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Elastic Photoproduction of π^0 Mesons from Deuterium at 270 Mev*

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The cross section for the process $\gamma+d\rightarrow\pi^0+d$ has been measured by observing the recoil deuteron. The recoil was first analyzed by a uniform magnetic field and then allowed to pass through a nuclear emulsion. The following values were obtained for the absolute cross section for π^0 production at a photon energy of 270 Mev and π^0 laboratory angles 124° and 168°: $d\sigma/d\Omega(124°)=3.2\pm0.9$ μb/steradian and $d\sigma/d\Omega(168°)=1.3\pm0.5$ μb/steradian. The fact that the cross section is the same order of magnitude as the cross section for the production of π^0's from hydrogen is evidence that constructive interference exists between the π^0 production from the proton and the π^0 production from the neutron. On the basis of approximate theories of meson production from deuterium, a lower limit is deduced for the isotropic part of the angular distribution of π^0's photoproduced from hydrogen.

INTRODUCTION

The π^0 mesons produced by the interaction of γ rays with deuterium come from two processes:

$$\gamma+d\rightarrow\pi^0+n+p, \tag{1}$$

$$\gamma+d\rightarrow\pi^0+d. \tag{2}$$

As discussed in the previous paper[1] the magnitude of the cross section for process (2) (often referred to as the "elastic" process) should be quite sensitive to the relative phase between the complex amplitudes for π^0 production from the neutron and from the proton. A measurement of the cross section should be able to distinguish between constructive and destructive interference of these amplitudes.

In addition to the information available from the absolute magnitude of the elastic cross section, some information concerning the angular distribution of the π^0 production from hydrogen may be obtained by observing the variation of the deuteron cross section with meson angle. Using the impulse approximation Chew and Lewis[2] have shown that, in the case of elastic production from deuterium, the angular distribution of the π^0 in the center-of-mass system will be proportional to $F(\theta)I^2(d)$. $F(\theta)$ is closely related to the angular distribution for π^0 production from a free proton. In particular, $F(\theta)=2+5\sin^2\theta$ if the free proton distribution is $2+3\sin^2\theta$ (the latter distribution being appropriate for π^0 production in a $J=\frac{3}{2}$ state). $I^2(d)$ is an integral involving only the ground-state wave function of the deuteron and is a function of the recoil deuteron momentum. It represents the probability that the recoiling proton and neutron remain together in the form of a deuteron. By using calculated values of $I^2(d)$, the observed angular variation of the elastic production from deuterium may be related directly to $F(\theta)$, and hence to the angular distribution for production from the free proton.

The absolute cross section of the elastic process has been measured at Cornell University at gamma-ray energies near 270 Mev by two groups using different methods. Both groups used the Cornell 315-Mev synchroton as the source of gamma rays and identified the process of interest by detecting the recoil deuteron. The first group, Wolfe, Silverman, and DeWire,[1] measured the absolute cross section at a meson angle of 110° in the laboratory by detecting the recoil deuteron with a scintillation counter telescope. In addition, they determined the relative cross sections at 130°, 110°, 93°, and 76° by observing one of the decay gamma rays of the π^0 meson in coincidence with the recoil deuteron. Their experiment is described in detail in the preceding paper.[1] The second method of measuring the cross section is the subject of this paper. In this method, the recoil deuterons were first analyzed by a uniform magnetic field and then detected by a nuclear emulsion. The absolute cross section was determined at deuteron recoil angles of 4° and 22° in the laboratory, corresponding to meson laboratory angles of 168° and 124°.

EXPERIMENTAL METHOD

The deuterium target consisted of a thin piece (0.37 g/cm^2) of heavy paraffin (CD_2). The effect of the carbon was subtracted by measurement of the recoil deuterons from a pure carbon target.

The momentum and angle of emission of the recoils were determined by the arrangement shown in Fig. 1. The experimental target and emulsion were situated in a uniform magnetic field of 18.2 kilogauss. The deuteron recoils produced in the target by the gamma-ray beam traversed a 40° arc before passing through a 200-micron nuclear emulsion inclined at an angle of 45° with the horizontal plane of the experimental system. Measurements on the track of a given recoil determined (1) the position at which the recoil entered the emulsion and (2) the horizontal incident angle of the recoil trajectory. For a recoil issuing from a point target these two observations uniquely determine both the momentum

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[1] Wolfe, Silverman, and DeWire, preceding paper [Phys. Rev. **99**, 268 (1955)].

