows directly from (13), since P is an integral from k to ∞ over a positive function. As mentioned previously, the preceding treatment is valid only for high γ energies ($k\gtrsim 500$ Mev), although it may give a reasonably accurate estimate down to $k\sim 200$ Mev. At still lower energies, it is not legitimate to replace $f(E_\pi,\theta_\pi)$ by $f(E_\pi,\theta_\gamma)$ in Eq. (6). In this case, the γ-ray spectrum involves an integral over the production cross section for a wide range of angles θ_π, and it is probable that a measurement of $P(k,\theta_\gamma)$ at several angles θ_γ would be necessary to determine $f(E_\pi,\theta_\pi)$ for any given θ_π. Moreover, the procedure for solving Eq. (6) for f would then be very complicated.[5]

[4] A similar expression has been obtained by Carlson, Hooper, and King [Phil. Mag. **41**, 701 (1950)] for the energy spectrum of π^0 mesons irrespective of angle in terms of the γ-ray spectrum.

[5] For the case of π^0 mesons produced by γ rays in hydrogen, Borsellino has obtained an expression for the energy distribution of the decay γ rays, $P(k,\theta_\gamma)$, on the assumption that the π^0 production cross section in the center-of-mass system of the γ and the proton is proportional to $A\cos^2\bar{\theta}_\pi+B$, where $\bar{\theta}_\pi$=center-of-mass angle of π^0 [see G. Cocconi and A. Silverman, Phys. Rev. **88**, 1230 (1952)].

In order to test the validity of the approximation made by using Eq. (10) for E_π, the integrand of Eq. (13) was evaluated exactly for $k=400$ Mev and 800 Mev. The exact integrand which replaces $[nNf(E_\pi,\theta_\gamma)]\times(2/E_\pi)$ of Eq. (13) is given by

$$I=[nNf(E_\pi,\theta_\gamma)]\left\{\sin\psi\frac{\partial\psi}{\partial E_\pi}\frac{\partial E_\pi}{\partial k}\frac{k^2}{\bar{k}^2}\right\}. \tag{16}$$

The exact expressions for $\partial E_\pi/\partial k$ and $\partial\psi/\partial E_\pi$ as obtained from Eq. (3) were calculated for the complete range of ψ, and the resulting values of the curly bracket of Eq. (16) were compared with the approximate value $2/E_\pi$. For $k=400$ Mev, $2/E_\pi$ is smaller than the exact value by an amount which decreases from 6 percent to 2 percent as ψ increases from 0° to $\psi_{\max}(=9.7°)$ along the lower branch of the E_π *vs* ψ curve (corresponding to the range of E_π from 411 Mev to 800 Mev). For the upper branch of the E_π *vs* ψ curve, the error is of the order of 2 percent. For $k=800$ Mev, the underestimate is $\lesssim 2$ percent for the lower branch of the E_π *vs* ψ curve. These errors are quite small and of the expected order of magnitude, being somewhat less than m_π^2/k^2.

In Eq. (13), the lower limit of the integral was taken as k. If the exact expression for $E_{\pi,\min}$ [Eq. (5)] is used, one obtains

$$P(k,\theta_\gamma)=2nN\int_{k(1+\bar{k}^2/k^2)}^\infty \frac{f(E_\pi,\theta_\gamma)}{E_\pi}dE_\pi. \tag{17}$$

Differentiation of (17) gives

$$\frac{\partial P(k,\theta_\gamma)}{\partial k}=-2nN\frac{f[k(1+\bar{k}^2/k^2),\theta_\gamma]}{k(1+\bar{k}^2/k^2)}\frac{d}{dk}\left[k\left(1+\frac{\bar{k}^2}{k^2}\right)\right]. \tag{18}$$

Since $\bar{k}^2/k^2\ll 1$, Eq. (18) becomes

$$f[k(1+\bar{k}^2/k^2),\theta_\gamma]=-\left(1+\frac{2\bar{k}^2}{k^2}\right)\frac{k}{2nN}\frac{\partial P(k,\theta_\gamma)}{\partial k}. \tag{19}$$

Upon replacing k by $k(1-\bar{k}^2/k^2)$ on both sides of Eq. (19), one finds

$$f(k,\theta_\gamma)=-\frac{k}{2nN}\left(1+\frac{\bar{k}^2}{k^2}\right)\frac{\partial P[k(1-\bar{k}^2/k^2),\theta_\gamma]}{\partial k}. \tag{20}$$

