ТНЕ

PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

Second Series, Vol. 99, No. 6

SEPTEMBER 15, 1955

Second Sound Attenuation in Rotating Helium II*

R. G. WHEELER,[†] C. H. BLAKEWOOD, AND C. T. LANE Sloane Physics Laboratory, Yale University, New Haven, Connecticut (Received June 10, 1955)

An experiment is described wherein liquid helium II is rotated in an annulus between a metal cylinder which rotates at constant speed and a concentric cylindrical cavity at rest. Second sound pulses, propagated parallel to the cylindrical axis and through a substantially uniform velocity field, are found to suffer an extra attenuation due to the motion of the fluid. If we define σ as the ratio of the received thermal pulse heights (fluid in motion versus fluid at rest), then for angular velocities of the cylinder ω and second sound path lengths l it appears that $\sigma = e^{-\beta\omega l}$. The coefficient β decreases somewhat with rising temperature in the range from about 1.2 to 2.0°K, having a value of $(8\pm0.8)\times10^{-3}$ rev⁻¹ cm⁻¹ sec at 1.39°K. The effect is discussed in the light of present knowledge concerning the nature of helium II.

INTRODUCTION

N 1941, Kapitza¹ reported a notable series of experiments aimed at providing a better understanding of the then obscure properties of liquid helium II. Among these was one which operated substantially as follows. A glass capillary was provided with a concentric glass rod, the annular space between the pair being filled with helium II. Various constant angular velocities could be imparted to the central rod while the capillary tube remained at rest. In effect, Kapitza measured the thermal conductivity of the liquid, in the annular space, as a function of the angular velocity of the central cylinder. The result reported showed a decrease in the thermal conductivity of He II, under rotation, as compared with the conductivity for the system at rest.

This interesting experiment does not appear to have been repeated and has, in fact, received very scant notice in the numerous review articles which have been written since that time. We therefore undertook to attempt a variation on the above experiment using a very similar geometrical arrangement but measuring the second sound velocity rather than the effective thermal conductivity. The second sound is also a

thermal property; it was, of course, undiscovered at the time of Kapitza's experiments, and we hazarded the guess that a positive effect might occur. After a little experimentation, however, it emerged that there was no measurable effect of rotation on the second sound velocity within a considerable range of angular velocities of the central cylinder.² We did, however, notice that an "extra" attenuation in the thermal pulses occurred as a function of the rotation. In what follows, we present some measurements on this rotational attenuation effect in the second sound.

EXPERIMENTAL METHOD

In order to make attenuation measurements which can be attributed only to the rotation of liquid helium, the technique used must be such that the "beam spreading" and the normal attenuation are eliminated. Also we desire to have the second sound propagated through a substantially uniform fluid velocity field. Finally we must use a signal frequency or pulse width which will give us a measurable attenuation, since the normal attenuation coefficient varies as the square of the signal frequency.

With these factors in mind, the rotation of the liquid was set up in the annulus between two coaxial cylinders, of which the inner was rotated while the outer one remained at rest. Second sound was propagated in this

^{*} Assisted by a grant from the National Science Foundation.

sertation.

¹ P. L. Kapitza, J. Phys. (U.S.S.R.) 4, 181 (1941).

² Our velocity measurements were such that a change in velocity of 0.1% could have been detected.



annulus parallel to the axis of rotation. The transmitter and receiver were annular strips whose width was about one-seventh the width of the annulus between the cylinders, thus propagating in as small a region of the annulus as technically possible. These two elements remained stationary and did not rotate with the fluid. A pulse technique was dictated by two considerations. The first being the fact that it was necessary to separate out, in time, the signals which travel various discrete paths from the transmitter to the receiver. Secondly, the pulse width had to be small enough so that the attenuation could be measured for reasonable path lengths.

With this geometry and the pulse widths used, the transmitter was of the order of 30 wavelengths wide, thus appreciable beam spreading is encountered. The apparent attenuation due to beam spreading may then be eliminated by comparing signals with rotation to those with no rotation holding all other parameters constant. This of course implies that the beam spreading is not a function of the rotation of the liquid. We believe this to be so since the velocity, hence the wavelength, does not change with rotation.

