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Disintegration of Hyperfragments*

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A systematic search has been made for events which appear to be due to the disintegration of a hyperfragment; that is, a nuclear fragment which contains a bound unstable particle. A total of 20 000 3-Bev proton stars and 500 1.5-Bev π^--meson stars from the Brookhaven Cosmotron were studied as well as 9000 cosmic-ray stars. Twenty-one events which are interpreted as hyperfragments were found in the proton plates, seven in the cosmic-ray plates and one in the π-meson plates. Of the 29 disintegrations, a π meson was ejected in only two cases and a K-meson in one event. In the remaining 26 disintegrations, only nuclear particles were emitted. The charge and energy distributions, as well as the angular distribution of the hyperfragments, have been measured. In four cases the energetics of the decay strongly indicate that the bound unstable particle was a Λ^0 particle. The binding of the Λ^0 particle was measured in these cases. In two additional cases the minimum energy release was significantly greater than the decay energy of a Λ^0 particle but possibly consistent with the decay energy of a charged hyperon. In one case a negative K-meson was ejected from a secondary star. It is impossible to determine whether this secondary star was due to a fragment disintegration or the nuclear capture of a stopped particle. This event cannot be explained in terms of established unstable particles.

I. INTRODUCTION

In 1952, a very unusual event was reported by Danysz and Pniewski[1] in which a nuclear fragment, from a cosmic-ray star, stopped in the emulsion and subsequently disintegrated with an energy release of 140 to 180 Mev. Two possible explanations for the event were given. It was suggested (1) that the fragment may have contained a negative π meson in an orbit around it and that after the fragment stopped the negative π meson was absorbed, or (2) that the fragment may have contained a bound Λ^0 particle which disintegrated after the fragment stopped in the emulsion.

Similar events have been reported by various investigators.[2–15] In a few favorable cases the energy release from the disintegration of the fragment was found to be very close to the decay energy of the free Λ^0 particle.[16] From the energetics of the disintegration, an estimate of the binding energy of the Λ^0 particle in the fragment was obtained, and in these cases the binding was found to be comparable to or less than that of a neutron in a similar stable nucleus.

The excited fragments of low Z appear to decay with the emission of a π meson while heavier fragments undergo nonmesonic decay. Cheston and Primakoff postulated that the nonmesonic decay may be the result of the stimulated decay of the Λ^0 particle in the nuclear fragment and calculated the ratio of mesonic to nonmesonic decay.[17] In several cases there is good evidence that the nuclear fragment stopped before it disintegrated, and therefore a lower limit to the lifetime of a bound Λ^0 particle was found. The data showed that even in nuclei containing as many as 10 nucleons, the lifetime, in comparison to that of the free Λ^0 particle, was not shortened by many orders of magnitude.

Following a suggestion of M. Goldhaber, we propose to call a nuclear fragment containing a bound hyperon or some other unstable particle, a hyperfragment, and in specific cases we shall use the normal symbol for the nucleus with a subscript designating the type of hyperon that is bound in it; *viz.*, a Λ^0 particle bound in a Be^9 nucleus would be designated as ${}_{\Lambda}Be^9$.

It seemed worthwhile to make a systematic search for hyperfragments in order to obtain additional information on the interaction of the Λ^0 particle in various nuclei. Although the observed energy release in many cases was found to be very close to the expected value from the decay of a bound Λ^0 particle, in other cases the data did not exclude other mechanisms. For example, in many events the total energy release cannot be estimated and may have been considerably greater or less than the expected value from a bound Λ^0 particle. In fact, three disintegrations have been found in the course of the work described in this paper, where the decay energy is significantly greater than the free decay energy of the Λ^0 particle.

II. PROCEDURE

A. Exposures

Since the most favorable mechanism for the production of hyperfragments was not known, it was decided to expose plates to various sources of high-energy particles.

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17 W. Cheston and H. Primakoff, Phys. Rev. **92**, 1537 (1953).

TABLE I. Frequency of hyperfragments.

	Cosmic rays	3-Bev protons	1.5-Bev π mesons
Number of stars observed	9000	20 000	500
Number of hyperfragments	7	21	1
Ratio of hyperfragments to total stars	8×10^{-4} [a]	10^{-3}	2×10^{-3}

[a] If only stars of 2 or more shower particles are included, this ratio is about doubled.