Equation (20) may be compared with (15). Both the presence of $(1+\bar{k}^2/k^2)$ and the fact that $\partial P/\partial k$ is evaluated at a smaller γ energy $[k(1-\bar{k}^2/k^2)$ instead of $k]$ contribute to increase $f(k,\theta_\gamma)$, since it is expected that $|\partial P/\partial k|$ increases with decreasing k. On the other hand, the underestimate of the integrand of Eq. (13) tends to give a value of f which is too high. Hence the replacement of the exact lower limit $E_{\pi,\min}$ by k is probably compensated to some extent by the use of the approximate integrand in Eq. (13). It can be concluded that the accuracy of Eq. (15) is of the same order and possibly somewhat better than the maximum errors of 6 percent and 2 percent in the integrand which were found for $k=400$ Mev and 800 Mev, respectively.

Swartz and DeWire[6] have made measurements of the γ-ray spectrum at 90° to the proton beam of the Brookhaven Cosmotron, at proton energies of 1, 2, and 3 Bev. If one assumes a target nucleon Fermi energy of 25 Mev, the cutoff energy of the γ rays is 471, 659, and 766 Mev, respectively, for the three proton energies. The observed spectra[5] at the three energies show a maximum at $\sim$100 Mev and extend up to energies in the range 300–600 Mev. At 100 Mev, $\psi_{\max}=42.5°$, and the above treatment does not apply. However, near the upper end of the spectrum ($k\gtrsim 300$ Mev), Eq. (15) can be used to give an estimate of the π^0 production cross section. For forward angles of observation ($\theta_\gamma \lesssim 45°$), Eq. (15) applies over a wider range of energies, since the cutoff k is then of order 1–3 Bev.

III. EXTENSION TO GENERAL TWO-BODY DECAY

It seems of interest to extend Eq. (15) to the case of an arbitrary two-body decay. The result may be of interest in cases of the decay of unstable particles in which it is easier to detect the secondaries than the primaries. As an example, one may consider the possibility of counter experiments to detect the θ^0 particles, in which the energy distribution of the π^+ and π^- mesons is measured.

We consider secondary particles of total energy[7] E in the laboratory system, which are moving at an angle θ to the incident beam. By a derivation similar to that of Eq. (3), one finds that the laboratory total energy E_p of the primary particle is related as follows to E and to the angle ψ between primary and secondary:

$$E_p=\frac{m_p(p^2\cos^2\psi+\bar{E}^2)}{E\bar{E}\pm p\cos\psi(\bar{p}^2-p^2\sin^2\psi)^{\frac{1}{2}}}, \qquad (21)$$

where p= momentum of secondary in laboratory system, m_p= mass of primary particle, $\bar{E}$ and $\bar{p}$ are the (constant) energy and momentum, respectively, of the secondary in the rest system of the primary. Equation (21) shows that ψ must be $\leqq\psi_{\max}$, where

$$\psi_{\max}\equiv\sin^{-1}(\bar{p}/p). \qquad (22)$$

E_p is double-valued for $\psi<\psi_{\max}$. These results are similar to those obtained for the π^0 decay [see Eq. (4) and Fig. 1].

The number of secondaries $P(E,\theta)$ per unit energy interval and per steradian is given by

$$P(E,\theta)=2\pi nN\sum\int_0^{\psi_{\max}} f(E_p,\theta_p) \times(\partial E_p/\partial E)(1/4\pi)J\sin\psi d\psi, \qquad (23)$$

where $f(E_p,\theta_p)$ is the differential cross section for producing primaries of energy E_p at an angle θ_p to the incident beam; J= Jacobian; the factor (2π) comes from the integration over the azimuthal angle; $(1/4\pi)$ gives the number of decays per steradian in the rest system,[8] and the sum sign indicates summation over the two branches of the E_p *vs* ψ curve. In the following, it is assumed that E is sufficiently high so that $\psi_{\max}$ is small ($\lesssim 10°$) and $f(E_p,\theta_p)$ can be replaced by $f(E_p,\theta)$, i.e., f can be evaluated at the angle of the secondary. Following the same procedure as in Sec. II, we set $\cos\psi\approx 1$ in Eq. (21), and neglect $\bar{E}^2$ and $\bar{p}^2$ in comparison with E^2 and p^2, except in the term $\bar{p}^2-p^2\sin^2\psi$. In this approximation, the difference between p and E is also neglected. Equation (21) becomes

$$E_p=\frac{m_pE}{\bar{E}\pm(\bar{p}^2-E^2\sin^2\psi)^{\frac{1}{2}}}. \qquad (24)$$

