# Photoproduction of Mesons in Deuterium

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The corrections from multiple scattering of the photoproduced meson to the usual impulse approximation to the elastic photoproduction cross section for neutral mesons in deuterium have been computed. The result obtained for gamma-ray energies of 285 Mev and 345 Mev is a depression of the cross section of about a factor of two at all angles and at both energies. This is in qualitative accord with experiment.

## 1. INTRODUCTION

THE theory of neutral photomeson production in deuterium has been considered by many authors.[1-4] These treatments were phenomenological and based on the impulse approximation; they therefore neglected the possibility of a final state interaction of the photoproduced meson with the deuteron. Since the meson-nucleon interaction is quite strong, it is expected that the cross section will be considerably modified. This interaction can be described as the consequence of multiple scattering, and has been treated in connection with scattering of mesons in deuterium.[5] In this paper we shall extend previous calculations to include the corrections to the impulse approximation which arise from this effect.

The method to be described retains the phenomenological features of the previous calculations by the following assumptions; first, a transition operator is used which yields a $(2+3\sin^2\theta)$ distribution for $\pi^0$ production from hydrogen. The physical basis for choosing this transition operator is the assumption that $\pi^0$ production takes place in the state with $I=\frac{3}{2}$ and $J=\frac{3}{2}$;[6] the distribution predicted seems to be in accord with experiment.[7] Following this model it is assumed that the scattering also has a resonance in this state. The parameters of the theory then may be determined by comparison with the measured transition amplitudes for photoproduction and scattering in hydrogen. Second, off-the-energy-shell scattering has been neglected. It will be seen that the nature of this approximation is such as to lead us not to expect agreement of more than qualitative nature between the theoretical and experimental results. There are several reasons for this neglect. First, there is some reason to believe that the range of the corrections due to multiple virtual scattering is less than that due to multiple real scattering, and therefore that most of the correction comes from the latter. Second, the present state of meson theory does not allow reliable predictions of the behavior of the scattering amplitudes off the energy shell. Since our treatment is phenomenological, it seems best to omit this type of calculation. Third, the difficulties of solving the multiple scattering problem including virtual mesons are very considerable, since numerical solution of coupled integral equations is required.

The procedure for the calculation is quite straightforward. A formal solution to the Schrödinger equation for the problem is constructed following the method of Chew and Goldberger[8] and of Watson[9] and making use of approximations similar to the impulse approximation to simplify the results. Finally the formal solution is evaluated in terms of the known expressions for the operators which are involved. The elastic differential cross section is computed for photoproduction of $\pi^0$ mesons. From these results it is also easy to obtain the usual impulse approximation to the cross section, so that the effects of the multiple scattering can readily be seen. The graphs and numerical results are discussed in Sec. III.

## 2. FORMAL METHODS

Notation: Let the two nucleons be numbered 1 and 2.

$h_{1,2}$ = interaction terms in the Hamiltonian between the meson field and the nucleon field.

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$h=h_1+h_2$.

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$\mathcal{H}_0$ = the sum of the free-field Hamiltonians.

$\mathcal{H}'=h+H$.

$\mathcal{H}$ = the total Hamiltonian $=\mathcal{H}_0+\mathcal{H}'$.

$\psi$ = the solution to the Schrödinger equation for the problem.

$\phi$ = the initial state (a deuteron plus a gamma ray).

$\frac{1}{a}=\frac{1}{E-\mathcal{H}_0+i\epsilon}$, the Green's function for the problem.

$T=\mathcal{H}'+\mathcal{H}'\frac{1}{a-\mathcal{H}'}\mathcal{H}'$, the transition operator for the problem.

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$t_i = h_i + h_i \frac{1}{a-h_i} h_i$, the transition operator for scattering at nucleon $i$.

$$T_i = h_i + H_i + (h_i + H_i)\frac{1}{a-h_i-H_i}(h_i+H_i)$$

$$\simeq \left[1+h_i\frac{1}{a-h_i}\right] H_i \left[1+\frac{1}{a-h_i}h_i\right],$$

if one neglects terms not linear in $H_i$, the transition operator for photomeson production from the $i$th nucleon alone.