Pellicle stacks and glass-backed plates were exposed to cosmic radiation in several skyhook balloon flights by the Air Force from Holloman, New Mexico. The flights ranged from 4 to 12 hours above 90 000 feet.

Groups of plates and pellicles were exposed to 3-Bev protons from the Brookhaven Cosmotron. The intensity of the beam was reduced by a factor of approximately 10^5 in comparison to the normal operating intensity of the machine, which gave a track density in the plates of about 10^5 particles per cm^2.

Emulsions were also exposed to the external 1.5-Bev π^--meson beam from the Cosmotron. The flux density in these plates was quite low and therefore less time was spent investigating the π-meson stars.

B. Selection Criteria

The emulsion was area-scanned for stars under a low magnification (60 to 100×). After a star was found, the gray and black tracks were followed until they either left the emulsion or stopped. Connected stars were then studied under a higher magnification.

Many double stars were found which were connected by a gray or black track. In many cases it could be shown that the secondary star was due to a collision of the particle from the primary star with the nucleus of an atom in the emulsion.

The following criteria were used to eliminate events which were not hyperfragments. In several cases the total charge of the particles from the secondary star was greater than the charge of the connecting particle, clearly indicating that the secondary star was produced by a collision and not by the disintegration of the fragment. In other events, δ rays were observed along the connecting track very near the secondary star,

TABLE II. Characteristics of hyperfragment stars.

Event No.	Type of exposure[a]	Classification of primary star	Range of connecting fragments in μ	No. of prongs in secondary star	Total charge of the secondary star	Total energy of charged particles	Residual momentum in Mev/c
1	C.R.	17+5?	55	2	4	105	370
2	C.R.	$20+1p$	19	3	6	147.5	0
3	3-Bev proton	$12+1p$	8	3	4	117	318
4	3-Bev proton	$9+1p$	77	2	2	6.5	93
5	C.R.	$11+0n$	193	3	4	19	250
6	3-Bev proton	$5+2p$	5	3	3	146	300
7	1.5-Bev π^-	$12+2\pi^-$	5	6	8	62	520
8	C.R.	$12+3p$	21	3	5	33	360
9	C.R.	$17+0p$	16	3	4	26	161
10	3-Bev proton	$15+2p$	19	3	5	17	384
11	3-Bev proton	$12+1p$	46	4	6	23	294
12	C.R.	$20+0h$[b]	5	3	5	123	425
13	3-Bev proton	$11+3p$	9	5	8	29	312
14	3-Bev proton	$14+0p$	7	5	8	57	534
15	3-Bev proton	$10+0p$	4	3	6	93	575
16	3-Bev proton	$10+0p$	7	3	3	28	200
17	3-Bev proton	$8+2p$	4	2	3	27	347
18	3-Bev proton	$13+1p$	7	5	6	46	220
19	3-Bev proton	$13+0p$	10	3	6	26	416
20	3-Bev proton	$13+2p$	4	3	6	12	166
21	3-Bev proton	$15+1p$	8	2	3	91	250
22	3-Bev proton	$13+2p$	8	3	5	11	157
23	3-Bev proton	$9+0p$	4	2	3	5	58
24	3-Bev proton	$9+0p$	10	4	6	56	320
25	3-Bev proton	$9+1p$	5	3	4	36	425
26	3-Bev proton	$9+3p$	1	3	5	7	100
27	C.R.	$22+9p$	192	4	3	83	342
28	3-Bev proton	$21+0p$	10	3	4	206	330
29	C.R.	$21+9n$	9	3	2	54	200

[a] C.R. ≡cosmic ray. [b] h≡heavy nucleus.

indicating that the connecting fragment did not come to rest before producing the star. The ionization along the connecting track, in many events, clearly indicated that the secondary star was the result of a collision. It is possible that a nonmesonic decay in flight of a hyperfragment may have been misinterpreted as a collision, although the number of such cases is undoubtedly small. The mesonic decay in flight of a hyperfragment should be easily detected. No such events have been observed.[13]

Occasionally, very slow negative π mesons are ejected from the primary stars and cause secondary stars. These can be identified by the large amount of multiple scattering near the π-meson star and occasionally by the presence of "Auger" electrons associated with the secondary star.