Upon introducing

$$x\equiv E/E_p, \qquad (25)$$

Eq. (24) gives

$$\sin^2\psi=E^{-2}(2m_p\bar{E}x-m_p^2x^2-m^2), \qquad (26)$$

$$\sin\psi d\psi=(m_p^2/E_p^3)[1-\bar{E}/(m_px)]dE_p, \qquad (27)$$

$$\frac{\partial E_p}{\partial E}=\frac{m_p\bar{E}x-m^2}{m_px^2(m_px-\bar{E})}, \qquad (28)$$

where m= mass of secondary particle.

The Jacobian J is given by $d\cos\bar{\psi}/d\cos\psi$, where $\bar{\psi}$ is the angle between primary and secondary in the rest system. By a straightforward calculation, one finds

$$J=\frac{p^2}{\gamma_p\bar{p}(p-v_pE\cos\psi)}, \qquad (29)$$

where v_p= velocity of primary and $\gamma_p=(1-v_p^2)^{-\frac{1}{2}}$. Equation (29) is exact. We now make the approximation used above, of treating terms such as m^2/E^2 and m_p^2/E^2 as small compared to 1. One obtains by means of (26):

$$E=(p^2+m^2)^{\frac{1}{2}}\approx p[1+m^2/(2p^2)], \qquad (30)$$

$$v_p\approx 1-m_p^2/(2E_p^2), \qquad (31)$$

$$\cos\psi\approx 1-\psi^2/2\approx 1-(2m_p\bar{E}x-m_p^2x^2-m^2)/(2p^2). \qquad (32)$$

[6] C. E. Swartz and J. W. DeWire, Bull. Am. Phys. Soc. 30, No. 1, 25 (1955), and private communication.

[7] Throughout this section, letters without a subscript refer to the secondaries; letters with subscript p refer to the primaries. Unbarred quantities pertain to the laboratory system, while barred quantities pertain to the rest system of the decaying particle.

[8] It is assumed that the decay is isotropic in the rest system of the primary particle.

Upon inserting (30)–(32) into (29), one finds

$$J=\frac{p^2 m_p x}{\bar{p}(m_p\bar{E}x-m^2)}. \tag{33}$$

Upon substituting (27), (28), and (33) in Eq. (23), one obtains

$$P(E,\theta)=\frac{nNm_p}{2\bar{p}}\int_{E_{p,\min}}^{E_{p,\max}}\frac{f(E_p,\theta)}{E_p}dE_p, \tag{34}$$

where $E_{p,\min}$ and $E_{p,\max}$ are the minimum and maximum values of E_p, given by

$$E_{p,\min}=m_pE/(\bar{E}+\bar{p})\equiv\alpha E, \tag{35}$$

$$E_{p,\max}=m_pE/(\bar{E}-\bar{p})\equiv\beta E, \tag{35a}$$

where α and β are coefficients defined by (35) and (35a). Upon differentiating both sides of (34), one obtains

$$\frac{\partial P(E,\theta)}{\partial E}=-\frac{nNm_p}{2\bar{p}E}[f(\alpha E,\theta)-f(\beta E,\theta)]. \tag{36}$$

It is convenient to introduce ρ and $Q(E,\theta)$ defined by

$$\rho\equiv\beta/\alpha=(\bar{E}+\bar{p})/(\bar{E}-\bar{p}), \tag{37}$$

$$Q(E,\theta)\equiv-\frac{2\bar{p}E}{nNm_p}\frac{\partial P(E,\theta)}{\partial E}. \tag{38}$$

Equation (36) can be written

$$Q(E,\theta)=f(\alpha E,\theta)-f(\beta E,\theta). \tag{39}$$

$Q(E,\theta)$ can be obtained from the observed intensity of secondaries. In order to solve Eq. (39) for f in terms of Q, we note that

$$Q(\rho E,\theta)=f(\alpha\rho E,\theta)-f(\alpha\rho^2E,\theta), \tag{40}$$

and there are similar equations for $Q(\rho^jE,\theta)$ for $j\geqq 2$. It is seen that $f(\alpha E,\theta)$ is given by

$$f(\alpha E,\theta)=Q(E,\theta)+Q(\rho E,\theta)+Q(\rho^2E,\theta)+\cdots. \tag{41}$$