[2] G. F. Chew and H. W. Lewis, Phys. Rev. **84**, 779 (1951).

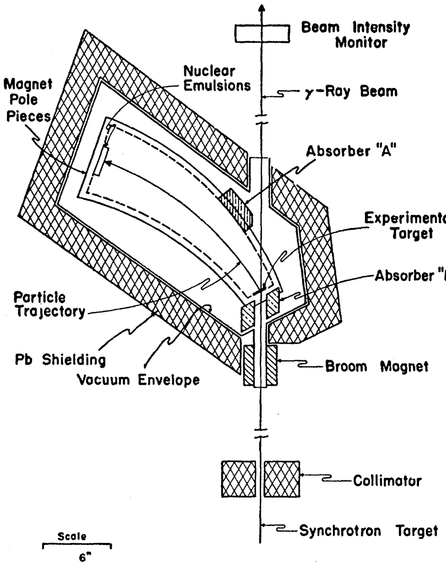

FIG. 1. Experimental arrangement (top view), showing γ-ray beam, target, magnet poles and nuclear emulsion detector. Target, recoil particle path, and emulsion are all inside a vacuum chamber maintained at fore-pump pressure.

and the angle of emission of the recoil. We assume that the deuteron recoils from the deuterium were all produced by the process $\gamma+d\rightarrow\pi^0+d$. Since this is a two-body interaction, the recoil momentum and emission angle determine the energy and angle of the π^0 and the energy of the incident photon. In Fig. 2, the deuteron energy is shown as a function of laboratory angle for an incident photon of 270 Mev. Actually, the finite size of the target introduces an uncertainty in the momentum and emission angle associated with a given track in the emulsion. The uncertainty in momentum was about ±5 percent and the uncertainty in angle of emission was ±2°.

The assumption concerning the identity of the recoils presupposes that (1) no other process produces deuteron recoils from deuterium in a significant quantity and (2) that the recoil particles which are not deuterons may be distinguished from the recoil deuterons. With regard to (1), at the gamma-ray energies we are dealing with, the only competing process to be considered seriously is the nuclear Compton effect: $\gamma+d\rightarrow\gamma'+d$. The absolute cross section for this process has not yet been measured; however, Ernstene, Keck and Tollestrup[3] have performed an experiment which sets an upper limit on this cross section. They suggest that the yield of recoil deuterons from the Compton effect is not more than a few percent of the yield from the elastic process at a deuteron laboratory angle of 50°. Even if one assumes a pessimistic angular distribution for the Compton cross section (i.e., one containing a large $\cos^2\theta$ term), their result places an upper limit for Compton effect contribution to our total yield of 4 percent and 10 percent at the 124° and 168° points, respectively.

Concerning (2), at the gamma-ray energies of interest, protons are the only other charged particles which can be photoproduced from deuterium with sufficient momentum to be observed in our system. Tracks made by protons and deuterons of the same momentum were easily separated because their grain densities differ by a factor of three. For this purpose Ilford C2 emulsions have a convenient sensitivity. In the momentum interval of interest, protons have energies greater than 50 Mev while deuterons have energies between 30 Mev and 60 Mev. In C2 emulsions the maximum energy which a particle can have and still produce an observable track is about 50 Mev for protons and about 100 Mev for deuterons. Therefore, in the selected momentum interval, proton tracks were nearly invisible while deuteron tracks were easily recognized. Figure 3 shows tracks of different particles all having the same $H\rho$.

Heavier particles, such as tritons or alpha particles, can be produced only in the carbon and therefore subtract out in the CD_2-C difference.

In order to obtain a valid CD_2-C difference, it is necessary that the scanning efficiency be the same for both the CD_2 and the C tracks. This condition was insured by using the same emulsion to detect the recoils from both targets. The tracks caused by recoils from one target were made distinct from the tracks belonging to the other target by rotating the emulsion plane 180° about its perpendicular axis when the targets were changed. Under the microscope the scanner sees the tracks from one target travel up the field of view and the tracks from the other target travel down the field of view as he racks down into the emulsion. Since both sets of tracks occur in the same block of emulsion and are scanned simultaneously, there should be no difference in scanning efficiency. Simultaneous scanning also reduces the total scanning time.