TECHNICAL DETAILS

Figure 1 is a somewhat schematic and much simplified picture of our experimental arrangement. This went through several evolutions and this was the final one. The central cylinder B was brass 1.588 cm in diameter and approximately 14 cm long. The concentric cavity Lwas of Lucite (for optical transparency) and was 3.175 cm in internal diameter³; hence the annular gap was 0.793 cm wide. The second sound transmitter T and receiver R were flat annular strips of carbon resistor board approximately 1.23 mm wide. These were constructed by painting two concentric circles, of slightly different diameter, on the resistor board using silver paint. These were each connected to an electrical lead and so the current passed through the resulting annular carbon strip in a radial direction. This permitted us to use carbon strip of high specific resistivity and yet end up with an element of moderate resistance at helium temperatures. These carbon resistor elements were constructed from *IRC* resistance strip, and had a dc resistance at 1.39° K of approximately 4200 ohms. The center of each strip was located at a distance of 1.08 cm from the axis of cylinder *B*; i.e., they were asymmetrically placed in the annular gap formed by *B* and *L*. Each strip was provided with small coaxial cable leads⁴ out through the top of the cryostat.

The brass cylinder was driven from a motor at the top of the cryostat via a stainless steel tubular shaft S the bearings (not shown) being of Kel-F plastic. The receiver was mounted in a fixed position near the bottom of the apparatus, but the transmitter being attached to the sliding Lucite cylinder C could be moved, within the cryostat, in a direction accurately parallel to the axis of B such that the distance between T and R could be set at any desired value up to 14 cm. This distance could be measured, under helium, by means of a cathetometer looking through slits in the Dewars.

The motor driving B consisted of one of a pair of Bendix synchos. This was mounted above the cryostat and coupled into the shaft in the Dewar by means of a simple magnetic clutch using Alnico magnets. Thus vacuum tightness of the cryostat was preserved without using a packing gland for shaft S. The twin syncho was driven at various required speeds by an electronically controlled, variable-speed dc motor. The average variation in speed obtained with this driving mechanism was less than 0.5 percent. Other things being equal, no difference in the results was observed for clockwise or anticlockwise rotation of B.

The transmitter was fed with single "square" electrical pulses of approximately 5- or 10- μ sec duration, but due to the thermal time constants of the transmitter the second sound pulse was spread out to about 20 μ sec. The electrical power in most runs was about 0.7 watt/ cm² at the transmitter but, due to beam spreading, the effective area soon becomes that of the entire annulus rather than that of the carbon strip. The receiver was pulsed with a small dc bias current and the resistance change resulting from the received second sound heat



FIG. 2. Block diagram of the electronic circuitry.

³ All measurements at room temperature.

pulse displayed, in the usual way, on an oscilloscope with a fast, triggered sweep and photographed with a Praktiflex 35-mm camera. The electronics was based on that employed previously by Fairbank and coworkers^{5,6} in this laboratory, but due to beam spreading we had to pay great attention to the signal-to-noise ratio.

Figure 2 shows a block diagram of our arrangement. In particular the preamplifier had to be designed and constructed with great care. We finally made use of a special Western Electric 417A triode⁷ for the input tube. These triodes when placed in a cascode circuit gave an equivalent input noise resistance of about 80 ohms. With this arrangement our signal to noise ratio, under optimum conditions, amounted to better than 40:1.

OBSERVATIONAL PROCEDURE

With the distance between T and R set at some chosen value and the temperature set at some given point (as judged on an Octoil-S manometer) we photographed a series of pulses, usually about 8 frames, with B at rest. Cylinder B was then set in constant rotation, at some given value, determined with a tachometer on the driving sychro. This invariably caused a rise in the temperature of the helium bath for obvious reasons. The bath temperature was then brought back to its previous value by manual manipula-



FIG. 3. Three photographs of oscilloscope traces showing the attenuation of second sound by rotation. Time (250 µsec sweep) is proceeding from left to right. In each case, the left hand pip is that of the direct second sound pulse followed, in order, by echo 2 and echo 3. The top trace is for the cylinder B at rest $(\omega=0)$; the middle trace for $\omega=7.2$ rev/sec and the bottom trace for $\omega = 13.2$ rev/sec. In all three cases the temperature is 1.39° K, the path length for the direct pulse l=5.70 cm and the input electrical pulse is of 10-µsec duration. Echo 1 is unresolved but some of our photographs show it as a small pip on the rear edge of the direct pulse.