We start from the Schrödinger equation for the problem written symbolically in integral form:

$$\psi = \psi_0 + \frac{1}{a}\mathcal{H}'\psi. \tag{2.1}$$

A formal solution to the Schrödinger equation can be written, following the method of Chew and Goldberger,[8] as

$$\psi = \left[1+\frac{1}{a}T\right]\phi, \tag{2.2}$$

where the transition operator $T$ is, to terms linear in the weak gamma-ray interaction $H_i$,

$$T = \left[1+h\frac{1}{a-h}\right] H \left[1+\frac{1}{a-h}h\right]. \tag{2.3}$$

We now wish to replace the interaction Hamiltonians which appear in this expression by the related (and observable) transition operators for scattering and photoproduction on a single nucleon. To do this, we proceed in the following way: first, using the definitions for $T_i$, we find that Eq. (2.2) for $T$ is formally equivalent to

$$T = \left[1+h\frac{1}{a-h}\right]\left[\left[1+h_1\frac{1}{a-h_1}\right]^{-1}\right.$$
$$\times T_1\left[1+\frac{1}{a-h_1}h_1\right]^{-1} + \left[1+h_2\frac{1}{a-h_2}\right]^{-1}$$
$$\left.\times T_2\left[1+\frac{1}{a-h_2}h_2\right]^{-1}\right]\left[1+\frac{1}{a-h}h\right]. \tag{2.4}$$

We introduce our first approximation by assuming that

$$\{1+[1/(a-h_1)]h_1\}^{-1}\{1+[1/(a-h)]h\}$$

and

$$\{1+[1/(a-h_2)]h_2\}^{-1}\{1+[1/(a-h)]h\}$$

are equal to 1. Physically, this means that we neglect one nucleon's influence on the photoproduction at the other nucleon before the gamma ray is absorbed. The exchange of these virtual mesons also serves to bind the deuteron, and we will include this effect by using the correct deuteron wave function. Next, eliminating the meson-nucleon interaction terms $h_i$ by a series of formal manipulations, we are led to the result,

$$T = a\left[1-\frac{1}{a}t_1\frac{1}{a}t_2\right]^{-1}\left[\frac{1}{a}T_1+\frac{1}{a}t_1\frac{1}{a}T_2\right]+(1\rightleftarrows 2). \tag{2.5}$$

We can interpret this equation for $T$ physically. If the denominator were expanded in a power series, the first terms in the expansion would be $T_1$ and $T_2$, the usual impulse approximation. The next two terms are $t_1(1/a)T_2$ and $t_2(1/a)T_1$. These terms represent the first correction to the impulse approximation due to one scattering at the other nucleon after photoproduction. The other terms represent all higher order corrections of this type.

## 3. COMPUTATION

We assume that the scattering takes place in $P$ states and that photoproduction takes place in $S$ and $P$ states as predicted in reference 5. Accordingly, we choose a form for $t_i$ in momentum space:

$$\langle q|t_i|q'\rangle = b_i\mathbf{q}\cdot\mathbf{q}'e^{-i(\mathbf{q}-\mathbf{q}')\cdot\mathbf{r}_i}, \tag{3.1}$$

where $b_i$ is a matrix in charge space and is also dependent on energy. We have also a momentum representation for $T_i$,

$$\langle q|T_i|\phi\rangle = (\alpha_i+\boldsymbol{\gamma}_i\cdot\mathbf{q})e^{-i(\mathbf{q}-\mathbf{K})\cdot\mathbf{r}_i}, \tag{3.2}$$

where $\alpha_i$ and $\boldsymbol{\gamma}_i\cdot\mathbf{q}$ are matrices in charge space which contain the spin and energy dependence of the photoproduction transition matrix $|\phi\rangle$ is the initial state of the system containing a gamma ray of momentum $\mathbf{R}$ and two nucleons at $\mathbf{r}_1$ and $\mathbf{r}_2$. The vector $\mathbf{q}$ is the momentum of the photoproduced meson.

To solve our problem we need a representation of the inverse operator $[1-(1/a)t_1(1/a)t_2]^{-1}$ which appears in Eq. (2.5). Let

$$y = \left[1-\frac{1}{a}t_1\frac{1}{a}t_2\right]^{-1}. \tag{3.3}$$

Then we find that $y$ satisfies the integral equation

$$y = 1 + yt_1\frac{1}{a}t_2\frac{1}{a}. \tag{3.4}$$

In momentum space $1/a$ is diagonal.

$$\left\langle q\left|\frac{1}{a}\right|q'\right\rangle = \frac{1}{E-\omega+i\epsilon}\delta(\mathbf{q}-\mathbf{q}'). \tag{3.5}$$

We find from Eqs. (3.1) and (3.5) that the matrix element of $t_1(1/a)t_2$ is

$$\left\langle q\left|t_1\frac{1}{a}t_2\right|q'\right\rangle=-e^{-i(\mathbf{q}\cdot\mathbf{r}_1-\mathbf{q}'\cdot\mathbf{r}_2)}\mathbf{q}\cdot\nabla_R\mathbf{q}'\cdot\nabla_R \times\int b_1b_2\frac{d^3q}{(2\pi)^3}\frac{e^{i\mathbf{q}\cdot\mathbf{R}}}{E-\omega+i\epsilon}, \quad (3.6)$$

