duced the secondary star was bound in a nuclear fragment or whether it stopped in the emulsion and was captured by a nucleus. Nevertheless, in either case it is necessary to postulate that the secondary star was produced by a "heavy" hyperon or a "heavy" K-meson.

It now seems clear that a Λ^0 particle can be bound in a nucleus and furthermore that at least one other unstable particle can also be bound. It is hoped that additional energetic events, which are more favorable, will be found, so that the particle or particles which are responsible can be identified.

Although there are data on the binding energy of a Λ^0 particle in a few nuclei, knowledge of the variation of the binding energy with nuclear charge and mass is inadequate. Such information would be valuable in an attempt to understand the interaction of the Λ^0 particle with nuclei.

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Wave Equation for Spin 0 in Hamiltonian Form

K. M. Case
H. M. Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan
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It is shown that the Hamiltonian form for spin 1 particles also describes spin 0 particles if the 5-dimensional representation of the Duffin-Kemmer algebra is used.

I. INTRODUCTION

SCHRÖDINGER[1] has recently shown that the field-free equations for a spin 1 (Proca) particle can be written in the Hamiltonian form

$$i\partial\Psi/\partial t=\mathcal{H}\Psi, \tag{1}$$

where

$$\mathcal{H}=\boldsymbol{\beta}\cdot\mathbf{P}+\kappa\beta_4. \tag{2}$$

Here the matrices β_1, β_2, β_3, β_4 form a ten-dimensional representation of the Duffin-Kemmer algebra.[2] The wave function Ψ is subject to the initial condition

$$(\mathcal{H}\beta_4-\kappa)\Psi=0. \tag{3}$$

As a consequence of Eq. (1) the condition (3) is maintained for all time.

It is well known[2] that the field-free equations for both spin-one and spin-zero particles can be written in the form

$$(\beta_\mu\partial_\mu+\kappa)\Psi=0. \tag{4}$$

The only difference is that for spin one we need to use a ten-dimensional representation and for spin zero a five-dimensional representation of the algebra defined by the relations

$$\beta_\mu\beta_\nu\beta_\rho+\beta_\rho\beta_\nu\beta_\mu=\delta_{\mu\nu}\beta_\rho+\delta_{\rho\nu}\beta_\mu. \tag{5}$$

This suggests that, similarly, Eqs. (1) and (2) also describe spin-zero (Klein-Gordon) particles with the appropriate replacement of the β matrices.

II. THE HAMILTONIAN FORM

If we describe a spin-zero particle by a scalar function ϕ and a four-vector ϕ_μ, the field-free equations are[3]

$$\partial\phi/\partial x_\mu=\kappa\phi_\mu,\quad \partial\phi_\mu/\partial x_\mu=\kappa\phi. \tag{6}$$

On splitting ϕ_μ into a space vector $\boldsymbol{\phi}$ and a space scalar ψ ($\phi_4=i\psi$), these equations become

$$\nabla\phi=\kappa\boldsymbol{\phi}, \tag{7}$$

$$\partial\phi/\partial t=-\kappa\psi, \tag{8}$$

$$\partial\psi/\partial t=-\nabla\cdot\boldsymbol{\phi}+\kappa\phi. \tag{9}$$

Differentiating (7) with respect to time and eliminating $\partial\phi/\partial t$ using (8) gives

$$\partial\boldsymbol{\phi}/\partial t=-\nabla\psi. \tag{7a}$$

If (7a), (8), and (9) are required to hold for all times (7) need only be required to be true initially. It will then automatically be satisfied at later times. Hence the field equations can be written, on multiplying by i, as

$$i\partial\boldsymbol{\phi}/\partial t=\mathbf{P}\psi,\quad i\partial\phi/\partial t=-i\kappa\psi,$$
$$i\partial\psi/\partial t=\mathbf{P}\cdot\boldsymbol{\phi}+i\kappa\phi, \tag{8}$$

where

$$\mathbf{P}=-i\nabla. \tag{9}$$

The admissible solutions are those which satisfy (7) initially.

Combining the quantities $\boldsymbol{\phi}$, ϕ, and ψ into a single

[1] E. Schrödinger, Proc. Roy. Soc. (London) **A229**, 39 (1955).
[2] N. Kemmer, Proc. Roy. Soc. (London) **A173**, 91 (1939).
[3] $x_4=it$. $\hbar$ and c are taken to be unity.

five-component wave function,

$$\Psi=\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi\\ \psi\end{bmatrix}, \tag{10}$$

we see that Eqs. (8) can be written as

$$i\partial\Psi/\partial t=\mathcal{H}\Psi. \tag{11}$$

Here $\mathcal{H}$ is the 5×5 matrix:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & P_1\\ 0 & 0 & 0 & 0 & P_2\\ 0 & 0 & 0 & 0 & P_3\\ 0 & 0 & 0 & 0 & -i\kappa\\ P_1 & P_2 & P_3 & i\kappa & 0\end{bmatrix}. \tag{12}$$

This is certainly of the form of Eq. (2). β_1 is obtained by putting $P_1=1$, $P_2=P_3=\kappa=0$ in (12). β_2, β_3, and β_4 are obtained similarly. It may be noted that, as for spin one, these matrices are all Hermitian. Direct computation shows that the condition of Eq. (3) is just Eq. (7).