Laboratories for his gift of several of these triodes.

FIG. 4. Illustrating the paths of three possible second sound echoes plus a reproduction of a typical oscilloscope trace showing their resolution in time.



tion using the oil manometer as the criterion.⁸ When conditions were steady a second series of about 20 pictures was taken. The motor driving B was then stopped, the bath temperature again brought back to its proper value, and a third set of approximately 8 "stills" taken. Thence either a new speed, a new path length or a new temperature was chosen depending on which parameter we were varying, and the above process repeated. Thus each point on our graphs represent the measurement of about 36 pictures of which roughly half were for B at rest. Occasionally, however, we took approximately three times as many pictures, per observational point, in an effort to achieve better experimental accuracy.

The reason for the great care exercised in the temperature control was, of course, due to the inherent temperature dependence of the sensitivity of the receiver. A separate resistance versus temperature measurement (using bath vapor pressure in the usual way) on our receiver showed, however, that a bath-temperature variation of a few millidegrees was entirely negligible in its effect on the size of our recorded pulse.

From each set of 36 pictures we then measured the ratio of the received pulse heights (cylinder B in motion as against cylinder B at rest). This quantity we call the "relative pulse height" (σ) in what follows. Figure 3 shows a sample of our pictures.

It has previously been stated that a pulse technique was dictated by our geometry. This is due to the fact that we observe many second sound echoes caused by reflections at B and L—some of our photographs showed as many as 6 echoes in addition to the direct beam. A continuous wave technique, such as that employed by Hanson and Pellam⁹ to study the normal attenuation in He II, would here be useless since it would not resolve any echo. As a matter of fact our pulse technique did not resolve them all. Thus, referring to Fig. 4, the paths for the first three possible echoes are indicated. Due to the asymmetry of our transducer system in the annular gap, signals for the

⁵ E. A. Lynton and H. A. Fairbank, Phys. Rev. 80, 1043 (1950).

⁶ J. C. King and H. A. Fairbank, Phys. Rev. 93, 21 (1954). ⁷ Our thanks are due Dr. W. A. Tyrrell of Bell Telephone

⁸ In order to check the validity of this procedure we made a dummy run using the receiver as an ordinary resistance ther-mometer. The result showed that we could reset the temperature with the oil manometer, when B was rotating, to within 3 milli-degrees of its value when B was at rest.

W. B. Hanson and J. R. Pellam, Phys. Rev. 95, 321 (1954).

direct path and echo 1 are not completely resolved, the first completely resolved echo has a path labelled as 2 in Fig. 4. It will be noticed that the next echo (path 3) has a larger amplitude than that due to path 2. This is due to the fact that there are two equal paths possible for this echo, namely the one labeled 3 in Fig. 4 plus an equal one (not shown) created by reflections first at L and then at B; thus twice the transmitted energy, compared to a single path, will be received. On the other hand there is only one possible path for echo 2, that shown in Fig. 4.

In presenting, in the next section, our experimental results we have found it convenient to use the following nomenclature: l= least distance in cm between transmitter T and receiver R. (This we call the "path length.") $S_0 =$ height of the recorded second sound pulse (via path l) when cylinder B is at rest. $S_{\omega} =$ height of the same pulse when cylinder B has a constant angular velocity. $\omega =$ angular velocity of cylinder B in rev/sec. $\sigma = S_{\omega}/S_0$, a dimensionless number called the "relative pulse height."