where $\mathbf{R}=\mathbf{r}_1-\mathbf{r}_2$. In evaluating this integral we shall retain only the contribution from the pole on the energy shell, i.e., we suppose that $b_1$ and $b_2$ have no poles which contribute appreciably to the integral. This is equivalent to the assumption that the scatterings on the energy shell give the principal contribution to the correction due to multiple scattering. The evaluation is then elementary and we find that

$$\left\langle q\left|t_1\frac{1}{a}t_2\right|q'\right\rangle=e^{-i(\mathbf{q}\cdot\mathbf{r}_1-\mathbf{q}'\cdot\mathbf{r}_2)}\left[\frac{\omega_E}{2\pi}\right] \times b_1b_2\mathbf{q}\cdot\nabla_R\mathbf{q}'\cdot\nabla_R\frac{e^{iq_ER}}{R}. \quad (3.7)$$

We have introduced $q_E$ as the momentum vector which conserves energy with the incident gamma ray and $\omega_E=(q_E^2+u^2)^{\frac{1}{2}}$. Where no confusion can result, $q_E$ will often be called $q$. If we introduce the functions $f$ and $g$ defined by

$$f(R)=\frac{1}{R}\frac{d}{dR}\frac{e^{iqR}}{R}, \quad (3.8)$$

and

$$g(R)=\frac{1}{R}\frac{d}{dR}f(R), \quad (3.9)$$

we obtain

$$\left\langle q\left|t_1\frac{1}{a}t_2\right|q'\right\rangle=b_1b_2\frac{\omega_E}{2\pi}(f\mathbf{q}\cdot\mathbf{q}'+\mathbf{q}\cdot\mathbf{R}\mathbf{q}'\cdot\mathbf{R}g). \quad (3.10)$$

This result can be substituted in Eq. (3.4). We introduce an auxiliary function,

$$\mathbf{S}(q)=\int\langle q|y\mathbf{q}'e^{-i\mathbf{q}'\cdot\mathbf{r}_1}|q'\rangle\frac{d^3q'}{(2\pi)^3}. \quad (3.11)$$

We find the equation for $\mathbf{S}$ by operating on Eq. (3.4).

$$\mathbf{S}=\mathbf{q}e^{-i\mathbf{q}\cdot\mathbf{r}_1}+\mathbf{S}\left[\frac{\omega_E}{2\pi}\right]b_1b_2\cdot\int\frac{d^3q'}{(2\pi)^3} \times(f\mathbf{q}'+g\mathbf{R}\mathbf{q}'\cdot\mathbf{R})\frac{e^{-i\mathbf{q}'\cdot\mathbf{R}}}{E-\omega'+i\epsilon}. \quad (3.12)$$

If we perform the integration and put $c_i=-(\omega_E/2\pi)b_i$ we find that

$$\mathbf{S}=\mathbf{q}e^{-i\mathbf{q}\cdot\mathbf{r}_1}+\mathbf{S}c_1c_2f^2+\mathbf{S}\cdot\mathbf{R}(2fg+g^2R^2)c_1c_2\mathbf{R}. \quad (3.13)$$

We can solve for $\mathbf{S}$ by scalar multiplication by $\mathbf{R}$. The resulting equation for $\mathbf{S}\cdot\mathbf{R}$ is

$$\mathbf{S}\cdot\mathbf{R}=\frac{1}{1-c_1c_2h^2}\mathbf{q}\cdot\mathbf{R}e^{-i\mathbf{q}\cdot\mathbf{r}_1}, \quad (3.14)$$

where we have introduced the abbreviation

$$h=f+R^2g. \quad (3.15)$$

By substitution in Eq. (3.13) we obtain an equation for $\mathbf{S}$. The solution for $\mathbf{S}$ is

$$\mathbf{S}=\frac{1}{1-f^2c_1c_2}e^{-i\mathbf{q}\cdot\mathbf{r}_1}\mathbf{q}+\mathbf{R}\frac{\mathbf{q}\cdot\mathbf{R}}{R^2}e^{-i\mathbf{q}\cdot\mathbf{r}_1} \times\left[\frac{1}{1-h^2c_1c_2}-\frac{1}{1-f^2c_1c_2}\right]. \quad (3.16)$$

If we define

$$P(q)=\int\langle q|ye^{-i\mathbf{q}'\cdot\mathbf{r}_1}|q'\rangle\frac{d^3q'}{(2\pi)^3}, \quad (3.17)$$

a similar calculation shows that

$$P=e^{-i\mathbf{q}\cdot\mathbf{r}_1}-i\mathbf{q}\cdot\mathbf{R}\frac{e^{-i\mathbf{q}\cdot\mathbf{r}_1}}{1-h^2c_1c_2}fhc_1c_2. \quad (3.18)$$

