exponentially with depth, which is not the case for the anomalous contribution to the field. In this note, therefore, the absorption will be expressed in terms of the "absorptivity" A, defined as the fraction of normally-incident electromagnetic energy absorbed by an effectively infinite depth of material.

A calculation on the same lines as that previously published for metals (e.g., Dingle[3]) shows that for semiconductors with not too low carrier concentrations:

$$A=\left(\frac{3\omega h^3}{2\pi^2 m^2 e^2}\right)^{\frac{1}{2}}\mathrm{Re}\left[\left\{\frac{i}{\mathcal{V}\left(\frac{\tau v}{1+i\omega\tau}\right)+\frac{3i\mathfrak{p}\omega h^3}{32\pi^2 m^2 e^2}}\right\}^{\frac{1}{2}}\right]$$

$$+\tfrac{3}{4}(1-p)\frac{\omega}{c}\,\mathrm{Re}\left[\frac{i\mathcal{V}\left[\left(\frac{\tau v}{1+i\omega\tau}\right)^2\right]}{\mathcal{V}\left(\frac{\tau v}{1+i\omega\tau}\right)}\right], \quad (1)$$

where Re stands for "the real part," $\mathcal{V}$ for the integration operator $\mathcal{V}\equiv(-\int_0^\infty df_0\cdot v^2)$ with f_0 = Fermi-Dirac distribution function and v = carrier velocity, ω = circular frequency, m = effective carrier mass, τ = relaxation time, p = fraction of carriers specularly reflected at the internal surface, and $\mathfrak{p}$ is the quantity defined in Dingle[2] taking into account the effects of displacement current, atomic polarization, and the presence of absorption bands near the frequency region investigated. The first term in (1) is that predicted by the DKZ theory, and the second that arising from the anomalous nature of the skin effect.

With the approximations (a) $p=0$, as found for metals,[2] (b) combined effect of displacement current, atomic polarization and absorption bands negligible, a supposition normally valid for frequencies in the infrared (except of course very close to absorption bands), and (c) $\omega\tau\gg 1$, a condition usually satisfied for frequencies above the far infrared, the second (anomalous) contribution to (1) reduces to

$$A_{\text{anom}}\simeq\left(\frac{2kT}{mc^2}\right)^{\frac{1}{2}}\frac{F_1(T_0/T)}{F_{\frac{1}{2}}(T_0/T)}, \quad (2)$$

where the F's are Fermi-Dirac integrals[7,8] and T_0 is the degeneracy temperature.

In the degenerate limit $T\ll T_0$, as for metals

$$A_{\text{anom}}\rightarrow\tfrac{3}{4}v_{\text{Fermi}}/c; \quad (3)$$

while in the nondegenerate limit $T\gg T_0$,

$$A_{\text{anom}}\rightarrow\left(\frac{8kT}{\pi mc^2}\right)^{\frac{1}{2}}\sim\frac{\text{thermal velocity of carriers}}{\text{velocity of light in vacuo}}. \quad (4)$$

For metals, this anomalous contribution to A may be several hundred times that from the DKZ term for good conductors at low temperatures.[2] The corresponding contribution for semiconductors will be much less important, since with a *low* impurity concentration the carrier system will be practically nondegenerate and the average carrier velocity therefore small [see (4)], while with a *high* impurity concentration the DKZ contribution will be large and therefore much more important than A_{anom}. Nevertheless, the anomalous contribution could still represent an appreciable correction to the DKZ theory.

[1] A. H. Kahn, Phys. Rev. **97**, 1647 (1955).
[2] R. B. Dingle, Physica **18**, 985 (1952); **19**, 311, 348 (1953).
[3] R. B. Dingle, Physica **19**, 729 (1953).
[4] T. Holstein, Phys. Rev. **88**, 1427 (1952).
[5] R. Wolfe, Proc. Phys. Soc. (London) **A68**, 121 (1955).
[6] F. Seitz, *Modern Theory of Solids* (McGraw-Hill Book Company, Inc., New York, 1940), pp. 630–640.
[7] J. McDougall and E. C. Stoner, Phil. Trans. **A237**, 67 (1938).
[8] P. Rhodes, Proc. Roy. Soc. (London) **A204**, 396 (1950).

