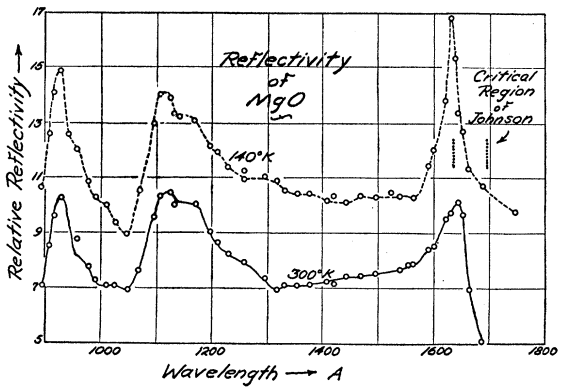

FIG. 2. Relative reflectivity of MgO as a function of wavelength.

peak is a truly fundamental absorption band with absorption coefficients of the order of 10^6 cm^{-1}, and not weak absorption due to impurity levels. This peak, indeed, confirms the results of Johnson[5] who made fluorescence and transmission measurements on thin crystals of MgO of known impurity content and also on samples of lower impurity content from the University of Missouri. Johnson found that in the region (indicated in Fig. 2) from 1635 A to 1695 A there is a rapid change in the excitation spectrum for fluorescence, and that at 1695 A the absorption coefficient is rising very rapidly. From these results, he concluded that there must be fundamental absorption in this region.

Before considering the shorter wavelength peaks, it should be pointed out that there is no resolvable splitting in the exciton peak of MgO at low temperature as we have observed in NaCl and KCl and as has also been seen in BaO.[6]

The temperature-independent peaks at 1120 A and 930 A are tentatively interpreted as absorption due to transitions from the uppermost filled band to the normally empty conduction band. This absorption band begins at about 1240 A; the band gap, then, would be about 10 ev and not 5.9 ev as reported recently by Saksena and Pant[7] who made studies on natural crystals of MgO.

In closing, it is interesting to compare this 10-ev band gap value with the approximate 11-ev value that Lempicki[8] gives as the threshold primary electron energy for secondary emission in MgO. This may be confirmation of his original supposition that the secondary electrons come from the uppermost filled band.

* Supported in part by the Office of Naval Research.
[1] Frederick Seitz, Revs. Modern Phys. **26**, 27 (1954).
[2] Nelson, Siegfried, and Hartman, Phys. Rev. **99**, 658(A) (1955).
[3] P. L. Hartman, Phys. Rev. **99**, 658(A) (1955).
[4] J. A. Krumhansl, "Photoexcitation in ionic crystals," Atlantic City Conference on Photoconductivity (to be published).
[5] P. D. Johnson, Phys. Rev. **94**, 845 (1954).
[6] R. J. Zollweg, Phys. Rev. **97**, 288 (1955).
[7] B. D. Saksena and L. M. Pant, J. Chem. Phys. **23**, 989 (1955).
[8] A. Lempicki, Proc. Phys. Soc. (London) **B66**, 278 (1953).

Theory of Acceptor Levels in Germanium*

WALTER KOHN† AND DANIEL SCHECHTER†
Carnegie Institute of Technology, Pittsburgh, Pennsylvania and Bell Telephone Laboratories, Murry Hill, New Jersey
(Received July 25, 1955)

IT is well known that holes in germanium can be bound by Group III impurities in so-called acceptor levels. The observed ionization energies of such holes are 0.0104 ev (B),[1,2] 0.0102 ev (Al),[2] 0.0108 ev (Ga),[2] and 0.0112 ev (In).[2] The fact that these energies differ by only 10% suggests that the binding is largely due to the long-range Coulomb potential of the acceptors rather than to the less well understood forces in their immediate vicinity.

We have therefore carried out a theoretical calculation using a Coulomb potential[3] $-e^2/\kappa r$ and "mass"-parameters determined by cyclotron resonance experiments.[4] The resulting theoretical ionization energy is 0.0089 ev, in rather good agreement with experiment.

