formed in high cross section from heavier target nuclei also, where it can hardly be a spallation residue.

It may be noted that, over the entire energy range, the N^{13} formation cross section is strikingly low compared to the other cross sections. In view of the similarity between C^{11}, N^{13}, O^{15}, and F^{18} as regards their positions relative to β stability and their β-decay energies, these low cross sections for N^{13} are, at first sight, surprising. Following a suggestion by Dr. D. H. Wilkinson, we attribute this apparent anomaly to the fact that all excited states of N^{13} are unstable with respect to heavy-particle emission[24] whereas each of the other nuclides observed has a number of excited states which can be de-excited by gamma emission only.

It is a pleasure to express our gratitude to the Cosmotron operating staff for carrying out the bombardments. Miss G. Vedder and Mrs. N. Hamilton helped with the activity measurements.

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[24] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. 24, 321 (1952).

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Elastic Photoproduction of π^0 Mesons from Deuterium*

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The average differential cross section for the reaction $\gamma+d=\pi^0+d$ has been measured for photons between 250 and 300 Mev at four angles. A difference measurement with deutero-paraffin and normal paraffin targets was employed. The recoil deuterons were detected by a counter telescope which discriminated against protons by recording the product of the energy and the specific ionization loss of the particles. The reaction was further identified by demanding a coincidence between the deuteron pulse and a pulse from a photon counter placed at an angle corresponding to the direction of the π^0. The values of the differential cross section for various angles of the π^0 in the laboratory system are as follows:

$\theta(\pi^0)$	$d\sigma/d\Omega$ (10^{-30} cm^2/steradian)	$\theta(\pi^0)$	$d\sigma/d\Omega$ (10^{-30} cm^2/steradian)
76°	4.2 ± 0.6	110°	2.5 ± 0.4
93°	3.2 ± 0.5	130°	1.2 ± 0.3

The stated errors are the standard statistical errors and apply to the relative cross sections at the various angles. The absolute cross-section scale is subject to an experimental error of 25 percent. The measured cross sections are in agreement with theoretical calculations based on the impulse approximation, with the assumption of equal amplitudes for π^0 production from the proton and neutron and constructive interference.

INTRODUCTION

Neutral pi mesons can be produced by the photon bombardment of deuterium in either of two reactions, the elastic process

$$\gamma+d=\pi^0+d$$

in which the deuteron recoils as a unit, and the inelastic process

$$\gamma+d=\pi^0+p+n$$

in which the deuteron is broken up into its component nucleons. A study of these two reactions together with a knowledge of the π^0 production from hydrogen,

$$\gamma+p=\pi^0+p,$$

can lead to information about the relative properties of the photoproduction from protons and neutrons.

An exact theoretical calculation of meson photoproduction from deuterium is beyond the scope of present-day physics although some approximate calculations based on simplified meson theories have been made.[1,2] To simplify the problem, one can make use of the impulse approximation, which states that meson production in a nucleus may be treated as the sum of interactions with the individual nucleons.[3–6] By using this method it has been shown[5] that the cross section for the elastic production of a π^0 meson with a deuteron recoil of momentum D may be written, neglecting spin effects, as

$$\sigma_{\rm el}=|A_n+A_p|^2\left(\int\psi_0^2\exp(\tfrac{1}{2}i\mathbf{D}\cdot\mathbf{R})d\mathbf{R}\right)^2, \qquad (1)$$

where A_n is the amplitude for π^0 production from the neutron, A_p is the amplitude for π^0 production from the proton, $\psi_0(R)$ is the ground-state deuteron wave function, and $\mathbf{R}$ is the relative coordinate between neutron and proton. The quantity

$$\left|\int\psi_0^2\exp(\tfrac{1}{2}i\mathbf{D}\cdot\mathbf{R})d\mathbf{R}\right|^2$$

can be interpreted as the probability that the deuteron will stick together if one of the nucleons is given an impulse D. In the same notation, the cross section of a

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* Assisted by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

† Now at Eastman Kodak Company, Rochester, New York.

[1] Heckrotte, Henrich, and Lepore, Phys. Rev. 85, 490 (1952).