Other essential features of the experimental arrange-

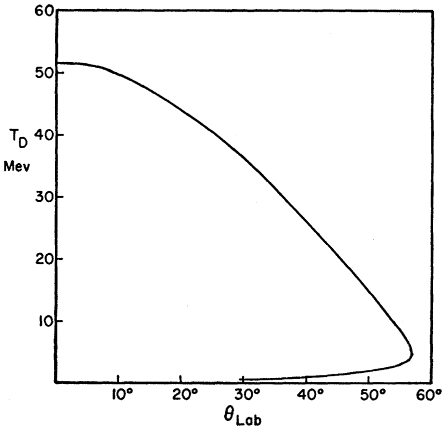

FIG. 2. Deuteron recoil energy as a function of laboratory angle for "elastic" processes produced by a 270-Mev γ ray.

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[3] Ernstene, Keck, and Tollestrup (private communication).

ment shown in Fig. 1 are the absorber A, the clearing magnet, and the vacuum envelope. Absorber A prevents particles from traveling from target to emulsion through the fringe field. The clearing magnet sweeps charged particles from the beam before they reach the experimental target, thereby considerably reducing the electron background in the emulsion. The vacuum envelope allows one to evacuate to forepump pressure the entire target-plate system as well as a part of the synchrotron beam path on either side of the target. The removal of the air reduces the background and is mandatory at the 4° position where electrons produced from the air behind the target may reach the emulsion.

Auxiliary experiments were performed to determine the no-target background and the amount of scattering-in. The no-target background was observed directly by exposing an emulsion in the usual experimental arrangement without the experimental target in position. No tracks which had acceptable values of incident angle and track length were found in this emulsion. Consequently an upper limit of 5 percent may be placed on the no-target background contribution to the total yield from the CD_2 target. In taking the difference between the CD_2 and C yields, the no-target background should subtract out, so no significant error is anticipated from this source.

Deuterons coming from the target may scatter from the polefaces of the magnet and strike the emulsion. Since the solid angle subtended at the target by the magnet polefaces is much larger than that subtended by the emulsion it seemed possible that the scattering-in from the polefaces might produce a serious error in the experiment. The scattering-in was observed directly by placing an absorber between the target and the emulsion in such a way that the emulsion was shielded from the target but both target and emulsion still had an unobstructed view of the poleface area. Thus, any particles which originated in the target and still reached the emulsion must have been scattered from the polefaces or other adjacent material. The number of such tracks was small enough to place an upper limit of 7 percent on the contribution of scattered-in tracks to the total number of CD_2-C difference tracks. In addition, about half of the exposures were made with "antiscattering-in" baffles on the polefaces. These baffles prevented scattering-in from a significant fraction of the poleface area. The results with and without the baffles in place were found to agree within the statistical uncertainty, confirming our direct observation that the scattering-in was not a large effect.

COLLECTION AND EVALUATION OF THE DATA

The scanners recorded only those tracks (about 3300 total were finally acceptable) with incident horizontal angles corresponding to momenta falling within limits set by the selected gamma ray energy interval. (At both emission angles the effective gamma energy interval was chosen to be 270±30 Mev. The deuteron recoils produced by these gamma rays via the elastic π^0 process had energies 40±10 Mev at 22°±4° and 50±10 Mev at 4°±4° in the laboratory.) The following information was recorded for each acceptable track: (1) α', the projection of the horizontal incident angle on the emulsion plane, (2) the xy coordinates of the track at the air surface, (3) the direction of travel of the track in the field of view, (4) the estimated gain density, and (5) Δy, the y component of the projected track length on the emulsion plane. The measurement of α' for a given track had a random uncertainty of ±1° (the total interval of acceptance was about 10° wide). The systematic error in the measurement of α', however, was not greater than 0.2°. Δy and the xy coordinates were determined with negligible uncertainty. The estimated grain density assigned to each track was subject to an estimated 30 percent total uncertainty. Coincidence scanning indicated that the absolute scanning efficiency was better than 95 percent.