From Eq. (3.14) we see that

$$\langle q|T|\phi\rangle=\int\frac{d^3q'}{(2\pi)^3}\langle q|y|q'\rangle\left[\langle q'|T_1|\phi\rangle +\int\frac{d^3q''}{(2\pi)^3}\langle q'|t_1|q''\rangle\frac{1}{E-\omega''+i\epsilon}\langle q''|T_2|\phi\rangle\right]. \quad (3.19)$$

Using the definitions of $T_1$ and $T_2$, we finally obtain

$$\begin{aligned}\langle q|T|\phi\rangle=&\left(1-i\mathbf{q}\cdot\mathbf{R}\frac{1}{1-h^2c_1c_2}fhc_1c_2\right)\alpha_1e^{i(\mathbf{K}-\mathbf{q})\cdot\mathbf{r}_1}\\&+\left\{\frac{1}{1-f^2c_1c_2}\mathbf{q}\cdot\boldsymbol{\gamma}_1+\frac{1}{R^2}\left(\frac{1}{1-h^2c_1c_2}-\frac{1}{1-f^2c_1c_2}\right)\times\mathbf{q}\cdot\mathbf{R}\boldsymbol{\gamma}_1\cdot\mathbf{R}\right\}e^{i(\mathbf{K}-\mathbf{q})\cdot\mathbf{r}_1}\\&+\left\{i\mathbf{q}\cdot\mathbf{R}\frac{1}{1-h^2c_1c_2}fc_1\alpha_2-\left[\frac{1}{1-f^2c_1c_2}fc_1\mathbf{q}\cdot\mathbf{r}_2+\frac{1}{R^2}\left(\frac{h}{1-h^2c_1c_2}-\frac{f}{1-f^2c_1c_2}\right)\times c_1\mathbf{q}\cdot\mathbf{R}\boldsymbol{\gamma}_1\cdot\mathbf{R}\right]\right\}e^{i(\mathbf{K}\cdot\mathbf{r}_2-\mathbf{q}\cdot\mathbf{r}_1)}+(1\rightleftarrows 2).\end{aligned} \quad (3.20)$$

To compute the elastic cross section for $\pi^0$ production we must average $\langle q|T|\phi\rangle$ over the square of the deuteron wave function.[1] We perform the average over the angles of $\mathbf{R}$ and introduce

$$\mathbf{l}=\tfrac{1}{2}(\mathbf{K}-\mathbf{q}), \tag{3.21}$$

and

$$\mathbf{m}=\tfrac{1}{2}(\mathbf{K}+\mathbf{q}). \tag{3.22}$$

Then Eq. (3.20) can be written as

$$\begin{aligned}\langle q|T|\phi\rangle=&\left(1-\mathbf{q}\cdot\nabla_l\frac{fh}{1-h^2c_1c_2}c_1c_2\right)\alpha_1\frac{\sin lR}{lR}\\&+\left[\frac{1}{1-f^2c_1c_2}\mathbf{q}\cdot\boldsymbol{\gamma}_1-\frac{1}{R^2}\left(\frac{1}{1-h^2c_1c_2}\right.\right.\\&\left.\left.-\frac{1}{1-f^2c_1c_2}\right)(\mathbf{q}\cdot\nabla_l)(\boldsymbol{\gamma}_1\cdot\nabla_l)\right]\frac{\sin lR}{lR}\\&+\left\{\frac{f}{1-f^2c_1c_2}\mathbf{q}\cdot\nabla_m c_1\alpha_2-\left[\frac{f}{1-h^2c_1c_2}c_1\mathbf{q}\cdot\boldsymbol{\gamma}_2\right.\right.\\&-\frac{1}{R^2}\left(\frac{h}{1-h^2c_1c_2}-\frac{f}{1-f_2c_1c_2}\right)c_1(\mathbf{q}\cdot\nabla_m)\\&\left.\left.\times(\boldsymbol{\gamma}_3\cdot\nabla_m)\right]\right\}\times\frac{\sin mR}{mR}+(1\rightleftarrows 2).\end{aligned} \tag{3.23}$$

We now compute the isotopic spin dependence of the transition operator. This is done in the Appendix. The results are

$$c_1=b\begin{bmatrix}2 & \sqrt{2}\\ \sqrt{2} & 1\end{bmatrix}, \tag{3.24}$$

and

$$c_2=b\begin{bmatrix}2 & -\sqrt{2}\\ -\sqrt{2} & 1\end{bmatrix}. \tag{3.25}$$

We also compute $\mathbf{q}\cdot\boldsymbol{\gamma}_1$ and $\mathbf{q}\cdot\boldsymbol{\gamma}_2$ in the appendix [Eqs. (5.8) and (5.9)].