Optical Absorption Spectrum of MgO by Reflectivity*

James R. Nelson
Department of Physics, Cornell University, Ithaca, New York

(Received August 1, 1955)

In the course of work to be published on absorption spectra of NaCl and KCl at 300°K and at 140°K by a reflectivity method,[1–3] a measurement was also made on a large single crystal of MgO obtained from the Norton Company (Canada). The apparatus for making this measurement appearing in Fig. 1 will be presented in greater detail in the forthcoming paper on alkali halides.

The result of this measurement on MgO is shown in Fig. 2. In discussing it recently,[4] Krumhansl has pointed out that he believes the narrowing of the 1640 A peak at 140°K is fairly good evidence of exciton absorption. Also, we feel from our absorption studies of NaCl and KCl both by transmission and reflectivity, that this

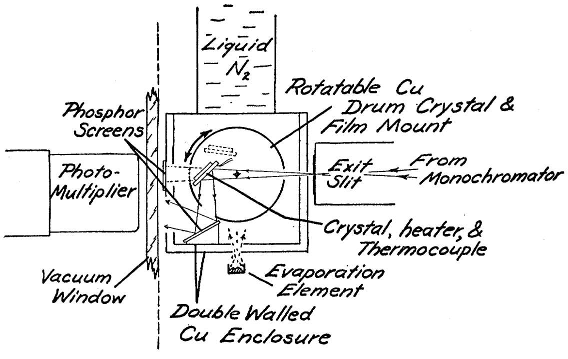

FIG. 1. Diagram of apparatus.

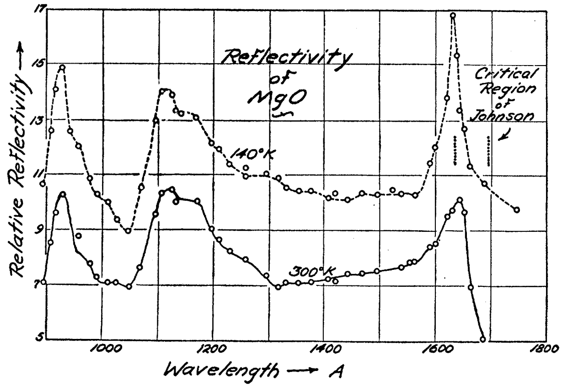

FIG. 2. Relative reflectivity of MgO as a function of wavelength.

peak is a truly fundamental absorption band with absorption coefficients of the order of 10^6 cm^{-1}, and not weak absorption due to impurity levels. This peak, indeed, confirms the results of Johnson[5] who made fluorescence and transmission measurements on thin crystals of MgO of known impurity content and also on samples of lower impurity content from the University of Missouri. Johnson found that in the region (indicated in Fig. 2) from 1635 A to 1695 A there is a rapid change in the excitation spectrum for fluorescence, and that at 1695 A the absorption coefficient is rising very rapidly. From these results, he concluded that there must be fundamental absorption in this region.

Before considering the shorter wavelength peaks, it should be pointed out that there is no resolvable splitting in the exciton peak of MgO at low temperature as we have observed in NaCl and KCl and as has also been seen in BaO.[6]

The temperature-independent peaks at 1120 A and 930 A are tentatively interpreted as absorption due to transitions from the uppermost filled band to the normally empty conduction band. This absorption band begins at about 1240 A; the band gap, then, would be about 10 ev and not 5.9 ev as reported recently by Saksena and Pant[7] who made studies on natural crystals of MgO.

In closing, it is interesting to compare this 10-ev band gap value with the approximate 11-ev value that Lempicki[8] gives as the threshold primary electron energy for secondary emission in MgO. This may be confirmation of his original supposition that the secondary electrons come from the uppermost filled band.