The calculation was based on the coupled effective mass equations which have recently been shown to arise when the band structure is degenerate.[5–7] In view of the fact that the spin-orbit splitting at the top of the valence band in Ge is about 30 times as large as the ionization energy of the acceptors,[8] the four coupled equations (V.15) of reference 7 are a good first approximation. Group-theoretical considerations suggest a trial function of the form (see reference 7):

$$ae^{-r/r_1}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}+be^{-r/r_2}\begin{bmatrix}z^2-\frac{1}{2}(x^2+y^2)\\0\\-(\frac{3}{2})^{\frac{1}{2}}(x^2-y^2)\\0\end{bmatrix}+ic\,e^{-r/r_2}\begin{bmatrix}0\\(x+iy)z\\xy\\0\end{bmatrix}.$$

The parameters were varied to maximize the ionization energy. The value 0.00883 ev was obtained with the following parameters: $r_1=43.3\times10^{-8}$ cm, $r_2=33.8\times10^{-8}$ cm, $a=1.71\times10^{9}$ cm$^{-\frac{3}{2}}$, $b=-2.29\times10^{21}$ cm$^{-7/2}$, $c=4.97\times10^{21}$ cm$^{-7/2}$.

With this wave function as a starting point, the original six coupled equations (see reference 7, V.13) were treated by a perturbation-variation method, which increased the ionization energy by about 1% to 0.00893 ev.

A variational calculation, such as the present one, always leads to a low ionization energy. We estimate that the true eigenvalue of the six coupled equations lies in the range 0.0094 ± 0.0005 ev. The remaining discrepancies with the experimental values may be ascribable partly to the breakdown of the theory in the immediate vicinity of the acceptor ions and partly to inaccuracies of the mass parameters used.

Kittel and Mitchell[6] have reported a theoretical value of 0.022 ev for the ionization energy. The large discrepancy with our result and with experiment is due to an incorrect transformation of the Hamiltonian operator (see their concluding sentence). A better order of magnitude estimate is obtained from a simple

Bohr model with an effective mass $m^*=0.34m$ derived from the mean curvature of the "heavy" hole band. This gives an ionization energy of 0.018 ev.

Calculations of the acceptor levels in silicon are in progress.

* Work supported in part by the Office of Naval Research.

† Permanent address: Carnegie Institute of Technology, Pittsburgh, Pennsylvania.

[1] P. Debye, Phys. Rev. **91**, 208 (1953).
[2] T. Geballe and F. J. Morin, Phys. Rev. **95**, 1085 (1954).
[3] κ is the static dielectric constant which was taken to be 16.0; see H. B. Briggs, Phys. Rev. **77**, 287 (1950).
[4] Dresselhaus, Kip, and Kittel, Phys. Rev. **77**, 287 (1950). B. Lax *et al.*, Phys. Rev. **93**, 1418 (1954); for the present calculation we have used the values $A=-13.0\hbar^2/2m$, $|B|=8.7\hbar^2/2m$, and $|C|=11.4\hbar^2/2m$ which were kindly communicated to us by Dr. Dexter and Dr. Zeiger of M.I.T.
[5] W. Kohn and J. M. Luttinger, Phys. Rev. **96**, 529 (1954).
[6] C. Kittel and A. M. Mitchell, Phys. Rev. **96**, 1488 (1954).
[7] J. M. Luttinger and W. Kohn, Phys. Rev. **97**, 869 (1955).
[8] H. A. Kahn, Phys. Rev. **97**, 1647 (1955).

Generation of $1/f$ Noise by Levels in a Linear or Planar Array

S. Roy Morrison

Physics Laboratory, Sylvania Electric Products, Bayside, New York

(Received June 9, 1955)

TRAPPING levels in a linear or planar array (at edge dislocations or the surface), or agglomerates of levels, will possess the following important property. A trapped charge will cause a potential barrier to further trapping of like charges, and a fluctuation in the trapped charge will produce a proportional fluctuation in the barrier height (although the energy of ionization from the levels will be constant). We will show that trapping with this simple barrier property may lead to $1/f$ noise.