The states used as basis vectors for this representation are the two states of the meson—two nucleon system with total isotopic spin equal to unity; $b$ is $\lambda^3e^{i\delta}\sin\delta$. To obtain this result we have introduced the assumption that scattering takes place in the state with isotopic spin equal to three halves. If we introduce dimensionless variables

$$\mathbf{x}=\mathbf{q}R, \tag{3.26}$$

$$\mathbf{y}=\mathbf{K}R, \tag{3.27}$$

$$\mathbf{w}=\mathbf{m}R, \tag{3.28}$$

$$\mathbf{z}=\mathbf{l}R, \tag{3.29}$$

then the transition matrix element for elastic $\pi^0$ production can be shown to be

$$\begin{aligned}\langle q|T_{MS}|\phi\rangle=&\frac{1}{K}\frac{2\pi}{(12)^{\frac{1}{2}}}\frac{v}{(wq)^{\frac{1}{2}}}\left\{\left[\left(\frac{\sin z}{z}+f\frac{\sin w}{w}\right)\right.\right.\\&\times\left(\frac{1}{1-f^2}+\frac{1}{2}\left(\frac{1}{1-h^2}-\frac{1}{1-f^2}\right)\right)\\&\times[-2a(z)+\mathbf{x}\cdot\mathbf{z}b(z)]-\frac{1}{2}\left(\frac{h}{1-h^2}\right.\\&\left.\left.-\frac{f}{1-f^2}\right)[-2a(w)+\mathbf{x}\cdot\mathbf{w}b(w)]\right]\\&\times[x_1(1)+x_1(2)]-\frac{i}{2}\frac{y}{x}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{\varepsilon}\\&\times\left[\mathbf{x}\cdot\mathbf{z}\left(\frac{1}{1-h^2}-\frac{1}{1-f^2}\right)b(z)\right.\\&\left.\left.+\mathbf{x}\cdot\mathbf{w}\left(\frac{h}{1-h^2}-\frac{f}{1-f^2}\right)b(w)\right]\right\}.\end{aligned} \tag{3.30}$$

The various symbols used in the equation are defined as follows: $v$ is a term which contains part of the energy dependence of the cross section. Its value is determined by comparison with the $\pi^0$ production cross section in hydrogen. The functions $f$, $g$, and $h$ are functions now of $x$, and are redefined as

$$f(x)=\frac{e^{i(x+\delta)}}{x}\sin\delta, \tag{3.31}$$

$$g(x)=\frac{1}{x}\frac{d}{dx}f(x), \tag{3.32}$$

$$h(x)=f(x)+x^2g(x). \tag{3.33}$$

We have also (reference 6):

$$x_i=\frac{1}{qK}\{2\mathbf{q}\cdot(\mathbf{K}\times\boldsymbol{\varepsilon})-i\boldsymbol{\sigma}_i\cdot[\mathbf{q}\times(\mathbf{K}\times\boldsymbol{\varepsilon})]\}. \tag{3.34}$$

Finally,

$$a(z)=\frac{1}{z}\frac{d}{dz}\frac{\sin z}{z}, \tag{3.35}$$

and

$$b(z)=\frac{1}{z}\frac{d}{dz}a(z). \tag{3.36}$$

The usual impulse approximation is obtained by putting $\delta=0$. Then $f=h=0$. We find that

$$\langle q|T_{IA}|\phi\rangle=\frac{1}{K}\frac{2\pi}{(12)^{\frac{1}{2}}}\frac{v}{(\omega q)^{\frac{1}{2}}}\frac{\sin z}{z}[x_1(1)+x_1(2)]. \tag{3.37}$$