III. EXPERIMENTAL RESULTS

A. General Features

A total of 9000 stars were observed in the plates exposed to cosmic rays; 20 000 in the plates exposed to 3-Bev protons and 500 caused by 1.5-Bev negative π mesons. The frequency of occurrence of hyperfragments is summarized in Table I.

The expected frequency of hyperfragments has been calculated by Jastrow[18] and found to be 2.5×10^{-3}. The experimental value is somewhat lower ($\sim10^{-3}$).

Table II gives the general characteristics of the connected stars which appear to be the result of the decay of hyperfragments. An estimate of the total charge from the secondary stars is given in column 6. In events 1, 4, 5, 27, the Z of the fragment could be estimated from the characteristics of its track, namely from the thin-down and δ rays. The Z of the fragment, in these cases, is found to be consistent with the total Z from the secondary star. In the remaining cases, the track of the fragment is so short that an estimate of the charge cannot be made. However, it seems reasonable to assume that the Z of the fragment is equal to the total charge of the particles from the secondary star. It is difficult, if not impossible, to determine the Z of a particle which produces a short track; hence the values given in column 6 may be in error by ±1 in some cases, and in a few cases the value is only a lower limit. The charge distribution of the hyperfragments is summarized in Fig. 1.

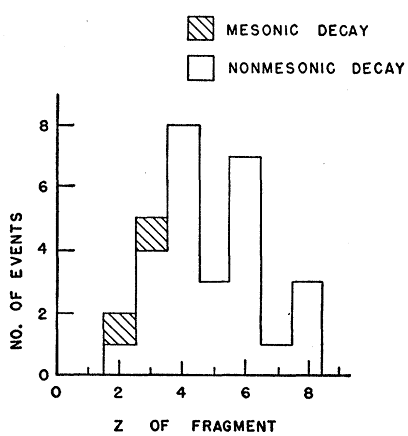

FIG. 1. Charge distribution of hyperfragments. In most cases the charge of the fragment was determined from the total charge from the secondary star.

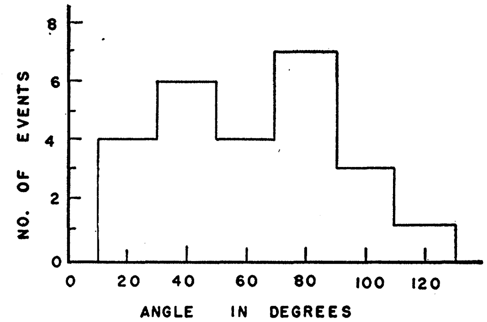

FIG. 2. Angular distribution of the hyperfragments, with respect to the incoming charge particle.

In those cases where the incident charged particle which produced the primary star could be identified, the angle was measured between the direction of the hyperfragment and the incoming particle. The results of the angular measurements are shown in Fig. 2. There is a strong tendency for the hyperfragments to be ejected in the forward hemisphere. This is probably due to the production of the Λ^0 particle in the forward direction[18,19] as well as the tendency for ejection of nuclear fragments in the forward direction.

The energy distribution of the hyperfragments, at the time of ejection from the primary star, is shown in Fig. 3.[20]

A measure of the kinetic energy of the charged particles from the secondary star is given in column 7 of Table II. Tracks which were produced by a particle of charge 1 were assumed to be protons in the evaluation of

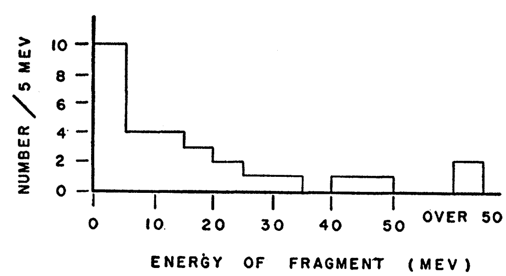

FIG. 3. Energy distribution of the hyperfragments. Although the range energy relationship is not well established for slow heavy particles, the errors will not appreciably change the distribution.

[18] Robert Jastrow, Phys. Rev. 97, 181 (1955).

[19] Fretter, May, and Nakada, Phys. Rev. 89, 168 (1953).

[20] The range-energy relations of Wilkins were used. J. J. Wilkins, Atomic Energy Research Establishment Report, Harwell $G/R664$ (unpublished).

TABLE III. Characteristics of secondary stars.