[2] N. C. Francis and R. E. Marshak, Phys. Rev. 85, 496 (1952).

[3] G. F. Chew, Phys. Rev. 80, 196 (1950).

[4] M. Lax and H. Feshbach, Phys. Rev. 81, 189 (1951).

[5] G. F. Chew and H. W. Lewis, Phys. Rev. 84, 779 (1951).

[6] M. Lax and H. Feshbach, Phys. Rev. 88, 509 (1951).

free proton is given by

$$\sigma_p = A_p^2.$$

The total cross section from deuterium (elastic plus inelastic) is given by

$$\sigma_d = A_n^2 + A_p^2 + 2\,\mathrm{Re}\,A_n^* A_p \int \psi_0^2 \exp(i\mathbf{D}\cdot\mathbf{R})d\mathbf{R}. \quad (2)$$

The last term of (2) is small except at threshold or for forward angles of π^0 production. For other cases the ratio σ_d/σ_p is a measured of the relative size of A_n and A_p. This ratio has been measured by Cocconi and Silverman[7] and by Andre[8] who find values of σ_d/σ_p close to 2 over a wide range of angle and energy. This result implies that $A_n^2 = A_p^2$. Adopting this equality and referring to (1), one can readily see that the elastic cross section is extremely sensitive to the relative phase of A_n and A_p. If $A_n = A_p$ (constructive interference) σ_{el} is relatively large but if $A_n = -A_p$ (destructive interference) σ_{el} is zero, in the aforementioned approximation.

To be more precise, the experiments show that A_n and A_p do not differ by more than 25 percent. If we assume this difference, then the elastic cross section can vary by a factor of 32 depending on whether one assumes constructive or destructive interference. Since this ratio is so large, it is apparent that an experimental measurement of the cross section of the elastic process should clearly distinguish between constructive and destructive interference.

EXPERIMENTAL PROCEDURE

1. Counter Geometry

The process $\gamma + d = \pi^0 + d$ is an elastic process in the sense that there are only two bodies involved at any one time. Therefore a measurement of the energy and angle of emission of one of the products completely determines the energy and angle of the other product and the energy of the photon that initiates the event. In this experiment the angle and energy of the recoil deuteron were measured. The target was deuterated paraffin 2 mm thick and the background count was observed from a normal paraffin target of the same chemical composition. In order to reduce this background, the process was further identified by demanding a coincidence between the deuteron counter and a photon counter placed in the direction of emission of the π^0. Figure 1 shows the experimental geometry for the case of π^0 emission at 110° and recoil deuteron at 28° to the direction of the incoming photon.[9]

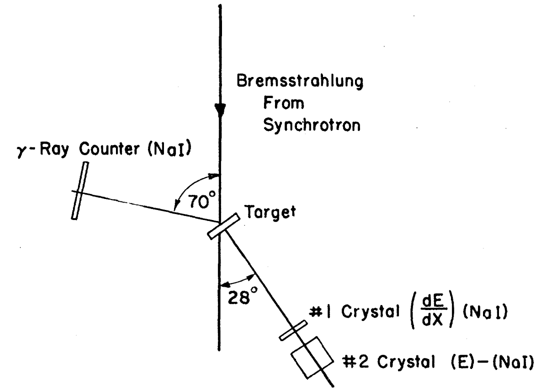

FIG. 1. Experimental arrangement showing γ-ray beam, target, deuteron detector and γ-ray counter. There is a $\frac{1}{4}$-in. Pb converter between the target and γ-ray detector (not shown).

The photon beam intensity was monitored by a thick-wall ionization chamber which had been calibrated previously[10] in terms of effective quanta in the bremsstrahlung spectrum.[11] The current from the monitor chamber was integrated electronically[12] to give the number of effective quanta accumulated during a run. From the measured spectrum[10] the number of incident photons could be computed.

The efficiency of the photon counter was kept constant throughout the experiment at each of the four angles studied. However, its absolute efficiency was not known, so that the measurements gave only the relative cross sections at each angle. To obtain the absolute cross section the measurement was repeated at one angle without the requirement of a photon coincidence. Recoil deuterons alone were detected and it was assumed that all deuterons of the required energy and angle in the $DC_2 - CH_2$ difference came from π^0 production.