The only remaining question is whether the β_i satisfy all the relations implied by (5). This is rather simple to see since the content of (5) is merely the following: Let n_μ be a unit four vector. Then

$$(n_\mu\beta_\mu)^3=n_\mu\beta_\mu. \tag{13a}$$

Alternatively, to check the relations (5) we need only check that

$$\mathcal{H}^3=(P_1^2+P_2^2+P_3^2+\kappa^2)\mathcal{H}, \tag{13b}$$

for arbitrary P_1, P_2, P_3, and κ. Direct computation shows this to be true.

III. REMARKS

Following Schrödinger, we may define the spin operators by

$$S_i=i\epsilon_{ikl}\beta_k\beta_l, \tag{14}$$

where ϵ_{ikl} is the alternating symbol and i, k, l run from 1 to 3. The sum of the squares of these three matrices is diagonal. The number 2 appears along the first three diagonal positions and zeros elsewhere. The subsidiary condition (3) shows that in the rest system only the eigenvalue 0 for the spin appears. The situation is much like that noted by Kemmer[2] in that an apparently nonzero spin appears at relativistic energies.

A similarly peculiar result is obtained on introducing an electromagnetic field. The same treatment as above leads to Eq. (11) again, where now

$$\begin{aligned}\mathcal{H}=eV+\kappa\beta_4+\beta\cdot(\mathbf{P}-e\mathbf{A})+(ie/\kappa)\mathbf{P}\cdot\boldsymbol{\beta}\beta_4,&\\ +(\ )=ie/2\kappa\ \epsilon j\kappa i\mathbf{H}j\beta\kappa\beta_4\beta_i&\end{aligned} \tag{15}$$

where V, $\mathbf{A}$, and $\mathbf{EH}$ are the scalar potential, vector potential, electric and magnetic field respectively. The non-Hermitian term in (15) is quite similar to the apparent imaginary electric dipole moment one obtains from the Dirac equation.

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Classification of the Fundamental Particles*

R. G. Sachs
University of Wisconsin, Madison, Wisconsin
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The decay and production processes of the pions, K-mesons, nucleons, and hyperons are classified in terms of selection rules for an integral quantum number, a, called the "attribute," which is assigned a definite value for each particle and assumed to be additive when particles are combined. No attempt is made to relate the attribute to other physical properties of the particles. The scheme suggests relationships between processes which have yet to be observed such as the associated production of a cascade particle with two (positive or neutral) K-mesons. When it is combined with the notion of isotopic spin (I) conservation, it suggests the existence of several new particles, the Σ^0 of Gell-Mann and Nishijima, a Ξ^0 and a neutral K-meson differing in its properties from the θ^0. Results of isotopic spin assignments suggest the rule (odd-even rule) that even-a fermions have half-integral I, odd-a fermions have integral I, and conversely for the bosons. There are also implications concerning the interactions between various particles: the range of the potential binding the Λ^0 to a nucleon should be of the order of the K-meson Compton wave length.

The classification is extended to include electrons, neutrinos, and muons with the result that their attributes must be half-integral. In order to exclude certain unobserved processes, it is necessary to assume that the neutrino is the source of the weak (Fermi) interaction of fermions, in contrast to the notion of the universal Fermi interaction. The existence of an antineutrino is strongly suggested. The $K_{\mu 3}$ and K_{e3} (considered as one particle) may be interpreted as a boson (K) or fermion (κ). In the former case, the decay schemes $K^\pm\to e^\pm+\nu$, $K^0\to 2\nu$, and $K^0\to\pi+\mu+\nu$ are expected to occur. In the latter case, production of the κ through the decay process $K\to\kappa+\nu$ is suggested.

Several unusual new events are classified in Sec. VI in order to illustrate the method. A table of thresholds for production of the various particles is included in an Appendix. No excuse is offered for the nonoccurrence of $\pi-e$ decay.

I. INTRODUCTION

THE purpose of this note is to introduce a scheme for classifying the fundamental particles in such a way as to correlate their modes of production, their observed decay rates, and the interactions between them. The classification is carried out in terms of a single quantum number called the "attribute" which is not given a specific physical interpretation. This scheme provides a useful way to summarize data, and to predict

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