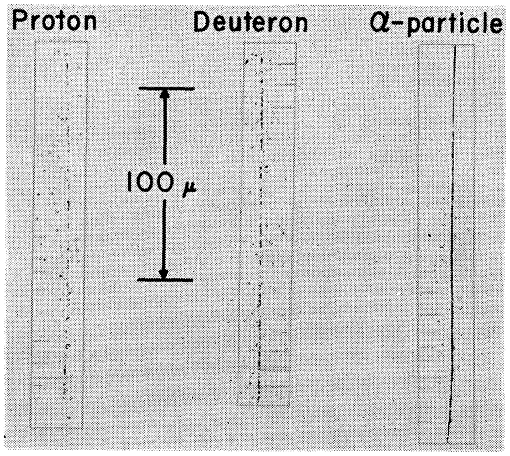

FIG. 3 Examples of proton, deuteron, and α-particle tracks all having the same momentum near the minimum of the acceptable momentum interval when they enter the emulsion at the top.

The distribution of the values of Δy and the estimated grain densities were investigated for the two groups of tracks obtained from each emulsion. The Δy distribution agreed in each instance with that expected for tracks made by recoils traveling directly from the target to the emulsion. The estimated grain densities were plotted as a function of the angle α', the plots showing the expected decrease in grain density with increasing momentum. The grain densities of tracks from CD_2 and tracks from C corresponding to the same momentum were found to be the same. The absolute values of the grain densities were in good agreement with grain densities of tracks known to be made by deuterons of the same energy. (The latter tracks were observed in 400μ emulsions exposed in the same experimental arrangement but at a dip angle of 5°. Since it was possible to observe both momentum and range the particle mass could be determined.) All tracks were

TABLE I. Summary of experimental data.

Deuteron lab angle Meson lab angle	22° 124°			4° 168°	
Emulsion No.	2C140	2C124	2C104–2C110	2C132A	2C145
Uncorrected N_D	126	695	453	236	415
totals N_C	89	501	296	183	343
Corrected N_D'	126	693	453	236	433
totals N_C'	88	500	290	183	345
Differences ΔN	38	193	163	53	88
Standard deviation of differences	±15	±35	±27	±20	±28
Normalized ΔN^*	67	80	95	37	44
differences	±26	±14	±16	±14	±14
$d\sigma/d\Omega$ at 270 Mev (cm² per steradian per incident photon per target nucleus)	$(3.2\pm0.9)\times10^{-30}$			$(1.3\pm0.5)\times10^{-30}$	

excluded from the final count which had estimated grain densities less than one-half the mean grain density attributed to deuteron tracks at the same incident angle. Since, for the same momentum, the grain density of proton tracks is only one-third that of deuteron tracks and since the estimated grain densities are accurate to within 30 percent, this selection procedure insured that all the deuterons and none of the protons were included in the final count.

Before the CD_2-C subtraction was performed some small corrections were applied to the total number of tracks N_D from the heavy paraffin, and to the total number of tracks N_C from the carbon. The corrections adjusted for errors arising from (1) the difference in energy loss of the recoils in the CD_2 and C targets, (2) a small difference in the amount of carbon in the two targets, and (3) any difference existing in angle of inclination of the emulsion during the CD_2 and C target runs. These corrections were never more than 5 percent each and, since they tend to cancel, the total correction was even less. In Table I the original totals N_D and N_C, the corrected totals N_D' and N_C' and the difference $\Delta N=N_D'-N_C'$ are shown for the various emulsions scanned. The differences ΔN^* are the ΔN's normalized to correspond to the same emulsion area and synchrotron exposure. ΔN^*, therefore, should be the same for all the emulsions at the same deuteron recoil angle. The ΔN^*'s were averaged at each of the two major angles to give the final cross sections shown in Table I. The uncertainties indicated in the final results represent the standard deviation of the total experimental uncertainty. The various contributions to the total uncertainty are shown in Table II. The ratio of the cross section at the two angles is independent of the uncertainties in synchrotron energy, the standard ion chamber calibration, and the number of target nuclei per cm³. The value of the ratio with its standard deviation is $R(168°/124°)=0.41\pm0.12$.