---

* Supported in part by the Office of Naval Research.

[1] Frederick Seitz, Revs. Modern Phys. **26**, 27 (1954).

[2] Nelson, Siegfried, and Hartman, Phys. Rev. **99**, 658(A) (1955).

[3] P. L. Hartman, Phys. Rev. **99**, 658(A) (1955).

[4] J. A. Krumhansl, "Photoexcitation in ionic crystals," Atlantic City Conference on Photoconductivity (to be published).

[5] P. D. Johnson, Phys. Rev. **94**, 845 (1954).

[6] R. J. Zollweg, Phys. Rev. **97**, 288 (1955).

[7] B. D. Saksena and L. M. Pant, J. Chem. Phys. **23**, 989 (1955).

[8] A. Lempicki, Proc. Phys. Soc. (London) **B66**, 278 (1953).

Theory of Acceptor Levels in Germanium*

WALTER KOHN† AND DANIEL SCHECHTER†

Carnegie Institute of Technology, Pittsburgh, Pennsylvania and
Bell Telephone Laboratories, Murry Hill, New Jersey

(Received July 25, 1955)

IT is well known that holes in germanium can be bound by Group III impurities in so-called acceptor levels. The observed ionization energies of such holes are 0.0104 ev (B),[1,2] 0.0102 ev (Al),[2] 0.0108 ev (Ga),[2] and 0.0112 ev (In).[2] The fact that these energies differ by only 10% suggests that the binding is largely due to the long-range Coulomb potential of the acceptors rather than to the less well understood forces in their immediate vicinity.

We have therefore carried out a theoretical calculation using a Coulomb potential[3] $-e^2/\kappa r$ and "mass"-parameters determined by cyclotron resonance experiments.[4] The resulting theoretical ionization energy is 0.0089 ev, in rather good agreement with experiment.

The calculation was based on the coupled effective mass equations which have recently been shown to arise when the band structure is degenerate.[5–7] In view of the fact that the spin-orbit splitting at the top of the valence band in Ge is about 30 times as large as the ionization energy of the acceptors,[8] the four coupled equations (V.15) of reference 7 are a good first approximation. Group-theoretical considerations suggest a trial function of the form (see reference 7):

$$ae^{-r/r_1}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}+be^{-r/r_2}\begin{bmatrix}z^2-\frac{1}{2}(x^2+y^2)\\0\\-(\frac{3}{2})^{\frac{1}{2}}(x^2-y^2)\\0\end{bmatrix}+ic\,e^{-r/r_2}\begin{bmatrix}0\\(x+iy)z\\xy\\0\end{bmatrix}.$$

The parameters were varied to maximize the ionization energy. The value 0.00883 ev was obtained with the following parameters: $r_1=43.3\times10^{-8}$ cm, $r_2=33.8\times10^{-8}$ cm, $a=1.71\times10^9$ cm$^{-\frac{3}{2}}$, $b=-2.29\times10^{21}$ cm$^{-7/2}$, $c=4.97\times10^{21}$ cm$^{-7/2}$.

With this wave function as a starting point, the original six coupled equations (see reference 7, V.13) were treated by a perturbation-variation method, which increased the ionization energy by about 1% to 0.00893 ev.

A variational calculation, such as the present one, always leads to a low ionization energy. We estimate that the true eigenvalue of the six coupled equations lies in the range 0.0094 ± 0.0005 ev. The remaining discrepancies with the experimental values may be ascribable partly to the breakdown of the theory in the immediate vicinity of the acceptor ions and partly to inaccuracies of the mass parameters used.

Kittel and Mitchell[6] have reported a theoretical value of 0.022 ev for the ionization energy. The large discrepancy with our result and with experiment is due to an incorrect transformation of the Hamiltonian operator (see their concluding sentence). A better order of magnitude estimate is obtained from a simple