2. Deuteron Detector

The deuteron counter telescope had to be capable of identifying deuterons against a proton background which sometimes exceeded the deuteron rate by a factor of ten. This was done by using a scintillation counter telescope consisting of a thin sodium iodide counter to measure the energy loss dE/dx of the particle and a thick sodium iodide counter in which the particle stopped and gave a pulse proportional to its energy E. The pulse heights in the two counters were multiplied electronically, the product EdE/dx being approximately proportional to $M^{0.8}Z^2E^{0.2}$, where M and Z are the mass and charge of the particle. Such a pulse spectrum would be expected to give good separation between protons and deuterons. This technique is described in

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[7] G. Cocconi and A. Silverman, Phys. Rev. **88**, 1230 (1952).

[8] C. C. Andre, University of California Radiation Laboratory Report 2425, November, 1953 (unpublished).

[9] All angles in this article refer to the laboratory frame.

[10] Corson, DeWire, McDaniel, and Wilson, *The Cornell 300-Mev Synchrotron* (Cornell University Press, Ithaca, 1953).

[11] The number of effective quanta Q is related to the spectrum $N(W)$ by $QW_0 = \int_0^{W_0} WN(W)dW$, where W_0 is the maximum energy of the spectrum.

[12] R. M. Littauer, Rev. Sci. Instr. **25**, 148 (1954).

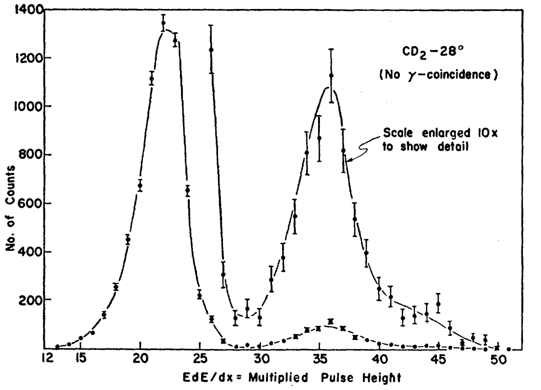

FIG. 2. The product (EdE/dx) pulse-height distribution from a deutero-paraffin target. No γ-ray coincidence was required. The peak at smaller pulse heights is due to protons and the one at larger pulse heights to deuterons. The calculated ratio of the two pulse heights is 1.7 and equals the measured ratio within the experimental error. There is some evidence for a triton group at the large pulse-height side of the deuteron distribution.

detail elsewhere,[13] and hence no further details will be given here.

The particle resolution of the deuteron counter is illustrated in Fig. 2 where the pulse-height spectrum from a deutero-paraffin target is shown. No photon coincidence was required in this run. In the measurements involving a coincidence with a photon the proton-deuteron separation was not so marked; an error of 10 percent was incurred in determining the number of deuteron counts in the spectrum. This error is included in the final errors in cross section.

At each angular setting the energy limits on the deuteron counter were adjusted to accept deuterons from bombarding photons of energies between 250 and 300 Mev. This was done by setting upper and lower biases on the energy pulse from the large crystal. The energy scale was established by observing the maximum pulse height obtained from the crystal exposed to protons, using the known thickness of the crystal and its corresponding proton energy.

Figure 3 shows the calculated effective bombarding photon spectrum at each angle where the experiment was performed. The ordinates are proportional to the product of the photon flux and the effective target thickness; hence the integrals under the curves are proportional to the number of incident photons per effective quantum multiplied by the number of target deuterons from which detectable events arise.

3. π^0 Detector

The emission of a π^0 meson was observed by counting one of the decay photons in a photon counter placed in the direction of emission of the π^0. The counter consisted of a one-cm thick NaI crystal preceded by a $\frac{1}{4}$-inch Pb converter and a 2-inch carbon plate to absorb low-energy charged particles. The crystal was 10 cm in diameter. The associated electronic circuit was biased so that a particle was required to lose 3 Mev in the crystal to be recorded.

The kinematics of the elastic process are such that the angle of emission of the π^0 is determined almost completely by the angle of the deuteron, independent of energy over a rather large range. Because of the relativistic contraction in the π^0 decay process, the efficiency of the π^0 counter is enhanced when it is placed directly in the path of the high-energy π^0.