TABLE II. Contributions to total experimental error.

Source	Percent uncertainty in final cross section 124°	168°
Statistical	12	24
No. of photons	9	9
No. of target nuclei	5	5
Solid angle	2	2

Our results and those of Wolfe *et al.*[1] are compared in Fig. 4 of the preceding paper with the theoretical curves calculated by Brueckner and Chappelear for the elastic π^0 process. The calculations assume constructive interference and a $(2+3\sin^2\theta)$ distribution in the center-of-mass system. The upper curve was calculated by using the impulse approximation; the lower curve was obtained in the same way except that corrections arising from the multiple scattering of the π^0 by the nucleon system were taken into account. It is apparent that both experiments are compatible with the theoretical angular distribution. Our absolute values are considerably higher than the theoretical curves and higher than Wolfe, Silverman, and DeWire's experimental values. However, in view of the uncertainties of both measurements and of the calculations as well, no particular significance should be attached to this apparent disagreement.

CONCLUSIONS

Two pieces of information are obtained from the experiment: (1) the order of magnitude of the absolute value of the cross section and (2) the ratio of the cross sections at 124° and 168°. Our result for (1) is in agreement with that found by Wolfe *et al.*, and we confirm their conclusion that the interference is constructive for elastic π^0 production near 300 Mev.[4]

Result (2), the ratio of the cross sections at 124° and 168° is, as we have pointed out, related to the angular distribution of the π^0 production from hydrogen. If this latter distribution is represented by $G(\theta)$, then in the impulse approximation the angular distribution for the elastic production from deuterium at a given γ-ray energy has the form $(d\sigma/d\Omega)\sim F(\theta)I^2(\theta)$, where $F(\theta)$ is closely related to $G(\theta)$ but with a relatively smaller isotropic part, and $I^2(\theta)$ is the probability that the deuteron remains bound after the π^0 production. Accepting this qualitative picture of the process, and taking $G(\theta)=a+b\sin^2\theta$, we may use our data to set a lower limit on a/b, the relative isotropic part of the angular distribution for π^0's photoproduced from the free proton, by the following arguments:

(1) The deuteron is less likely to remain bound when it recoils with higher momentum, i.e., when it goes more forward and the meson goes more backward (168°). So without using any detailed calculation of $I^2(\theta)$ we can safely say that $I^2(168°)\leq I^2(124°)$, and we will use the extreme value $I^2(168°)=I^2(124°)$ leading to a minimum a/b in our limit calculation.

(2) The effect of scattering by the "spectator" nucleon is not known for sure, but the qualitative features of the calculation by Brueckner and Chappelear[5]

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[4] C. C. Andre [University of California Radiation Laboratory Report No. 2425, 1953 (unpublished)] also found constructive interference for the elastic cross section near threshold at 150 Mev.

[5] See the preceding paper for discussion of the theoretical curves [Wolfe, Silverman, and DeWire, Phys. Rev. 99, 268 (1955)].

show that, in the laboratory angle interval of interest, the primary effect is to suppress the cross section and not to change the angular distribution. Consequently we ignore the effect of such scattering in calculating a minimum a/b.

(3) The isotropic part of the free proton angular distribution will, if anything, be larger than the isotropic part of the elastic deuteron angular distribution (before modification by the deuteron form factor). Again we take the extreme case of no difference in the two distributions in calculating a lower limit for a/b.

On the basis of these arguments, and using our measured cross-section ratio $R(168°/124°)=0.41\pm0.12$, we calculate $a/b\geq 0.35^{+0.25}_{-0.06}$.

If we take the impulse approximation at its face value, use Chew and Lewis[2] values for $I^2(\theta)$, and make no scattering corrections, we calculate $a/b=0.80^{+0.70}_{-0.20}$.

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Energy Distribution of γ Rays from π^0 Decay*

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It is shown that the γ-ray energy distribution resulting from the decay of π^0 mesons produced in a target bombarded by a high-energy particle beam is related in a simple manner to the differential π^0 production cross section, for sufficiently high energies of the γ's ($\gtrsim$500 Mev). An expression is obtained for the π^0 production cross section in terms of the γ-ray energy distribution. This result is extended to the case of an arbitrary two-body decay, for which an expression is obtained for the production cross section of the primaries in terms of the energy distribution of the secondaries emitted in the decay.

