APPENDIX. DISCUSSION OF EQ. (3.8)

For simplicity let us look at the part of Eq. (3.8) related to the x-component. Then the problem is to evaluate the value of the function

$$g(\kappa)=\sum_{k\epsilon C} e^{2\pi ik\kappa/K}, \qquad \text{(A.1)}$$

where k and κ are the same as in Eq. (2.15). In the complex number plane, $g(\kappa)$ is the vector connecting the origin to the end point of the succession of vectors of the summand. When one assumes that points belonging to the group C are scattered at random, the problem is looked upon as random walk in the complex number plane, and the probability $\bar{W}(\mathbf{r})d\mathbf{r}$ of finding the vector $g(\kappa)$ lying within the interval $(\mathbf{r}, \mathbf{r}+d\mathbf{r})$ from the origin is given by

$$\bar{W}(\mathbf{r})d\mathbf{r}=(\pi n)^{-1}\exp(-|\mathbf{r}|^2/n)d\mathbf{r}. \qquad \text{(A.2)}^{28}$$

In this equation n is the total number of arrows considered, i.e., $n=N^{\frac{1}{3}}p(C)$. As $p(C)$ is independent of N, when N becomes larger $\bar{W}(\mathbf{r})$ approaches to the delta function, $\delta(\mathbf{r})$, closer and closer. Therefore for very large N, Eq. (3.8) is justified.

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[28] See, for instance, S. Chandrasekhar, Revs. Modern Phys. **15**, 1 (1943).

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Photoelectric Mixing of Incoherent Light*

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Beats have been obtained between incoherent light sources by mixing Zeeman components of a visible spectral line at a photosurface. Periodicity in emission was observed through the excitation of a 3-cm cavity. Because of incoherence between the spectral lines and incoherence between the beats from different photocathode areas, the signal-to-shot-noise ratio at the cavity is only 3×10^{-5} but the beats were modulated optically, while maintaining constant total intensity and our receiver was able to yield a signal-to-noise ratio of two at the indicator. The basic idea is that, in the photoelectric process, the emission probability for electrons is proportional to the square of the resultant electric field amplitude, implying an interference between light originating in independent sources. This is a point of view which does not appear to be tested in any other experiment involving quantum effects. The experiment also demonstrates that any time delay between photon absorption and electron release must be significantly less than 10^{-10} second.

I. INTRODUCTION

THE combination of two wave trains of slightly different frequencies is equivalent to a wave of the average frequency modulated by the difference frequency. This is evidenced in the phenomenon of acoustical beats and is responsible for the operation of superheterodyne radio receivers. The periodic variation in intensity which occurs at a fixed point on the image of a Michelson interferometer when one of the mirrors is moving may also be interpreted as beats between the light reflected from the stationary mirror and light which has had its frequency changed by reflection from the moving mirror.[1–3] However, the problem of beating incoherent light waves, i.e., light waves which originate in different sources, is quite different, and since the publication of the original suggestion,[4,5] it has been argued[6] that the observation of such beats is impossible. These arguments, when examined carefully, really provide reasons why beats between incoherent light sources are difficult, rather than impossible, to detect. Were optical lines much sharper than they are, or possible to produce, without great broadening, in much greater intensity than present techniques permit, beats between incoherent light waves would be easy to observe.

Following the publication of the original suggestion[4] for this experiment, Ruark,[7] calling attention to an

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[1] A. Righi, J. Physique 2, 437 (1883) describes an ingenious production of light beats. To demonstrate that light which has passed through a rotating Nichol prism may be resolved into two circularly polarized beams, one increased and one decreased in frequency with respect to the incident light, he performed an experiment, in its essence a double-slit interference experiment, in which one slit was illuminated by light of reduced frequency and the other slit by the increased frequency, both altered to be plane polarized in the same plane. The moving fringe pattern he observed, and interpreted as beats, is, in principle, no different from those produced by moving a mirror of a Michelson interferometer.

[2] E. Rüchardt, Optik 6, 238 (1950), raises an objection to this point of view based on a requirement for coherence over a beat period. His objection is adequately answered in reference 3.

[3] C. V. Fragstein, Optik 8, 289 (1951).

[4] Forrester, Parkins, and Gerjuoy, Phys. Rev. **72**, 728 (1947).

[5] Gerjuoy, Forrester, and Parkins, Phys. Rev. **73**, 922 (1948).

[6] L. R. Griffith, Phys. Rev. **73**, 922 (1948).

[7] A. Ruark, Phys. Rev. **73**, 181 (1948).

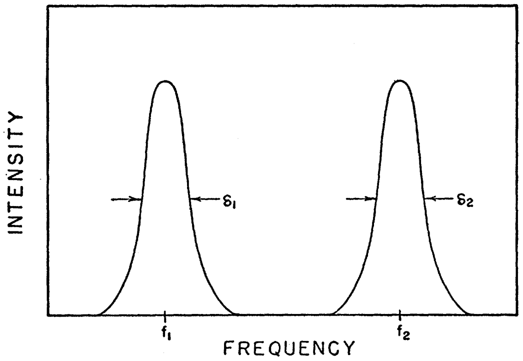

FIG. 1. Spectrum required for the production of sharp beats.

earlier paper,[8] pointed out that waves of different frequencies are not necessarily incoherent, a point of view with which we are in complete agreement. When the mirror of a Michelson interferometer is moved, or in Righi's experiment,[1] the two frequencies are thoroughly coherent. When a periodic modulation of a wave occurs, the sidebands generated in the process have phases which are related and are therefore coherent. We wish to make a special point of the fact that the lines we are mixing are regarded as completely incoherent with respect to each other; i.e., that the relative phase shift of the two lines is completely random and occurs at a rate limited only by the widths of the two lines.