RESULTS

Figure 5 is a plot of σ versus *l* for $\omega = 15$ rev/sec at a temperature of 1.39° K. The relationship is seen to be accurately exponential, albeit the curve is a rather flat one. In order to determine the form of the function $\sigma = F(\omega, l)$ at constant temperature we made several runs, of the above nature, for several different values of the speed ω . A plot of $\log \sigma$ versus *l* (Fig. 6) yields a family of straight lines all extrapolating to the value $\sigma = 1$ at l=0. A plot of the slope of these lines versus ω (Fig. 7) yields another linear relationship. From this it appears, therefore, that the functional relationship between σ , ω , and *l* is

$$\sigma = e^{-\beta \omega l},$$

where β is a constant which we shall hereafter call the rotational attenuation coefficient.

All the above analysis has been carried out at one temperature, namely, 1.39° K and the question as to



FIG. 5. Relative pulse height (σ) versus path length (l) at 1.39°K for $\omega = 15$ rev/sec. The electrical input pulse was 10 μ sec duration. The circles are the experimental points and the solid curve is a plot of $\sigma = e^{-\beta\omega l}$ with $\beta = 7.6 \times 10^{-3}$ rev⁻¹ cm⁻¹ sec.



FIG. 6. Semilog plot of relative pulse height (σ) versus path length (l) at 1.39°K for three different speeds of cylinder B. The electrical input pulse was of either 5 or 10 µsec duration

whether β is temperature dependent remains, for the moment, open. At 1.39°K, and for the geometry used in the experiment, $\beta = (8 \pm 0.8) \times 10^{-3} \text{ rev}^{-1} \text{ cm}^{-1} \text{ sec.}$

In order to test the truth of the above hypothesis somewhat further, Fig. 8 shows a plot of σ versus ω at l=5.70 cm and T=1.39°K. The curve is, within our accuracy, an exponential, as expected, yielding, for this particular run, a $\beta=8.3\times10^{-3}$ rev⁻¹ cm⁻¹ sec. To answer the question concerning the possible temperature dependence of the coefficient β we performed the following kind of experiment. With l and ω fixed, at some chosen values, we measured σ as a function of temperature in the range from about 1.2 to 2.0°K. Assuming the truth of the above relationship between σ , ω , and l, we then compute β at each temperature. The result is shown in Fig. 9.

We found it necessary to use two different transmitters to cover the above temperature range. The reason for this lies in the fact that, as the lambda point is approached, the specific heat of liquid helium increases sharply. Hence more power must be employed around 2°K than was used at, say, 1.2°K in order to produce the same temperature pulse height in the two cases. Our original transmitter, whose resistance at helium temperatures was about 4000 ohms, had too high a resistance to give sufficient pulse amplitude for accuracy beyond about 1.8°K. We therefore constructed a second one, identical in dimensions, but with a lower resistance so that more power could be fed into the helium from our square-wave generator.

This necessary increase in power, at the higher temperatures, leads to some difficulties connected with the formation of shock wave effects. As is well known,¹⁰ shock effects are disturbing to both velocity and attenuation measurements in second sound. We were careful, therefore, to limit our powers to values such that measurable signals were produced without the formation of noticeable shock wave effects. Nevertheless, above 2°K, we were unable to produce usable

¹⁰ K. R. Atkins, Phil. Mag., Suppl. 1, 169 (1952).

signals at any power available to us. We attribute this largely to the rapid increase in the natural attenuation which is known to occur above about 2°K,⁹ coupled with the decrease in sensitivity of our receiver with rising temperatures.

DISCUSSION

A considerable effort, both theoretical and experimental, has been devoted to what we have called the "normal" attenuation of the second sound in helium II. The theory is principally due to Khalatnikov¹¹ and in this analysis the attenuation is caused by three types of dissipative mechanism, e.g., (i) the viscosity of the normal component of the liquid, (ii) the so-called second or bulk viscosity and (iii) dissipation due to diffusive thermal flow in the normal component. These various factors are computed by Khalatnikov on the basis of the assumed roton and phonon spectra forming the normal component. This theory seems to be in substantial agreement with the latest experimental observations of Hanson and Pellam⁹ and Atkins and Hart,¹² although some of the essential parameters in the theory require empirical determination for the numerical values.