This barrier property has been associated with the Elovich equation[1-3] which describes in numerous cases the rate of irreversible adsorption. Thus, the application of the model to the many apparently disconnected cases of $1/f$ noise is possible.

Denoting by N the trapped charge in excess of equilibrium, we obtain from an analysis of the model:

$$dN/dt=B(e^{\beta N}-1), \tag{1}$$

where β is related to the "capacitance" between the levels and the bulk material, B represents the equilibrium rate at which charge crosses the barrier, and is very sensitive to temperature. The first term is thus the trapping rate, the second the ionization rate.

From Eq. (1),

$$N=-\beta^{-1}\ln(1-Ae^{-\gamma t}), \tag{2}$$

with A the constant of integration, $\gamma=-\beta B$.

If one assumes that the distribution $g(N)$ of N over the levels is Gaussian, with mean $\bar{N}$ and standard deviation ξ, the autocorrelation function can be calculated. The use of this distribution function is not critical to the theory. Then the spectral distribution of noise becomes:

$$G(\omega)=-4\int_{N=-\infty}^{\infty}\int_{\tau=0}^{\infty}N\beta^{-1}\ln\{1-Ae^{-\gamma\tau}\}\times g(N)\cos\omega\tau d\tau dN, \tag{3}$$

where $A=1-e^{-\beta N}$.

For large barriers, γ can be very low. For example, surface traps on germanium, as detected by field effect measurements,[4,5] have decay times about a minute at 20°C, yielding $\gamma=10^{-2}$ sec^{-1}.

If one assumes $\omega\gg\gamma$ as a lower limit, the contribution to $G(\omega)$ from the negative values of N turns out to be small, as the decay rate is slow.

For positive N, when the decay is fast, the logarithmic term in (3) can be expanded and the integration over t performed. Replacing the resulting summation by an integration, thus neglecting terms in γ/ω^2, and using the rough approximation that $\mathrm{Ci}(x)\sin x-\mathrm{Si}(x)\cos x+\frac{1}{2}\pi\cos x$ is a step function, zero for $x>1$ and $\pi/2$ for $x<1$, we obtain

$$G(\omega)=\tfrac{1}{2}\pi(\beta\omega)^{-1}\left\{\int_0^{\infty}Ng(N)dN-\int_0^{\beta^{-1}\ln(\omega/\gamma)}N(2\pi\xi^2)^{-\frac{1}{2}}\times\exp[-(N-\bar{N})^2/2\xi^2]dN\right\}; \tag{4}$$

and if $N-\bar{N}\ll\sqrt{2}\xi$ the second integral is a slowly varying function of ω and the $1/\omega$ distribution of noise is obtained. The requirement sets an upper limit on the $1/\omega$ spectrum, namely $\ln(\omega/\gamma)<\beta(\sqrt{2}\xi+\bar{N})$. It is interesting that the temperature-sensitive quantity γ does not appear in the expression for $G(\omega)$, except in the insensitive logarithmic form.

We have here developed the frequency spectrum of the trapped charge. Noise will appear in bulk conductivity measurements since charge trapped represents a decrease in current carriers. In contact or rectifier studies, the noise may arise from fluctuations in the barrier height; for small fluctuations the barrier height is proportional to the trapped charge. Thus the concepts presented may be extended to carbon contact devices and metal films, as well as semiconducting devices.

A more detailed discussion of the model and the experimental results on the field effect which led to this analysis will be published later.

The author is indebted to Professor H. Callen and Professor P. H. Miller, Jr., of the University of Pennsylvania for their helpful suggestions.

[1] H. A. Taylor and N. Thon, J. Am. Chem. Soc. **74**, 4169 (1952).
[2] D. Melnick, Thesis, Physics Department, University of Pennsylvania, 1954 (unpublished).
[3] S. R. Morrison, Advances in Catalysis **7**, 62 (1955).
[4] R. H. Kingston and A. L. McWhorter, Phys. Rev. **98**, 1191(A) (1955).
[5] J. Bardeen and S. R. Morrison, Physica **20**, 873 (1954).