Imagine a source of light emitting two spectral lines arising from two sets of atoms and therefore incoherent. The line widths δ_1 and δ_2, shown in Fig. 1, can arise in many ways, e.g., as a natural width or from Doppler broadening. Whatever the source of the broadening, this curve may be thought of as a plot of the squares of the amplitudes of the Fourier components of the electromagnetic field which is the light. The combination of these two lines should show beats between all of the Fourier components of one line and all of the components of the other (and also between components of a single line which can be ignored because they occur in a different frequency range). The variation in amplitude with time is shown in Fig. 2(a) where the effect of a line width which is not negligible is displayed by irregularities in the wave envelope. The beat frequencies will vary approximately from $f_2-f_1-\delta$ to $f_2-f_1+\delta$ but will nevertheless be a well-defined beat pattern providing that

$$f_2-f_1\gg\delta, \tag{1}$$

which is equivalent to stating that the coherence time $1/\delta$ should be long compared to the beat period $1/(f_2-f_1)$.

Line widths, in the visible region, for allowed transitions, run about 10^8 cps but because it is hard to avoid a great deal of broadening when high intensity is sought, 10^9 cps is a more reasonable figure to use in making estimates. The inequality (1) then requires that f_2-f_1 be of the order of 10^{10} cps or greater to produce a sharp beat frequency. Fortunately 10^{10} cps is a very convenient region of the electromagnetic spectrum in which to detect energy and is a frequency separation easily produced by the Zeeman effect.

A nonlinear device is required for the generation of the difference frequency. For light waves a solution is provided by the photoelectric effect, in which, over a small range of frequencies, the current is proportional to light intensity, or electric field amplitude squared. Indeed, the assumption that the probability of emission of an electron is proportional to the square of the total electric field intensity is really the fundamental assumption of this experiment. It implies an interference between light originating in different atoms, which many physicists find contradictory to their ideas about the nature of interference.

The validity of this approach to the photoelectric effect, rather than, for example, a squaring of the amplitudes of the Fourier components before addition (which would yield a time-independent current), although extremely basic, does not appear to be tested, at least with anything like this degree of directness, in any other experiment. Furthermore, it seems that in no other experiment involving a quantum effect as detector, is an effect observed which must be interpreted as interference between independently generated waves, including wave functions for, say, electrons.

It is worth noting that the generation of the beat frequency depends on what would ordinarily be called a one-quantum process. Any effect which required that photons act in pairs, or successively, to produce emission is so small compared even to the observed effect, as to be considered, at present, indiscernible.

Makinson[9] has made a calculation which is a more rigorous treatment of the photoelectric effect than we attempted. Since he describes the electric fields classically and permits the addition of waves of different frequencies, it is a calculation which we would expect

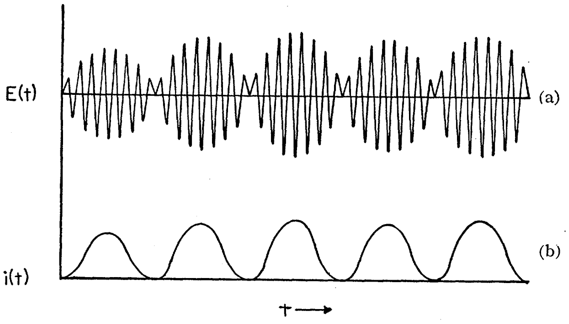

FIG. 2. Electric field variation (a) due to an admixture of two spectral lines and the accompanying photoelectric current (b). The irregularities show the effect of line widths which are not negligibly small.

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[8] Breit, Ruark, and Brickwedde, Phil. Mag. 3, 1306 (1927).

[9] R. E. B. Makinson, Phys. Rev. 75, 1908 (1949).

to reduce to our own. However, in his application of his results to the problem of light beats he finds the phase of the beats to vary with the velocity of the emitted electrons which, he states, would result in a very small percentage modulation, whereas the starting point in our calculations is that the photocurrent from a sufficiently small area is 100 percent modulated, as shown in Fig. 2(b). Makinson's conclusion should be based on a demonstration that the variation in phase with velocity is of the order of π or greater. As far as we are aware, the magnitude of this phase variation has not been calculated.

II. ORDER OF MAGNITUDE CALCULATIONS

A. Comparison between Signal and Shot Noise

It is shot noise which provides the basic obstacle to the observation of the process as outlined and a very simple picture suffices for an estimation of the signal-to-shot noise ratio which can be expected. In Fig. 2(b) the photocurrent, resulting from a spectrum made up of two lines satisfying the criterion (1), is shown. Shot noise which would cause variations in current not mirroring those in the field intensity is neglected for the moment. Unfortunately, beats from widely separated areas of the cathode cannot be expected to be in phase, unless we use perfectly plane light. In a more realistic case we can expect the relative phase of each of the light waves and, therefore, the phase of the beats to be constant over an area of the order of the size of the diffraction pattern of a single point on the source,[10] i.e., λ^2/Ω, where Ω is the solid angular spread in the light at the photocathode. From this area the average peak value of the ac current, at the beat frequency, is equal to the average or dc current, so that the mean square value of the ac current from this elementary area is

$$\langle i_0^2\rangle = \tfrac{1}{2}i_{\text{dc}}^2 = \tfrac{1}{2}(I\lambda^2/A\Omega)^2, \tag{2}$$

where I is the total photocurrent from a cathode of total area A. If we consider the signal from each area of coherence (meaning an area of the size λ^2/Ω) to be randomly phased with respect to all others, then we must increase the ac current for the entire cathode by a factor $\sqrt{n}$, where $n = A\Omega/\lambda^2$ is the number of times the area of coherence goes into the entire cathode area. Thus for the entire cathode, the mean square current is

$$\langle i^2\rangle = \langle i_0^2\rangle A\Omega/\lambda^2 = I^2\lambda^2/2A\Omega. \tag{3}$$

The dc current is proportional to n, so that, if n is very large, the current has the character of dc current modulated by the minute amount given by Eq. (3). Inevitably associated with a dc photocurrent I is the mean square shot noise current $\langle i_n^2\rangle = 2eI\Delta f$. To compare this with the signal, Δf should be taken as the frequency interval throughout which the signal is spread, approximately the line width δ times $\sqrt{2}$, leading to

$$\langle i^2\rangle/\langle i_n^2\rangle = \lambda^2 I/4\sqrt{2}e\delta A\Omega, \tag{4}$$

an equation which we can use in determining the order of magnitude of the expected signal-to-noise in the emitted current. The experimental signal-to-noise ratio, aside from errors arising from the character of the approximations made in deriving Eq. (4), will be worse than this due to receiver noise. However when circuit noise is minimized and I made as large as inequality (1) allows, the shot noise does become the major source of noise.