Since $\alpha>1$, $Q(E,\theta)$ involves only values of $f(E_p,\theta)$ for $E_p>E$. In view of $\rho>1$, the sum of Eq. (41) will therefore have a finite number of terms, limited by the energy of the incident beam. Thus if j_m is the largest j for which $Q(\rho^jE,\theta)>0$, we have

$$f(E,\theta)=\sum_{j=0}^{j_m}Q(\rho^jE/\alpha,\,\theta). \tag{42}$$

This equation gives the differential production cross section f in terms of Q, which in turn is related to the energy distribution of the secondaries by Eq. (38). In Eq. (42), E and θ are the energy and angle of the primary. The errors involved in the use of (42) are of order $(m_p\alpha/E)^2$, i.e., m_p^2 divided by the square of the lowest energy of the secondary for which Q must be evaluated.

As an example, for the θ^0 decay, $\bar{E}=247$ Mev and $\bar{p}=204$ Mev/c give $\alpha=1.095$, $\beta=11.5$, and $\rho=10.5$. Equation (42) becomes

$$f(E,\theta)=Q(0.913E,\theta)+Q(9.58E,\theta)+\cdots. \tag{42a}$$

It is seen that for 3-Bev incident protons, in the region of validity of the formula ($E\gtrsim 1.5$ Bev), only the first term of (42a) is present. As a check on the approximations made in the integrand of (34), $(\partial E_p/\partial E)J\times\sin\psi(\partial\psi/\partial E_p)$ was calculated exactly [using (21) and (29)] for pions of kinetic energy 2 Bev arising from θ^0 decay. For the lower branch of the E_p *vs* ψ curve, it was found that the approximate value $m_p/(\bar{p}E_p)$ of Eq. (34) underestimates the exact value by $\lesssim 5$ percent. This is approximately $m_p^2/E^2=0.053$.

Equation (42) can also be used for the pions from Λ^0 decay. In this case, $\alpha=4.10$, $\beta=15.5$, and $\rho=3.78$ give

$$f(E,0)=Q(0.244E,\theta)+Q(0.922E,\theta)+Q(3.49E,\theta)+\cdots. \tag{42b}$$

For the $\pi-\mu$ decay, we have $\alpha=1$, $\beta=\rho=1.71$, so that

$$f(E,\theta)=Q(E,\theta)+Q(1.71E,\theta)+Q(2.91E,\theta)+\cdots. \tag{42c}$$

IV. CONCLUSIONS

It has been shown that the differential cross section for π^0 production by a high-energy beam incident on a target can be obtained directly from a measurement of the energy distribution of the decay γ rays at various angles to the beam. This result is given by Eq. (15), which holds when the total energy of the π^0 is $\gtrsim 500$ Mev, although it may give reasonable estimates down to ~ 300 Mev. The energy distribution of the γ rays can be measured with a total absorption γ-ray spectrometer[9] of the type used by Swartz and DeWire.[6] Hence in this high-energy region, the experimental arrangement and interpretation are expected to be less complicated than at bombarding energies in the range[1] of 200–400 Mev, where it is advantageous to observe coincidences of the decay γ rays or the range of the recoil protons for the case of photoproduction in hydrogen. Equation (15) is obviously independent of the nature of the incident particles which produce the π^0's. However, this treatment is not applicable to the low-energy end of the π^0 spectrum.

The results for the π^0 decay have been extended to a general two-body decay, for possible application to counter experiments to detect K mesons or hyperons.[10] In this case, Eqs. (38) and (42) give the differential production cross section of the primaries in terms of the energy distribution of the secondaries arising from the decay.

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[9] A. Kantz and R. Hofstadter, Phys. Rev. **89**, 607 (1953); R. S. Foote and H. W. Koch, Rev. Sci. Instr. **25**, 746 (1954); R. M. Sternheimer, Atomic Energy Commission Reports AECU-2982, 2983, and 2984 (unpublished).

[10] S. L. Ridgway and G. B. Collins, Phys. Rev. **98**, 247(A) (1955).