Even though the photon counter provides no data on the kinematics of the π^0 production process, its use presents two advantages: (1) The reaction is more clearly defined. (2) The recoil deuterons from the carbon in the targets are nearly entirely eliminated. This provides a considerable improvement in the statistics of subtracting the count from the normal paraffin target.

On the other hand, the use of the photon counter involves the distinct disadvantage that it is very difficult to measure its absolute efficiency for counting π^0 mesons. This efficiency is the product of two factors, the first being the probability that at least one of the decay photons strikes the counter, the second the probability that a photon striking the counter will be counted. The first factor was calculated directly from the kinematics of the π^0 decay and varied from 0.59 at 76° for the π^0 to 0.40 at 130°. The second factor, the so-called intrinsic efficiency, cannot be computed with any confidence and requires a rather elaborate pro-

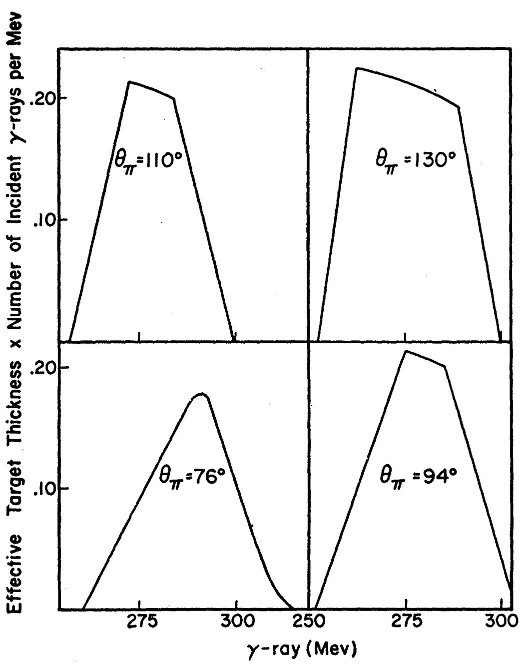

FIG. 3. Effective incident photon spectrum at each angle. The ordinates are proportional to the photon flux and effective target thickness. The integrals under the curve yield the quantity (Nt) used in the calculation of the absolute cross section (see text).

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[13] Wolfe, Silverman, and DeWire, Rev. Sci. Instr. (to be published).

cedure to measure it. Instead of this factor was determined, and the absolute cross section scale established, by repeating the run at 110° for the π^0 without the photon counter. The ratio of the difference counting rates divided by the computed first factor then gives the value of the intrinsic efficiency. This procedure implies that all the deuterons in the correct energy range in the D-paraffin, H-paraffin difference come from the elastic π^0 production. The intrinsic efficiency was found to be 0.78 ± 0.18. This value is consistent with the efficiency of a similar counter used by Silverman and Stearns[14] in measuring the photoproduction of π^0 mesons from hydrogen. Its efficiency was determined by using the 170-Mev monoenergetic photons from the Cornell synchrotron.[15]

EXPERIMENTAL RESULTS

1. Calculation of the Cross Section

At each angle the value of the differential cross section was computed from the experimental data using the following formula:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\pi^0}=\frac{C\,d\Omega_{\mathrm{D}}/d\Omega_{\pi^0}}{\Delta\Omega_{\mathrm{D}}(Nt)\epsilon f},$$

where $d\sigma/d\Omega|_{\pi^0}$ is the cross section per steradian in the lab system; C is the number of counts in the D-paraffin, H-paraffin difference per 10^{11} effective quanta; $\Delta\Omega_{\mathrm{D}}$ is the solid angle subtended by the deuteron counter; $d\Omega_{\mathrm{D}}/d\Omega_{\pi^0}$ is computed from the tables of Malmberg and Koester,[16] (Nt) is the product of the number of incoming photons per 10^{11} effective quanta and the effective thickness of the target in deuterons per cm^2—it is found from the integrals of the curves in Fig. 3 and the monitor reading; ϵ is the intrinsic efficiency, measured to be 0.78; and f is the probability that a π^0 decay photon strikes the photon counter. Numerical values used in this formula are given in Table I.