Accordingly it would seem that the observed rotational attenuation requires some new dissipative mechanism in addition to those listed above, but we are unable to hazard a guess as to what the precise nature of this might be. In this connection, however, an interesting experiment due to Hollis-Hallett¹³ gives results which probably have some bearing on ours. Hollis-Hallett has adapted the rotating cylinder viscometer to the measurement of the viscosity of the normal component of helium II. The geometry of this device is very similar to that employed in our experiments except for the difference that the outer cylinder rotates and the central one is at rest. In a classical fluid the torque on the fixed cylinder is found to be directly proportional to the velocity of the rotating one which



FIG. 7. A plot of the slopes of the lines in Fig. 6 versus the angular velocity ω .

¹¹ I. M. Khalatnikov, J. Exptl. Theoret. Phys. (U.S.S.R.) 20, 243 (1950); 23, 34 (1952).





FIG. 8. Relative pulse height (σ) versus angular velocity of cylinder *B* at 1.39°K for a constant l=5.7 cm. The electrical input pulse was of 10-µsec duration. The circles are the experimental points and the solid curve is a plot of $\sigma = e^{-\beta\omega t}$ with $\beta = 8.3 \times 10^{-3}$ rev⁻¹ cm⁻¹ sec.

is, of course in agreement with predictions of the Navier-Stokes equation which in turn shows that the proportionality constant contains the first viscosity. In helium II, however, Hollis-Hallett finds that the torque is not so linearly related even for velocities of as little as 0.1 cm/sec. The observed torque, at any velocity, was larger than that required by a linear law or, stated another way, there appears to be an "extra viscosity" over and above the classical one such as our experiment would also seem to require.

The hydrodynamics of helium II is based on the classical Navier-Stokes equation, for both normal and superfluid, with added terms. These include the observed pressure gradient in the superfluid induced by heat flow (fountain effect) and, in addition, the postulated "mutual friction" between normal and superfluid of Gorter and Mellink.¹⁴ At first sight this latter term might seem to account for the extra viscosity but, as Hollis-Hallett shows,¹³ it does not. The prediction of the adjusted Navier-Stokes equation is still for a torque linear with the velocity. It appears therefore that the hydrodynamic equation for He II are still incomplete.¹⁵

The apparent failure of the adjusted Navier-Stokes equation, discussed above, could also have some bearing on our experiment in another way. Thus it appears from our work that the transverse velocity of presumably the normal fluid (perhaps also the superfluid) increases the normal attenuation of second sound. But we do not, at the moment, know the magnitude of these velocities with any degree of certainty—the best we can do is to compute the velocity profile in the annulus from the Navier-Stokes equation thus treating the helium as if it were a classical liquid. This gives for the angular velocity of the fluid $\omega(r)$ at distance r from the

¹⁴ C. J. Gorter and J. H. Mellink, Physica 15, 285 (1949).

¹⁵ In addition to the Gorter-Mellink formulation of the hydrodynamic equations there are a number of others which, however, have been subjected to less experimental testing than these above. For details see J. G. Daunt and R. S. Smith, Revs. Modern Phys. 26, 172 (1954).



FIG. 9. Rotational attenuation coefficient β versus temperature.

axis of rotation of cylinder B:

$$\omega(r) = \omega \left(\frac{a^2}{b^2 - a^2}\right) \left(\frac{b^2}{r^2} - 1\right),\tag{1}$$

wherein a and b are the radii of cylinders B and L respectively and ω is the angular velocity of cylinder B.¹⁶ It may well be, however, in lieu of an actual velocity measurement in the annular gap, that the above relation does not express the true facts. We are in a position to make some test of this by considering the attenuation of the echoes indicated in Fig. 4, and from our data for a 5-cm path length have measured the relative attenuation of echoes 2 and 3 as a function of the average fluid velocity,¹⁷ computed from relation (1) above. The result,

¹⁶ The transient term, for our geometry, which multiples the expression for the steady-state velocity distribution is given by

$\{1-\exp(-\lambda^2\mu t/\rho)\},\$

where ρ and μ are the density and viscosity and λ is a geometrical factor obtained from the boundary conditions ($\lambda^2 \cong 16 \text{ cm}^{-2}$). Using the normal component density and viscosity at 1.4°K, this "time constant" is of the order of 40 seconds. We looked for such a time effect; however our experimental procedure limited us to variations in time occurring about 20 seconds after the system had started rotating. No such transient effect in the amplitude of the second sound pulse with rotation was observed. It must be stated, however, that the solution of the Navier-Stokes equation which yields the above transient term is probably too approximate. We have to neglect the centripetal force and make the even worse assumption that the velocities are sufficiently small to neglect second order terms.