Using the green line $\lambda = 5.461\times10^{-5}$ cm of Hg^{202}, and, after passing the light through an optical system containing a filter to remove all Hg lines but this one and a polaroid to select the appropriate components of the Zeeman spectra, we were able to get a photocurrent of 3.85×10^{-6} ampere from an $A\Omega$ of 0.17 cm^2-steradian, under conditions for which $\delta = 8\times10^8$ cps.[11] These numbers in Eq. (4) lead to an expected signal-to-noise ratio of approximately 10^{-4}.

B. Observability of this Effect

The detection of an effect so heavily overwhelmed by noise demands that a modulation be imposed on the signal in such a way that the noise remains unmodulated. In a technique which is now common[12] the modulated signal, after detection, is passed through a very narrow band amplifier, preferably phase selective, before registering on the indicator. A system of this kind gives a signal-to-noise ratio at the final indicator which is,[13] approximately,

$$S/N = \epsilon(\Delta_1 f/\Delta_2 f)^{\frac{1}{2}}, \tag{5}$$

where ϵ is the input signal-to-noise ratio, $\Delta_1 f$ the band width of the input, and $\Delta_2 f$ the band width of the narrow-band amplifier.

Unfortunately, it is not feasible to choose $\Delta_1 f$ equal to the spectral line width, approximately 1000 megacycles per second. It was found difficult to make a very low-noise microwave receiver with a band width greater than 7 megacycles. Since we arrived at $\epsilon = 10^{-4}$ in Sec. IIA, we require here, for $S/N = 1$ that $\Delta_2 f = 0.07$ cps. Actually effects not taken into account in this rough calculation, such as the effect of extraneous light on the photocathode, worsen the situation so that, in practice, a response time of 250 seconds, implying $\Delta_2 f \approx 0.004$, was able to give a signal-to-noise ratio of only two at the final indicator. A great deal of patience is required to obtain data under these conditions and pains have to be taken to see that effects of amplifier drift do not give spurious signals.

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[10] Discussed at greater length in Sec. IVB.

[11] Factors governing the choice of this spectral line, and conditions under which these numbers were obtained are discussed in Sec. IIID.

[12] R. H. Dicke, Rev. Sci. Instr. **17**, 268 (1946).

[13] Easily seen to be an approximation to the exact Eq. (17), as shown following Eq. (22). A qualitative argument is presented by R. H. Dicke (reference 12).

The accurate quantitative calculation from which Eqs. (4) and (5) may be found as approximations, is deferred to Sec. IV so that it can be developed with specific reference to the circuitry presented in Sec. IIID.

III. EXPERIMENTAL EQUIPMENT AND PROCEDURES

A. Modulation Technique

While producing the modulation of the signal it is necessary that the shot noise remain unmodulated to the order of one part in 10^5. It is not possible, for example, to produce the desired modulation by modulating the magnetic field responsible for the line separation because the light intensity is too dependent on the magnetic field intensity. It did prove possible to modulate with constant total intensity by taking advantage of the difference in the polarization of the central and outer components of the Zeeman spectrum with the optics shown in the upper left of Fig. 3. Ignoring, for the moment, the optics to the left of the light source, light from the source passes through a rotating half-wave plate whose effect on a polarized beam of light is to cause a rotation of the plane of polarization at twice the rate at which the retardation plate rotates. The action on polarized light of the rotating $\lambda/2$ plate followed by a polaroid is to produce an interruption of the light at four times the rotation frequency. Viewed perpendicular to the magnetic field, the light is composed of the π components polarized parallel to the magnetic field, and the σ components, polarized normal to the magnetic field so that the effect of the rotating $\lambda/2$ plate plus polaroid B is to produce an alternation between the π and σ components at the photosurface. If the magnetic field is set so that beats between σ components occur at the frequency of the cavity, then this will result in a modulation of these beats.

If the total light from the source is unpolarized this modulation will occur with no variation in total light intensity or photoelectric emission. However, it was found that our sources were never unpolarized to a degree even approaching one part in 10^5. Even in the absence of a magnetic field a light source is usually polarized by an amount of the order of 1 percent. In a magnetic field the polarization of our microwave excited electrodeless discharges was of the order of 10 percent,[14] or 10^4 times too large. An adjustable tipped glass plate which could compensate for 6–8 percent polarization was placed between the source and the rotating $\lambda/2$ plate as shown in Fig. 3. The remainder of the polarization compensation was accomplished by making use of the light leaving the source in the reverse direction. This light was polarized, adjusted in intensity and plane of polarization and sent back through the source so as to produce zero signal on the compensation monitoring oscilloscope and zero signal on the recording meter which is the output of the compensation monitoring circuit chain.

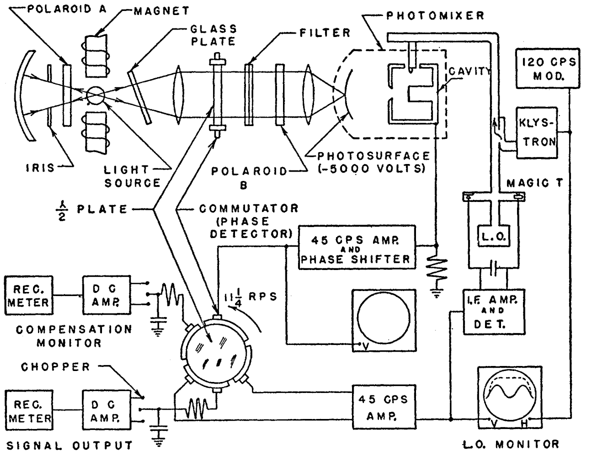

FIG. 3. Basic apparatus for the photoelectric mixing experiment.