I. INTRODUCTION

INFORMATION about the π^0 meson production in a target bombarded by a high-energy particle beam can be obtained from a measurement of the energy distribution of the γ rays from the π^0 decay at various angles to the beam. At incident energies in the range of 200–400 Mev,[1] the interpretation of the γ-ray spectrum is very complicated, because at each angle of observation, a wide range of angles of the π^0's is involved. However, with increasing energy of the incident particles and of the resulting γ rays from π^0 production, the maximum possible angle between the observed γ and the decaying π^0 becomes very small, and it can be assumed that the π^0 differential production cross section remains approximately constant over the small range of π^0 angles involved. It will be shown that in this high-energy region (γ energy$\gtrsim$500 Mev), the π^0 cross section can be expressed in a simple manner in terms of the γ-ray energy spectrum. A similar expression will also be obtained for an arbitrary two-body decay for the production cross section of the primaries in terms of the energy distribution of the secondaries which are emitted in the decay.

II. RELATION BETWEEN π^0 PRODUCTION CROSS SECTION AND γ-RAY ENERGY SPECTRUM

The velocity v_π of the π^0 in the laboratory system is related as follows[2] to the laboratory angle ψ between the observed γ and the π^0:

$$\bar{k}=\gamma_\pi k(1-v_\pi\cos\psi), \tag{1}$$

where k is the energy of the γ-ray in the laboratory system, $\bar{k}$ is its energy in the π^0 rest system, and $\gamma_\pi=(1-v_\pi^2)^{-\frac{1}{2}}$. Upon squaring Eq. (1) and solving for v_π, one obtains

$$v_\pi=\frac{k^2\cos\psi\pm\bar{k}(\bar{k}^2-k^2\sin^2\psi)^{\frac{1}{2}}}{k^2\cos^2\psi+\bar{k}^2}. \tag{2}$$

The total energy E_π of π^0 is given by

$$E_\pi=\frac{m_\pi(k^2\cos^2\psi+\bar{k}^2)}{k[\bar{k}\pm\cos\psi(\bar{k}^2-k^2\sin^2\psi)^{\frac{1}{2}}]}, \tag{3}$$

where m_π=mass of π^0. It is seen that for a given ψ, there are in general two values of E_π. Moreover, since the expression under the radical must be positive, ψ is

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* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] See, for example, A. Silverman and M. Stearns, Phys. Rev. **88**, 1225 (1952); G. Cocconi and A. Silverman, Phys. Rev. **88**, 1230 (1952); Goldschmidt-Clermont, Osborne, and Scott, Phys. Rev. **89**, 329 (1953); Phys. Rev. **97**, 188 (1955); Walker, Oakley, and Tollestrup, Phys. Rev. **89**, 1301 (1953); Marshall, Marshall, Nedzel, and Warshaw, Phys. Rev. **88**, 632 (1952); R. H. Hildebrand, Phys. Rev. **89**, 1090 (1953).

[2] It is assumed that the units are such that $c=1$.

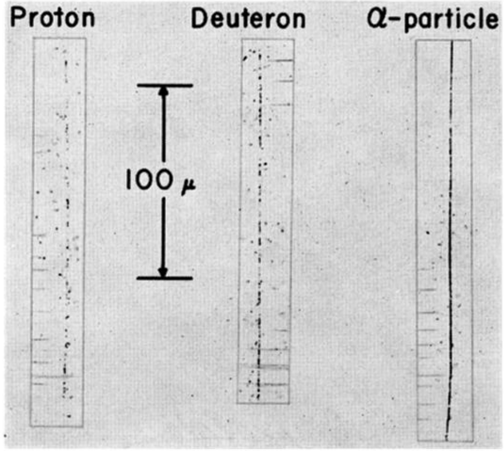

FIG. 3 Examples of proton, deuteron, and α-particle tracks all having the same momentum near the minimum of the acceptable momentum interval when they enter the emulsion at the top.