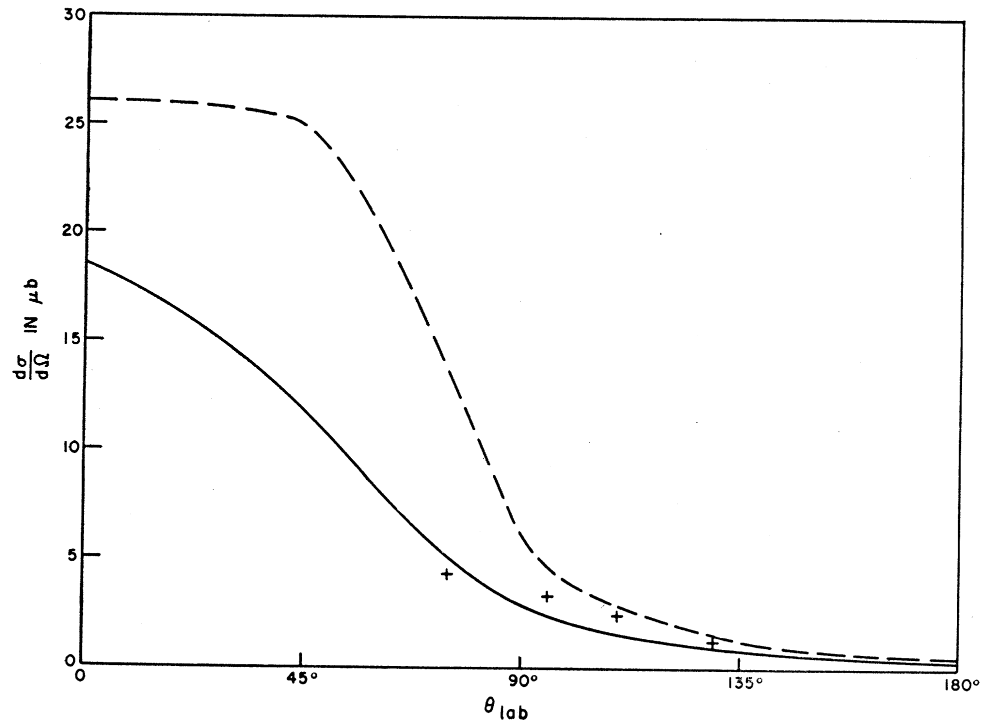


FIG. 1. Photoproduction cross section as a function of emitted meson angle in the laboratory system at a gamma-ray energy of 285 Mev. The dashed line is the impulse approximation, the solid line is the impulse approximation with multiple scattering corrections, and the crosses are experimental points from Silverman *et al.* (reference 10).

In obtaining these results we have made use of the assumption of reference 6, that photoproduction of neutron mesons takes place in a state with $J=\frac{3}{2}$ (magnetic dipole). We have not included contributions from production of charged mesons in $S$-states. This contribution has been evaluated numerically and found to be small. We introduce a simplified form of $\langle q|T_{MS}|\phi\rangle$.

$$\langle q|T_{MS}|\phi\rangle=\frac{1}{K}\frac{2\pi}{(12)^{\frac{1}{2}}}\frac{v}{(\omega q)^{\frac{1}{2}}}\times[A[x_1(1)+x_1(2)]+B(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{\varepsilon}]. \quad (3.38)$$

Let the averages over $\varphi_D{}^2(R)$ be denoted by $\bar{A}$, $\bar{B}$, and $\langle(\sin z)/z\rangle_{\text{Av}}$. Then the cross section is

$$\frac{d\sigma_{MS}}{d\Omega}=\frac{|v|^2}{9}\frac{1}{K^2}[|\bar{A}|^2(5\sin^2\theta+2)+|\bar{B}|^2-2\cos\theta\,\mathrm{Re}(A^*B)]. \quad (3.39)$$

We obtain the usual impulse approximation by averaging and squaring Eq. (3.37). We find

$$\frac{d\sigma_{SA}}{d\Omega}=\frac{|v|^2}{9}\frac{1}{K^2}(5\sin^2\theta+2)[\langle(\sin z)/z\rangle_{\text{Av}}]^2. \quad (3.40)$$

If we use the Hulthén wave function,

$$\varphi_D(R)=\left(\frac{28}{9}\alpha\right)^{\frac{1}{2}}\left(\frac{e^{-\alpha R}-e^{-7\alpha R}}{R}\right), \quad (3.41)$$

where $\alpha=45.5$ Mev, we find that

$$\langle(\sin z)/z\rangle_{\text{Av}}=\frac{28}{9}\frac{\alpha}{l}\left[-\tan^{-1}\left(\frac{2\alpha}{l}\right)+2\tan^{-1}\left(\frac{8\alpha}{l}\right)-\tan^{-1}\left(\frac{14\alpha}{l}\right)\right]. \quad (3.42)$$

The functions $\bar{A}$ and $\bar{B}$ must be evaluated numerically. This was done at gamma-ray energies of 285 and 345 Mev. These energies correspond to meson phase shifts for scattering of about 45° (135 Mev) and 90° (200 Mev). In Fig. 1 we show the results of the calculation in the laboratory system at $K=285$ Mev. The experiments of Silverman *et al.*[7] are also plotted. We see that the multiple scattering tends to depress the cross section by roughly the same amount at all angles. The magnitude of the cross section was fixed by fitting to the $\pi^0$ cross section in hydrogen.[10]

In Fig. 2 we also plot the results at $K=345$ Mev in the center-of-mass system. The results are approximately the same, a depression of the cross section at all angles.

We have also computed the cross section for a transition operator which yields a $\sin^2\theta$ distribution for $\pi^0$ production in hydrogen. The result is shown in Fig. 3 for $K=285$ Mev. The correction factor due to multiple scattering is about the same as in Fig. 1.

[10] A. Silverman and M. Stearns, Phys. Rev. **88**, 1225 (1952).