B. Photomixer

It is in the photoelectric mixing tube, or photomixer, shown in Fig. 4, that the beat frequency is generated. Because of the small signal, even under the most favorable conditions, it is important that the photosurface be as sensitive as possible. The other requirements for the tube can be quantitatively obtained from the calculations of Sec. IVA, but it is easy to understand qualitatively what they are. The cathode must have the largest possible area and see the largest solid angle so that the photocurrent can be large. The signal-to-shot-noise ratio is not affected by this increase in total current but it is important in order to get above detector noise. For the same reason it is important that the resonant cavity in which the current pulses are converted to electrical energy receive almost all of the current and have as high a Q and R (shunt resistance) as possible. The latter requirement calls for a large gap in the cavity which in turn requires high-energy electrons in order that transit time be held to less than half a period.

The obvious photosurface is the $SbCs_3$ photosurface because of its great sensitivity and because it can be laid down on the glass in a semitransparent layer having a very large light-gathering power. For electron focusing purposes the front surface of the tube was ground to the shape of a sphere of radius of curvature 2 in. Contact to the ½-in. diameter cathode was made by means of a layer of platinum laid down on the glass as shown. The electrode following the cathode was a sheet of molybdenum formed to be a sphere concentric with the cathode.

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[14] The large degree of polarization in a magnetic field has an interesting probable explanation. Electrons moving along the magnetic field are quickly swept out of the discharge. Electrons moving at right angles to H are confined to tight spirals. The plasma can be expected, then, to be made up predominantly of electrons whose velocity is normal to H. These can be expected to excite radiation polarized normal to H, the σ lines. These are observed to predominate.

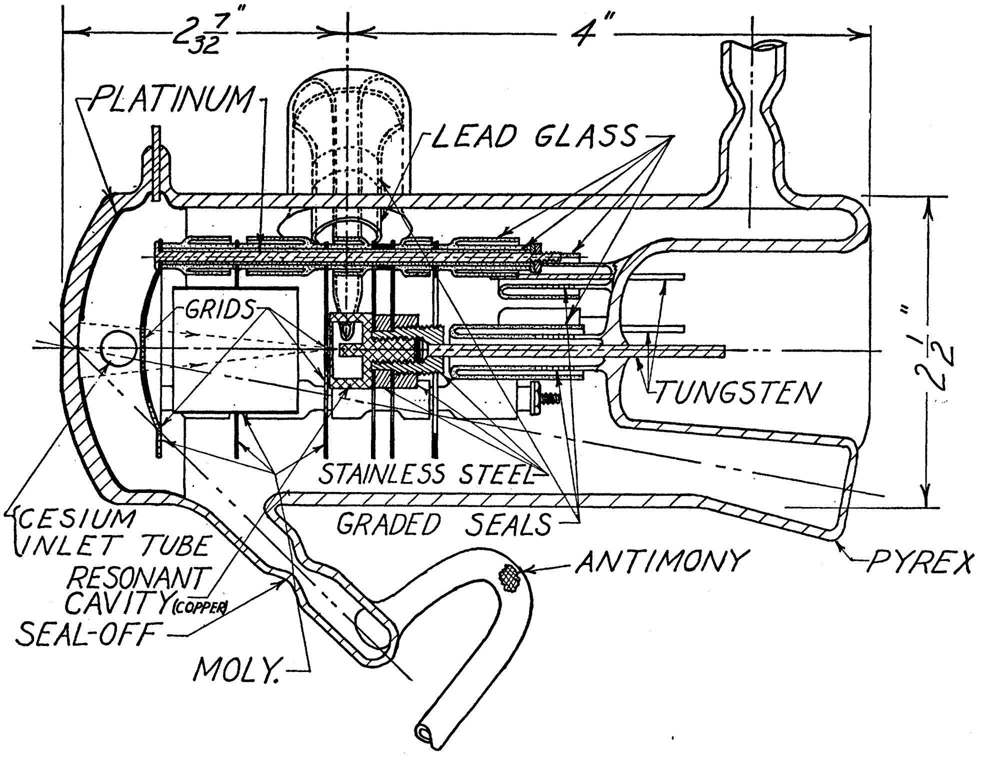

FIG. 4. Photoelectric mixing tube (photomixer).

A grid strung across the $\frac{3}{8}$-in. diameter hole was woven of 0.0005-in. diameter tungsten wire spaced 0.020 in. between wires. The purpose of the cylindrical electrode is to permit slight adjustments in focal length, and of the flat plate to shield the cavity from electrons which originate elsewhere than at the photosurface and to make possible a focusing prior to examination of the microwave power delivered to the cavity. The entire electrode structure except for the cathode is assembled, as shown in Fig. 4, on three columns of Pb glass beads held together with platinum tie rods, and carried on a single 0.100 in. diameter tungsten lead.

The requirement of high voltage, in order to minimize transit time, and the use of Cs in the photosurface manufacture, almost proved incompatible. After the admission of Cs it becomes impossible to hold high voltage across Pyrex. The Cs appears to form a conducting layer which does not leave the glass at temperatures up to 150°C, the highest temperature to which the tube could be subjected after the photosurface was laid down. However the use of lead glass[15] in strategic locations makes it possible to hold high voltages. Each lead-in had a graded seal and a $\frac{1}{2}$-in. length of lead glass and the support structure was made of lead glass beads held together by a tie rod made of platinum to match the coefficient of expansion of the glass.

Serious technical difficulties associated with the making of seals for bringing energy out of the cavity were eliminated by folding the envelope down into a hole in the side of the cavity, so that a pick-up loop on the end of a coaxial line could be inserted externally. The portion of the glass inside the cavity was a very thin-walled bubble and did not load the cavity measurably. A low-loss glass had been planned for this application but Pyrex was found satisfactory. Here, again, to prevent electrical leakage a lead glass shield had to be used. This method of withdrawing energy from the cavity not only eliminated the problem of difficult vacuum seals but provided the very great advantage of permitting external adjustment of the coupling to the cavity.