neglect second order terms. ¹⁷ Referring to Fig. 4 it is apparent that whereas the direct pulse passes through a substantially constant velocity field, echoes 2 and 3 (if relation 1 is true) do not. We therefore define an average velocity field $\langle \omega \rangle_{W}$ for these cases as

$$\langle \omega \rangle_{\mathrm{AV}} = \frac{1}{\tau_0} \int_0^{\tau_0} \omega(r) d\tau,$$

where τ_0 is the path length, between transmitter and receiver, for the echo. This quantity can be computed from the known geometry of the apparatus. Although the rotational attenuation depends also on the length of path it turns out that the paths for echoes 2 and 3 are, respectively, only about 2 and 5 percent longer than the direct pulse path in our apparatus. Hence this fact is ignored. The average velocities as computed from relation (1) are as follows:

direct pulse:	$\langle \omega \rangle_{\rm Av} = 0.376 \omega$,
echo 1:	$\langle \omega \rangle_1 = 0.636 \omega$,
echo 2:	$\langle \omega \rangle_2 = 0.152 \omega$,
echo 3:	$\langle \omega \rangle_3 = 0.333 \omega$.

together with that for the direct path, are shown in Fig. 10, and in this graph the flat exponential has been drawn as a straight line. It will be seen that the result appears reasonable. Thus the curves for the direct pulse and echo 3, which pass through average velocity fields of the same order of magnitude, are only slightly different but the attenuation effect in echo 2 which, according to calculation, passes through a smaller velocity field is much reduced. We were unable to make measurements on echo 1 since it is not resolved in our data. This does not, of course, establish the correctness, or otherwise, of the velocity profile as deduced above. It does, however, indicate that the velocity cannot be nearly uniform across the annulus.

From this it appears probable that the coefficient β , as defined by us, would depend on the position of the transducer system in the annular gap in which the helium rotates. The figure $(8\pm0.8)\times10^{-3}$ rev⁻¹ cm⁻¹ sec at 1.39°K thus refers to a position such that the center of the system is distant 1.08 cm from the axis of the



FIG. 10. The relative pulse height (σ) versus angular velocity of central cylinder for the direct beam and two echoes. Temperature 1.39°K with a 5-cm direct beam path length.

cylinder *B*. We made no measurements to test this point. In addition we have no information about the possible dependence of β on frequency. Our method is such that we cannot substantially alter the width of the input pulse (and thus alter the fundamental pulse frequency) and at the same time resolve the direct pulse and the numerous echoes.

It is tempting to ascribe our results to some kind of turbulent motion occurring in the superfluid, especially in view of the geometrical arrangement which we have employed. In a classical liquid it is known that the motion is turbulent for all angular velocities of the inner cylinder for a geometry such as ours.¹⁸

Note added in proof.—In a communication appearing after this paper was written, H. E. Hall and W. F. Vinen [Phil. Mag. 46, 546 (1955)] have briefly reported what appears to be an entirely different kind of experiment also showing an effect of rotation on the second sound attenuation.

¹⁸ H. Lamb, *Hydrodynamics* (Cambridge University Press, Cambridge, 1916), fourth edition, p. 655.



FIG. 3. Three photographs of oscilloscope traces showing the attenuation of second sound by rotation. Time (250 μ sec sweep) is proceeding from left to right. In each case, the left hand pip is that of the direct second sound pulse followed, in order, by echo 2 and echo 3. The top trace is for the cylinder *B* at rest (ω =0); the middle trace for ω =7.2 rev/sec and the bottom trace for ω =13.2 rev/sec. In all three cases the temperature is 1.39°K, the path length for the direct pulse *l*=5.70 cm and the input electrical pulse is of 10- μ sec duration. Echo 1 is unresolved but some of our photographs show it as a small pip on the rear edge of the direct pulse.