2. Values of the Cross Section

The differential cross sections measured in this experiment are given in Table II. The quoted errors are the standard statistical errors including the uncertainty in separating the pulses due to protons and deuterons. It must be remembered that the absolute values are all based on the additional run at $\theta(\pi^0)=110°$ taken without the photon counter, for which there is an uncertainty of 25 percent. Thus the errors given in Table II refer to the relative values at the various angles and there is an additional 25 percent uncertainty in the absolute cross section scale.

TABLE I. Numerical data used in computing the cross section. The various terms are discussed in the text. The number of counts C and the product (Nt) are normalized to 10^{11} effective quanta.

Deuteron angle	42.5°	35°	28°	20°
Photon counter angle	76°	97°	106°	128°
$\Delta\Omega_{\mathrm{D}}$(sterad)	0.00087	0.00089	0.00089	0.00112
C	7.5 ± 0.8	8.1 ± 0.8	5.8 ± 0.7	3.9 ± 0.8
f	0.59	0.56	0.53	0.42
$d\Omega_{\mathrm{D}}/d\Omega_{\pi^0}$	0.296	0.245	0.209	0.184
(Nt) (10^{33} cm^{-2})	1.38	1.67	1.36	1.78

TABLE II. Differential cross section for the elastic photoproduction of π^0 mesons from deuterium for various angles of emission of the π^0.

$\theta(\pi^0)$	76°	93°	110°	130°
$d\sigma/d\Omega\|_{\pi^0}$ (10^{-30} cm^2)	4.2 ± 0.6	3.2 ± 0.5	2.5 ± 0.4	1.2 ± 0.3

3. Auxiliary Measurements

A number of additional experiments were performed to test the assumption that the recorded counts actually came from the elastic π^0 production process. These were:

(*a*) Preliminary measurements were made with targets of D_2O and H_2O 0.5 cm thick. The results were in agreement with those quoted above. At 115° for the π^0 (30° for the deuteron) the cross section was observed to be 2.4 microbarns per steradian with an uncertainty of about 30 percent.

(*b*) Using the water targets and the same biases as in (*a*), the deuteron counter was moved from 30° to 60°. At this angle no deuterons could be expected from the elastic production. The D_2O rate at 30° was 44 ± 4 events per 10^{11} effective quanta while the rate at 60° was 5 ± 2. The H_2O rate at 30° was 12 ± 2.

(*c*) At the 30° position a one-cm carbon absorber was placed in front of the telescope. Such an absorber is just sufficient to exclude a deuteron from the elastic process from the energy acceptance interval of the counter. The D_2O-H_2O difference rate without absorber was 18 ± 2 per 10^{11} effective quanta. The D_2O rate with absorber was 2.4 ± 1.4.

(*d*) With the deuteron counter at 30°, the synchrotron energy was reduced from 315 Mev to 250 Mev. At this energy no deuteron with sufficient energy to be recorded should be produced. When this was done the difference counting rate dropped from 18 ± 2 to 0.5 ± 2 counts per 10^{11} effective quanta.

Measurement (*d*) gives an indication of the possible contribution of the deuteron Compton effect $(\gamma+d=\gamma+d)$ to our experiment. This process would record in our counting system in a similar way to the elastic π^0 production. If indeed we had been measuring only the Compton process at the higher energy then it is estimated that the counting rate at 250 Mev would have been about 7 per 10^{11} effective quanta. This estimate is made on the basis of a constant Compton cross section between 210 and 280 Mev.

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[14] A. Silverman and M. Stearns, Phys. Rev. **88**, 1225 (1952).

[15] J. W. Weil and B. D. McDaniel, Phys. Rev. **86**, 582 (1952).

[16] J. H. Malmberg and L. J. Koester, Jr., *Tables of Nuclear Reaction Kinematics at Relativistic Energies* (Physics Department, University of Illinois, Urbana).