To correct for the effects of stray magnetic fields and misalignments, the tube was mounted between two pairs of Helmholz coils so that a variable field in any direction perpendicular to the electron beam could easily be generated.

C. The Light Source

Equation (4) establishes as a figure of merit for a light source photosurface combination $\lambda^2 I/\delta A\Omega$ or $\lambda^2 g\mathcal{I}/\delta$, where g is the photoelectric sensitivity and $\mathcal{I}$ the intensity of the spectral line used. It has been found that the quantity $\mathcal{I}/\delta$ increased with $\mathcal{I}$ and that it is desirable to make the light source as bright as possible, consistent with the condition (1). Although it is not necessarily so, under certain circumstances δ will vary as $1/\lambda$. In that case the figure of merit becomes $\lambda^3 g\mathcal{I}$.

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[15] V. K. Zworykin and E. G. Ramberg, *Photoelectricity* (John Wiley and Sons, Inc., New York, 1949), p. 88.

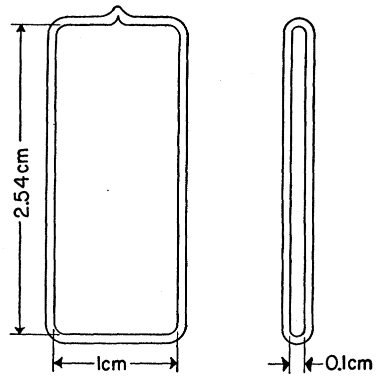

FIG. 5. Light source.

The semitransparent $SbCs_3$ photosurface has its maximum sensitivity at about 5000 A.[16] The factor $\lambda^2 g$ has its maximum at 5300 A and $\lambda^3 g$ at 5400 A. These figures immediately suggest the use of the Hg 5461 A line (see Fig. 5). In addition many other factors point to the use of this line. Mercury is an excellent choice because its large mass minimizes Doppler effect, its availability in separated isotopes makes it possible to eliminate hyperfine structure as a source of broadening, its sparse spectra contains very intense lines which are easily separated from each other and its convenient form and vapor pressure makes its use in discharge tubes extremely convenient. The green line not only occurs close to the optimum wavelength but is very brilliant and there is a great practical convenience in working with the wavelength which has become the standard for retardation plates, nonreflecting coatings, etc. There are two drawbacks to this line. The ease with which the line reverses prevents the use of deep columns at high intensity; and the Zeeman spectra for this line, shown in Fig. 6, are such that only 60 percent of the energy in the σ components is effective in producing beats. It is not inconceivable that a careful examination of the lines λ4046 or λ4358 might make them intrinsically somewhat better than λ5461 but the difficulty of separating the blue and violet lines with filters of sufficient transmissivity rules strongly in favor of the green line. The filter used was made up of 3 mm of Jena glass BG 20 and 2 mm of Jena Glass GG 11 "A" which passed 97 percent of the green line, excluding the effect of surface reflections, while effectively eliminating all of the other mercury lines. To minimize surface reflections the filter components and a polaroid were cemented together with Canada balsam.

In line with recent light source developments,[17–20] our light source was a microwave-excited electrodeless discharge using a Raytheon Microtherm CDM-2 with a maximum rating of 125 watts (but measuring only 100 watts) as a source of power. The advantages of exciting at 2450 Mc/sec were very great as compared to lower frequency excitation. With this supply we were able to get greater intensity and much more stable operation than with a 70-megacycle 2-kilowatt generator we had used previously; and the lifetimes of the tubes, limited by blackening of the walls of the quartz tubes, were much longer at the higher frequencies.[21]

The tubes were made of quartz and shaped as shown in Fig. 5, with light drawn off through the broad faces. The discharge was narrow in one dimension to minimize self-absorption and of large area to produce the maximum total amount of light. With this source we were able to get intensities of 6×10^{-4} watt per cm^2 per steradian[22] through an optical system containing the green filter and a polaroid under conditions for which the line width, using Hg^{202}, was approximately 10^9 cps. This was obtained with 100 percent of power to the source and cooling of the source by directing jets of air at the two 1-in. long edges. The cooling air was directed at the edges, rather than the faces so that the condensed mercury would not obscure the light.

It was possible, by cutting down on the cooling, to get intensities as high as 2.4×10^{-2} watt per cm^2 per steradian through the filter and polaroid, but under circumstances in which the line widths were too great for this experiment.

D. Circuits

The essential circuitry is shown in block form in Fig. 3. The power delivered to the cavity by the electrons is withdrawn by a coupling loop on the end of

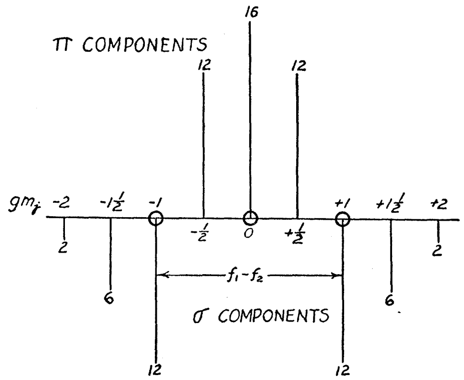

FIG. 6. Zeeman pattern for the 5461 A line of Hg, a $^3S_1\rightarrow{}^3P_2$ transition. π lines, polarized with the magnetic field, are drawn above and σ lines below. Heights of lines represent relative intensities and the circles indicate the positions in the normal triplet.

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[16] V. K. Zworykin and E. G. Ramberg (reference 15), p. 98.

[17] W. F. Meggers and F. O. Westfall, J. Research Natl. Bur. Standards **44**, 447 (1950).

[18] Kerr and Des Lattes, Office of Naval Research Technical Report No. 10, Contract Nonr, 248, T. O. 8 (unpublished).

[19] Zelakoff, Wykoff, Aschenhand, and Loomis, J. Opt. Soc. Am. **39**, 12 (1949).