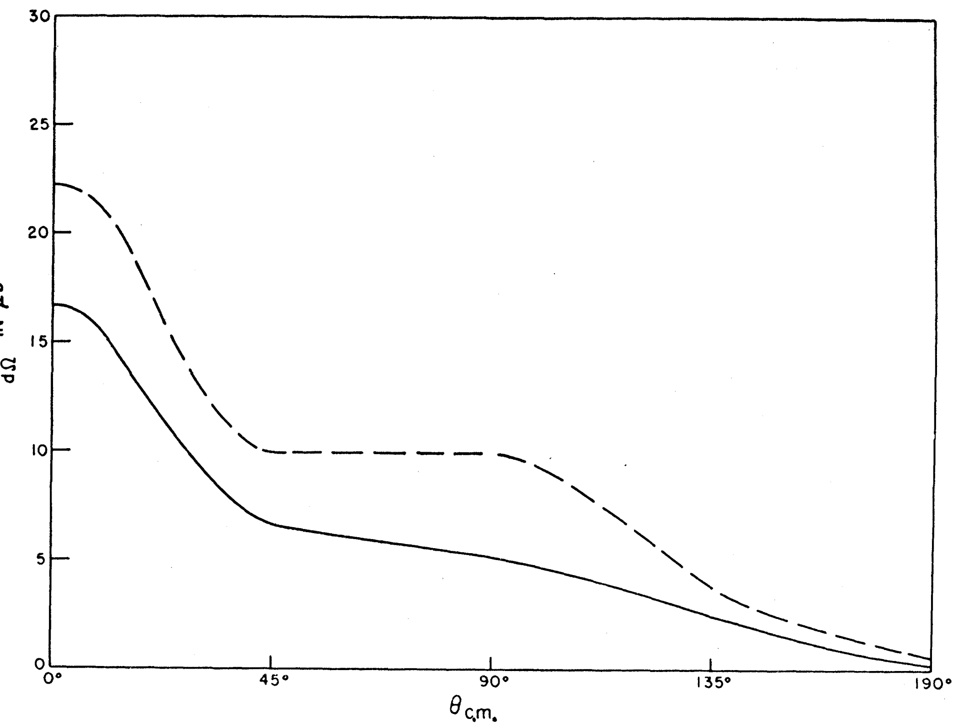


FIG. 2. Photoproduction cross section as a function of emitted meson angle in the center-of-mass system at a gamma-ray energy of 345 Mev. The significance of the curves is the same as in Fig. 1.

## 4. CONCLUSIONS

The possibility that the observed depression of the photoproduction cross section for mesons in deuterium can be explained in part by inclusion of the effects of multiple scattering in the calculations seems to be substantiated by these results. We see that measurement of the differential cross section at angles further forward is suggested, in order to look for a quantitative check on the theory. We also see that little information can be obtained as to what order of radiative transition is involved from measurements at these energies. The quantitative agreement with experiment is only fair.

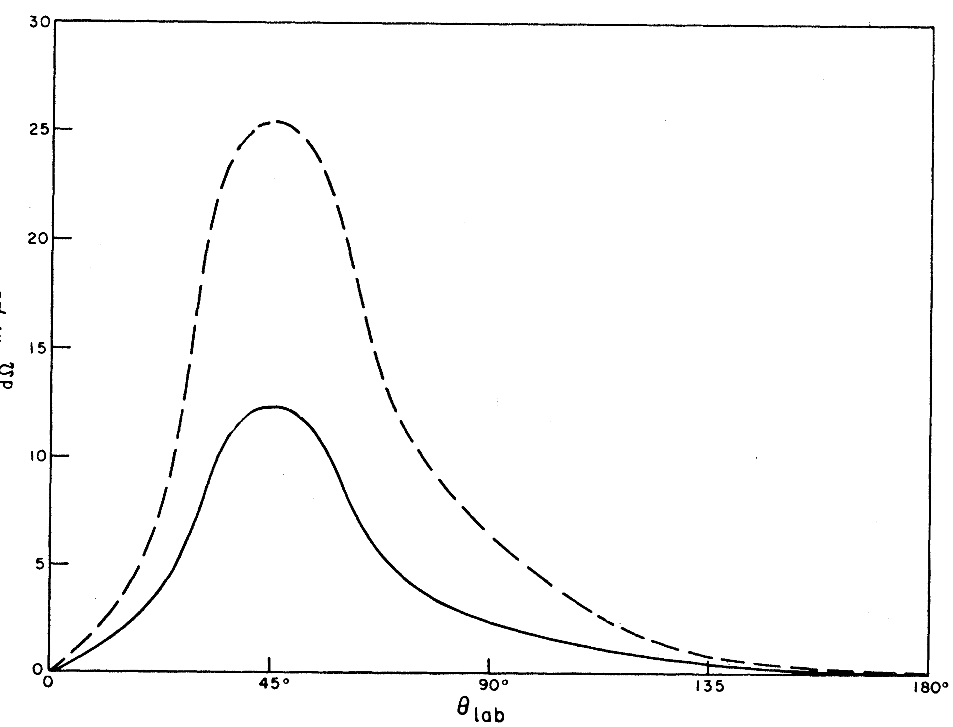


FIG. 3. Photoproduction cross section as a function of emitted meson angle in the laboratory system at 285 Mev with a transition operator yielding a $\sin^2\theta$ distribution. The significance of the curves is the same as in Fig. 1.