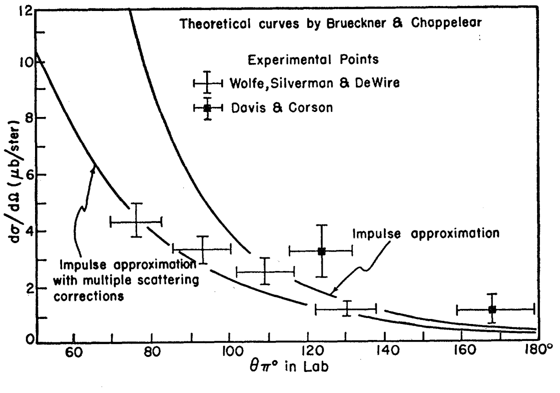

FIG. 4. Experimental results together with those of Davis and Corson. The solid curves are calculated by Brueckner and Chappelear using the impulse approximation with and without multiple scattering corrections.

Ernstene, Keck, and Tollestrup[17] have performed an experiment to measure the deuteron Compton effect. Their results, when applied to our experiment, indicate that the Compton effect is probably less than 15 percent of the elastic production.

DISCUSSION OF RESULTS

Our experimental results are plotted in Fig. 4. The curves are the results of calculations of Brueckner and Chappelear.[18,19] The upper curve is the result from the impulse approximation if one assumes a $2+3\sin^2\theta$ center-of-mass distribution of π^0 mesons from free protons and constructive interference, $A_n=A_p$. The lower curve is based on a modification of the impulse approximation which takes into account the scattering of the outgoing π^0 by the other nucleon of the deuteron.[20] The experimental points fit the corrected curve better, although except for the point at 76° the data are compatible with either curve.

There is some uncertainty in the theoretical curve at large π^0 angles where the deuteron comes off with high energy. The probability that the deuteron comes off bound is then dependent on the high-momentum components in the deuteron wave function, which are not too accurately known. Other uncertainties in the theoretical curves arise from a lack of knowledge of the relative magnitude of the spin flip and non-spin flip processes in π^0 production, and to the lack of precise information on the values of A_n and A_p. In view of these uncertainties, the excellent agreement between theory and experiment may be fortuitous. However the experimental results do rule out the possibility that the interference between neutron and proton waves is destructive, for this would give cross sections of the order of 1/30 the measured cross section or less. On the other hand, the experiment is not sufficiently sensitive to point definitely to constructive interference. For instance, a phase difference of 90° between A_n and A_p would lead to cross sections only a factor of two below the measured values. More reliable comparison with the impulse approximation could be made if the experimental cross sections were known in the forward direction since the recoil momentum of the deuteron is low and the form factor can be calculated more reliably. However, measurements in this region would have required a thinner target and the counting rates were very low. Further, the detection of deuterons of very low energy becomes difficult by this method since the first crystal must be made thin compared with the range of the deuteron.

Two other experimental results[8,21] confirm the main conclusion of this paper. Davis and Corson[21] have measured the elastic π^0 production from deuterium by detecting recoil deuterons in photographic plates after magnetic analysis. Their results are shown in Fig. 4 and are seen to be in reasonable agreement with the results of this paper. Andre[8] has measured the ratio of the total cross sections (elastic+inelastic) for hydrogen and deuterium near threshold. His measured ratio at 90° laboratory angle is

$$\frac{(d\sigma/d\Omega)_{\mathrm{D}}}{(d\sigma/d\Omega)_{\mathrm{H}}}=4.2\pm1.3.$$

From Eq. (2) one finds that the calculated ratio is 3.2 for constructive interference and 0.8 for destructive interference.

We wish to thank K. A. Brueckner and J. Chappelear for permission to quote their results prior to publication. We are also indebted to R. H. Dalitz for several discussions concerning the theoretical implications of this work.

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[17] Ernstene, Keck, and Tollestrup (private communication).

[18] K. A. Brueckner, Phys. Rev. **89**, 834 (1953).

[19] K. A. Brueckner and J. Chappelear (private communication).

[20] At the most forward angle the theoretical curves are changed only by a few percent if a $\sin^2\theta$ distribution is used instead of a $2+3\sin^2\theta$ distribution. At 130°, however, the theoretical values decrease by about 35 percent.

[21] H. L. Davis and D. R. Corson, following paper [Phys. Rev. **99**, 273 (1955)].