[20] E. Jacobsen and G. R. Harrison, J. Opt. Soc. Am. **39**, 1054 (1949).

[21] The light source is described much more fully in a separate paper being prepared for another journal.

[22] Intensities quoted are values estimated from the photocurrent from a 929 and the bandbook value of photosensitivity at λ5461.

a coaxial line coupling into a waveguide which feeds the power to a magic *T*. In the *T* this power is mixed with local oscillator power at a pair of crystals in a technique now common in microwave detection. To avoid the difficult transformation problems encountered in changing a balanced signal from the crystals to a one-sided input for the intermediate frequency (i.f.), the crystals are Microwave Associates IN23CMR, a matched pair of crystals of opposite polarity with respect to the cartridge. This retains the advantage of balanced detection, i.e., the cancellation of local oscillator noise, and simplifies the problem of an appropriate input transformer for the i.f. amplifier. Since the transformer is one of the most important components in achieving a low-noise, wide-band amplifier, this is an important feature of the detector. The i.f. amplifier is a 30-megacycle amplifier with a noise figure of 1.0 db and a band width of 7 Mc/sec with a gain of about 80 db. The i.f. detector output is fed into a 4-cycle wide 45-cps amplifier whose output was rectified by the phase detector which was a mechanical switch made up of four 90° segments, rotating with the half-wave plate which produced the signal modulation, and three brushes at 45° intervals connected as shown in Fig. 4. The phasing was adjusted to produce a maximum output for a signal of the proper phase by rotating polaroid *B*. The rectified signal was passed through a low-pass *RC* filter with an *RC* of 250 seconds, which sets the band pass of the 45-cps detecting circuitry at approximately 1/250 cps. The filter output is fed to a low frequency (0.1-cps) periodic chopper at the input of the balanced dc amplifier which eliminates dc amplifier drift as a source of spurious signal. The effect of the chopper is to discriminate between signals originating prior to the dc amplifier and dc amplifier drift in a manner which is made clear in the photographs of Fig. 7. The positive and negative portions of the chopper cycle are unequal in length to discriminate positive from negative signals. This is particularly important when working near noise level, as in trace 3, Fig. 7, in order to be able to observe, when frequent reversals occur, whether the trace is not more often in one direction than another.

The system has inherent in it a method of checking to see whether an apparent signal originates anywhere else in the circuitry. A rotation of polaroid *B* by 90° should reverse the sign of a signal which originates in the light source, but leave unaffected a signal originating between the photosurface and the chopper.

Because of klystron drift a device for maintaining the local oscillator frequency at 30 Mc/sec from the cavity frequency is necessary. A 120-cps frequency-modulated signal is reflected from the cavity as shown in Fig. 3. Far from the cavity resonance, the signal displayed on the oscilloscope trace mirrors the i.f. response, as shown by the dotted curve on the oscilloscope. When the local oscillator is properly set with respect to the cavity, the oscilloscope trace shows a dip whose width is characteristic of the cavity *Q* and whose depth is characteristic of the coupling to the cavity. This simple circuit, then, provides us with a guide to i.f. alignment, a monitor for holding the local oscillator at the correct frequency with respect to the cavity, and a guide for proper adjustment of the cavity coupling. The 120-cps signal and its harmonics are unable to pass through the narrow-band 45-cps amplifier.

The polarization compensation monitor chain is fed by the 45 cps component of the current to the resonant cavity, and is essentially identical to that portion of the main detector chain following the i.f. detector. However, it contains a phase shifter so that the two chains can be set to amplify exactly in phase.

IV. CALCULATIONS

A. Precise Calculations for an Arbitrary Area of Coherence

Because some interesting conclusions can be drawn from an accurate comparison of the observed signal with the theoretical signal strength, and because the detecting system is one which has a wide application, it seems pertinent to present an outline and the results of a precise calculation.

If two spectral lines of equal intensity and equal width, and with Gaussian shapes, are split into Fourier components which are added and squared, representing the action of the photosurface, that part of the current per unit area which arises from differences in frequencies in the two lines is

$$i=[(2\ln 2)/\pi]^{\frac{1}{4}}(i_{\mathrm{dc}}/\sqrt{\delta})\sum_k \exp[-(\ln 2)(\omega_0-\omega_k)^2/\delta^2] \times\cos(\omega_k t+\phi_k)(\Delta\omega)^{\frac{1}{2}}, \quad (6)$$

where ω_0 is the difference in frequency between line centers, δ is the line width, i_{dc} is the photocurrent per unit area, ϕ_k is a random phase and $\Delta\omega$ is the band width of a single component of the sum. (All frequencies are here expressed in radians/sec.) The current is proportional to cathode area only over a small area $\gamma\lambda^2/\Omega$, referred to as the area of coherence, where Ω is the solid angular spread in the light striking the cathode, λ is the wavelength, and γ is a numerical constant of the order of one.[23] Since the beats from separate areas are randomly phased, the rms beat current from the entire cathode is proportional to the square root of the number of such areas in the cathode surface. This leads to a mean square current, in the frequency range ω to $\omega+\Delta\omega$, including here the effect of the modulation at a frequency ω_M,

$$\langle I^2\rangle=\frac{1}{2}\left(\frac{\ln 2}{2\pi}\right)^{\frac{1}{2}}\frac{\alpha_2{}^2 I_c{}^2\gamma\lambda^2}{\delta\alpha_1 A\Omega} \times\exp\left[-\frac{2(\ln 2)(\omega_0-\omega)^2}{\delta^2}\right](\Delta\omega)\cos\omega_M t, \quad (7)$$

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[23] This point is amplified in Sec. IVB.

where I_c is the dc current to the cavity, α_1 is the fraction of the photocathode area from which electrons enter the microwave cavity and α_2 is the fraction of the photoelectrons due to light from the two lines being mixed.