This can be explained by remembering that only scattering on the energy shell was considered. Scattering off the energy shell, which is expected to be important for small nucleon separations, would raise the cross section, perhaps enough to obtain such agreement.

## 5. APPENDIX

The most general form for the operators $c_i$ in charge space is

$$c_i = a + b\boldsymbol{\tau}_i \cdot \mathbf{I}, \tag{5.1}$$

where

$$\mathbf{I} = i\mathbf{U} \times \mathbf{U}^\dagger. \tag{5.2}$$

Reference 8 contains a complete discussion of these points. $a$ and $b$ are scalar functions in charge space, independent of the isotopic spin. $\mathbf{I}$ is an operator in charge space which has the properties of an isotopic angular momentum. The components $U_i$ and $U_i^\dagger$ annihilate and create the $i$th component of the meson wave.

If scattering takes place in the state with $I=\frac{3}{2}$, we must have

$$a = 2b. \tag{5.3}$$

Then

$$c_i = b(2 + \boldsymbol{\tau}_i \cdot \mathbf{I}). \tag{5.3}$$

To obtain a representation for $c_i$ we choose a set of basis vectors as follows:

Representation 1

| | | | |
|---|---|---|---|
| I | $t_0^0 w_1^0$ | 2 nucleon singlet | $+\pi^0$ |
| II | $t_1^0 w_1^0$ | 2 nucleon triplet | $+\pi^0$ |
| III | $t_1^{-1} w_1^1$ | 2 neutrons | $+\pi^+$ |
| IV | $t_1^1 w_1^{-1}$ | 2 protons | $+\pi^-$ |

Using these states for basis vectors we can construct the matrix for $c_1$. It is

$$c_1 = \begin{bmatrix} 2 & 0 & 1 & -1 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix}. \tag{5.5}$$

If we choose a representation in which the total isotopic angular momentum is a constant of the motion,

Representation 2

| | $t$ | $l$ | $I$ |
|---|---|---|---|
| I | 0 | 1 | 1 |
| II | 1 | 1 | 0 |
| III | 1 | 1 | 1 |
| IV | 1 | 1 | 2 |

where $t$ = isotopic spin of 2 nucleons; $l$ = isotopic spin of mesons; $I$ = total isotopic spin. We can transform $c_1$ into this representation by use of the Clebsch-Gordan coefficients.[11] Then we find

$$c_1 = \begin{bmatrix} 2 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 0 \\ \sqrt{2} & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}. \tag{5.6}$$

Similarly we obtain

$$c_2 = \begin{pmatrix} 2 & -\sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix}. \tag{5.7}$$

We need retain only two rows and columns (the two states with $I=1$) since the photoproduction takes place in the state with $I=1$ and the scattering operators do not change $I$. We can also compute $\langle q|T_1|\phi\rangle$ and $\langle q|T_2|\phi\rangle$. We find

$$\langle q|T_1|\phi\rangle = \frac{e^{-i(\mathbf{q}-\mathbf{K})\cdot\mathbf{r}_1}}{K} \frac{2\pi}{(12)^{\frac{1}{2}}} \frac{x_1(1)v}{(\omega_q)^{\frac{1}{2}}} \begin{bmatrix} 1 \\ 1/\sqrt{2} \end{bmatrix} \tag{5.8}$$

and

$$\langle q|T_2|\phi\rangle = \frac{e^{-i(\mathbf{q}-\mathbf{K})\cdot\mathbf{r}_2}}{K} \frac{2\pi}{(12)^{\frac{1}{2}}} \frac{x_1(2)v}{(\omega_q)^{\frac{1}{2}}} \begin{bmatrix} 1 \\ -1/\sqrt{2} \end{bmatrix}. \tag{5.9}$$

This result is obtained by writing a transition operator which leads to the results of reference 6, and then computing the matrix elements for it in representation 2. To obtain this result we have assumed that production in $S$ states does not contribute. This is not correct, but numerical evaluation shows that this neglect is unimportant for the $\pi^0$ production cross section.

By reference to Eq. (3.2) we can obtain $\boldsymbol{\gamma}_i \cdot \mathbf{q}$ which we need in computing our cross section.


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[11] E. U. Condon and G. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, London, 1935).