Accompanying this current is the unavoidable shot noise given by

$$\langle I_n^2\rangle = eI_c\Delta\omega/\pi, \tag{8}$$

where e is the electron charge. The currents given by Eqs. (7) and (8) deliver power to the cavity according to

$$P=\frac{\langle I^2\rangle R}{1+4Q^2(\omega-\omega_c)^2/\omega_c^2} \tag{9}$$

where R, Q, and ω_c are the shunt resistance, Q-factor and resonant frequency of the cavity, leading to a power per unit frequency range into the microwave receiver, including receiver noise referred to its input,

$$\frac{dP_1}{d\omega}=\frac{1}{1+4Q^2(\omega-\omega_c)^2/\omega_c^2}+K_F+\frac{K_B\cos\omega_M t}{1+4Q^2(\omega-\omega_c)^2/\omega_c^2}, \tag{10}$$

where

$$K_B=\frac{1}{2}\left(\frac{\pi\ln 2}{2}\right)^{\frac{1}{2}}\frac{\gamma\lambda^2}{e}\frac{I_c\alpha_2^2}{\delta\alpha_1 A\Omega} \tag{11}$$

is the signal-to-shot-noise ratio at the cavity and

$$K_F=FkT(1+\beta)/2eI_cR\beta \tag{12}$$

is the ratio of the contribution of receiver noise to that of shot noise at the central frequency. F is the noise figure of the microwave receiver and β, the coupling coefficient, is the ratio of power withdrawn from the cavity to power dissipated in the cavity. Q and R are both inversely proportional to $(1+\beta)$.

The effect of the superheterodyne detection and i.f. amplification is to lower all frequencies and multiply the power by $G_1(\omega)$, the i.f. power gain, leading to

$$\frac{dP_2}{d\omega}=G_1(\omega)\left\{\frac{1}{1+4Q^2(\omega-\omega_{\text{i.f.}})^2/\omega_c^2}+K_F+\frac{K_B\cos\omega_M t}{1+4Q^2(\omega-\omega_{\text{i.f.}})^2/\omega_c^2}\right\}. \tag{13}$$

An expression for the output of the detector[24] following the i.f. may be obtained by repeating the process which led to Eq. (6), i.e., representing the power P_2 as the square of the voltage expressed in terms of the Fourier components. If we retain only terms $\omega\approx\omega_M=2\pi\times 45$, we get for the input to the 45-cps amplifier:

$$V=2Y\textstyle\sum_j\cos(\omega_j t+\phi_j)(\Delta\omega)^{\frac{1}{2}}+K_B X\cos\omega_M t, \tag{14}$$

where

$$X=\int_0^\infty\frac{G_1(\omega)d\omega}{1+4Q^2(\omega-\omega_{\text{i.f.}})^2/\omega_c^2}, \tag{15}$$

and

$$Y=\left\{\int_0^\infty G_1^2(\omega)\left[K_F+\frac{1}{1+4Q^2(\omega-\omega_{\text{i.f.}})^2/\omega_c^2}\right]^2 d\omega\right\}^{\frac{1}{2}}. \tag{16}$$

The effect of the phase detector plus the low-pass filter is to give, finally, a signal-to-rms-noise ratio[25] at the indicator:

$$S/N=K_B X/2YZ, \tag{17}$$

where

$$Z=\left\{\int_0^\infty[G_2(\omega)/G_2(0)]^2 d\omega\right\}^{\frac{1}{2}}, \tag{18}$$

$G_2(\omega)$ being the voltage gain of the low-pass filter.

In going from Eq. (13) to Eq. (14) we assumed, above, a square-law detector. Our detector, as operated, is more nearly linear than square-law but, according to Selove,[26] this will not alter Eq. (17).

For the low-pass filter shown in Fig. 3,

$$S/N=(RC/2\pi)^{\frac{1}{2}}K_B X/Y, \tag{19}$$

where R is the parallel impedance of the filter resistance and dc amplifier input.

For a flat i.f. response over a band width B,

$$X=B(\tan^{-1}s)/s, \tag{20}$$

where $s=QB/\omega_c$ is the ratio of i.f. to cavity width, and

$$Y=B\left[\frac{\tan^{-1}s}{s}\left(\frac{1}{2}+2K_F\right)+\frac{1}{2}\frac{1}{1+s^2}+K_F^2\right]^{\frac{1}{2}}, \tag{21}$$

so that

$$\frac{S}{N}=\frac{K_B(RCB')^{\frac{1}{2}}(\tan^{-1}s)/s}{\left[\frac{\tan^{-1}s}{s}\left(\frac{1}{2}+2K_F\right)+\frac{1}{2}\frac{1}{1+s^2}+K_F^2\right]^{\frac{1}{2}}}, \tag{22}$$

where $B'=B/2\pi$ is the i.f. band width in cycles per second. For $K_F=0$ and $s\ll 1$, this reduces to $S/N=(RCB')^{\frac{1}{2}}K_B$ in confirmation of Eq. (5).

B. Area of Coherence

There remains to be calculated the constant γ, giving the ratio of the effective area of coherence to λ^2/Ω. It is plain that the area of coherence must be of the order of the size of a diffraction image from the following considerations: Over a diffraction pattern a large phase shift occurs but, since the diffraction pattern is essentially identical for two frequencies very close together, the relative phase shift and therefore the phase of the

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[24] In ordinary microwave circuitry this would be the second detector. In this application it is the third, the first being the photosurface.

[25] This result is easily seen to be quite similar to that of R. H. Dicke (reference 12) as expressed in his Eq. (20).

[26] W. Selove, Rev. Sci. Instr. **25**, 120 (1954).

beats remains constant over a diffraction pattern, if the two frequencies originated at a single source point. Of course, the area of a single diffraction pattern contains many overlapping patterns but it is still true that the beats will not be completely random until we get to two image points separated by the size of the diffraction image. The area of the central image is $3.67\lambda^2/\Omega$ so that the effective area of coherence is certainly less than this, the intensity in the secondary diffraction rings being very small.

Gerjuoy[27] has made a calculation of the signal strength based on an integration over the entire cathode using an expression for the field intensity which considers the light as spread uniformly over an area A with a solid angular spread Ω, at each point, and derived an expression for the intensity which an appropriate comparison with the derivation of the preceding section yields a value of $\gamma=1$.

Another, and very interesting, approach to γ may be made through a work of Landé.[28] He derives an expression for the number of degrees of freedom in a beam of light acting for a specified time, which can be converted, for a narrow band of frequencies, into an expression for the number of independent oscillators required to duplicate a light beam of given area and angular spread. If this number of oscillators is equated to the number of areas of coherence, we are again led to $\gamma=1$.

V. EXPERIMENTAL RESULTS

Several recording meter traces are shown in Fig. 7. The signal size is gotten from a comparison of trace 1 with trace 3, corresponding to a rotation of the polaroid B, Fig. 3, through 90°. The signal does not merely change sign in this process, as it should, indicating that some signal originates in the circuitry and this is borne out by trace 2 taken for zero light intensity. The source of the spurious signal apparently lies in a mechanical coupling between the rotating $\lambda/2$ plate and the microwave circuitry. At least it is true that pains taken to isolate these and to make the rotation as quiet as possible reduced this to the low level shown from a very much higher one. The signal obtained in this manner, divided by the rms deviation from the mean was taken for several magnetic field variations and is shown plotted in Fig. 8. The lengths of the lines drawn through each point are the probable error in the results and are less than the rms values of the deviations out of the dc amplifier because the trace was observed for a time long compared to RC for the low-pass filter. Because of darkening of the light source tubes, and other effects, it was not possible to duplicate the conditions for each point and the signal heights have all been corrected to 3.85 μa.

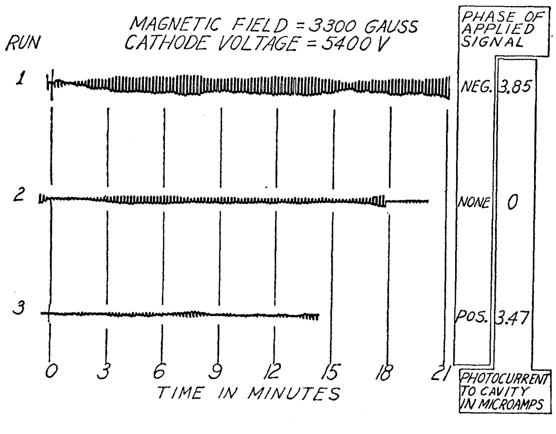

FIG. 7. Photograph of meter traces.

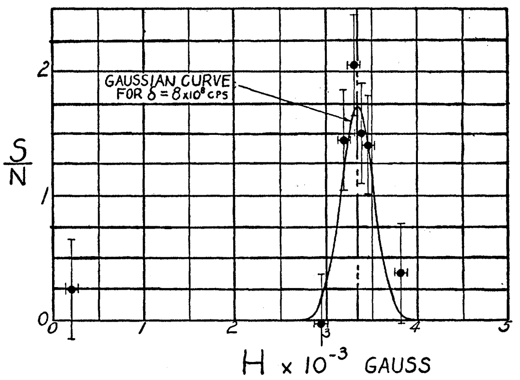

FIG. 8. Final signal-to-noise ratio *vs* Zeeman field. Cavity current $=3.85\ \mu$a. Accelerator voltage $=5400$ v.

The relative contribution of circuit noise and shot noise is contained in Eq. (22) and a measurement of the variation in total noise with cavity current yielded, through this equation, a value of $K_F=0.206$ at $I_c=3.85\ \mu$a. [This value in Eq. (12) yields a noise figure for the microwave receiver of approximately 8, which is consistent with that expected from the measured i.f. noise figures and checks the value roughly measured directly.]

For the calculation of K_B by Eq. (11), $\delta/2\pi$ was taken as 8×10^8 cps. This value is obtained from Fig. 7 if it is realized that the width, at half-maximum, of the best spectrum is $\sqrt{2}$ times the spectral line width. The fraction of photocurrent which entered the cavity was $\alpha_1=0.6$, and for the spectral line used (see Fig. 6) $\alpha_2=0.6$. The factor $A\Omega$ was approximately 0.17, leading to $K_B=2.63\times10^{-5}$.

When these values of K_B and K_F together with $RC=240$ sec, $B'=6.8\times10^6$ cps and $s=1$ are used, Eq. (23) leads to

$$(S/N)_{\text{calculated}}=0.83. \tag{23}$$

This is one-half of the observed S/N shown in Fig. 8, close enough to be regarded as good agreement. Furthermore, a spatial variation in emission over the cathode

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[27] E. Gerjuoy, University of Pittsburgh (unpublished).

[28] A. Landé, *Handbuch der Physik* (Verlag Julius Springer, Berlin, 1928), Vol. 20, p. 453.

area, due either to light source or photocathode inhomogeneity, will cause an S/N greater than that calculated on the basis of uniform emission and it is likely that the nonuniformity in emission was considerable.

VI. CONCLUSIONS

The agreement between calculations and observations is regarded as a verification of the basic premise discussed in the introduction, i.e., that photoelectric emission is proportional to the square of the total wave amplitude, implying an interference between independently generated light waves, and as a generalization, perhaps, the same kind of interference between other waves such as quantum mechanical wave functions. An alternative to this conclusion would be that there existed some coherence between components of a Zeeman spectrum. However, from a quantum-mechanical point of view two Zeeman lines originate in two distinct sets of atoms with different orientations in the magnetic field and are as independent as if they originated in different atomic species. Furthermore, it seems unlikely that an effect due to a completely unrelated cause would produce so nearly the same size signal as the one we calculated.

A secondary conclusion is that any delay between photon absorption and electron emission must be significantly less than 10^{-10} second, since a decay time equal to the beat period would decrease the signal by a factor of 6.4. This time is so short compared to the lifetimes of allowed transitions in the visible that it becomes significant. For example, the possibility that an electron could be raised to some metastable state from which it might decay to the free electron state is ruled out at least for the antimony-cesium photosurface used in this experiment.

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