Event No.	Track No.	Identity[a]	Range (μ)	Energy (Mev)
1	1	α	1011	57
	2	α	748	48
2	1	p		104 ± 6
	2	Li^7	45	17.5
	3	He^3	315	26
3	1	α	75.4	11.9
	2	p		98.5
	3	p	262	6.5
4	1	d or	6	0.6
		t		0.5
	2	p	226	5.9
5	1	R	7.9	
	2	p	308	7.1
	3	p	540	9.8
6	1	R	3.2	
	2	p		120
	3	p		25
7	1	p	455	8.9
	2	α	29	6.3
	3	p	1700	19.0
	4	α	96	13.8
	5	p	407	8.4
	6	p	206	5.6
8	1	α	136	17.2
	2	α	10.5	3.0
	3	p	806	12.4
9	1	R	5.4	
	2	p	344	7.5
	3	p	1417	17.1
10	1	R	6.1	
	2	p	349	7.5
	3	α	43.5	8.3

Event No.	Track No.	Identity[a]	Range (μ)	Energy (Mev)
11	1	α	95	13.7
	2	α	24	5.6
	3	p	71.0	2.9
	4	p	14.3	1.0
12	1	R	5.2	
	2	α	198	21.0
	3	p		100
13	1	R	3.9	
	2	p	525	9.8
	3	p	67	2.8
	4	α	14.0	3.8
	5	α	66	10.9
14	1	α	32	6.8
	2	α	41	8.0
	3	p	1545	18.0
	4	α	63	10.6
	5	p	905	13.3
15	1	α	437	34.5
	2	α	980	54.5
	3	α	17	4.3
16	1	p	488	9.3
	2	p	785	12.3
	3	p	17	6.8
17	1	α	58	10.1
	2	p	1380	17.0
18	1	p	171	5.0
	2	p	65	2.7
	3	α	8.0	2.4
	4	p	200	5.6
	5	p	4000	30
19	1	R	6.6	
	2	α	12	3.3
	3	α	182	20.5

Event No.	Track No.	Identity[a]	Range (μ)	Energy (Mev)
20	1	α	17.1	4.6
	2	α	9.3	2.6
	3	α	21.8	5.2
21	1	R	4.0	
	2	p		90
22	1	R	2.6	
	2	p	31.7	1.7
	3	α	44.4	8.4
23	1	R	1.3	
	2	p	144	4.5
24	1	α	48	8.9
	2	α	140	17.5
	3	p	761	12.0
	4	p	1545	18.0
25	1	R	2.6	
	2	p	2220	22.5
	3	p	865	12.9
26	1	R	2.6	
	2	p	225	5.9
	3	R	1	
27	1	p	62	2.7
	2	α	109	15.0
	3	p		20
	4	π^- or p		$45(\pi)$
28	1	R	2	
	2	p		20
	3	p or		185
		K		98
29	1	R	1.9	
	2	p	583	10.2
	3	K^-	12 200	43.8

[a] R denotes a recoil particle.

the energy. The energy of the particles which produce short recoil tracks is negligibly small and therefore the assignment of their charge and mass are unimportant in an estimation of the total energy of the charged particles. For the above reasons, the quantities given in column 7 of Table II may be slightly low.

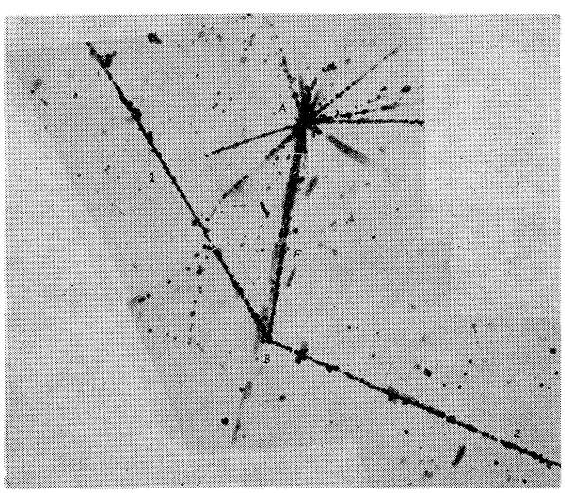

FIG. 4. Photograph of event 1. The secondary star B has two outgoing particles, tracks 1 and 2, which stop in the emulsion.

The range and the nature of each particle from the secondary stars are given in Table III. Tracks shorter than about 10 microns were assumed to be nuclear recoils, and no attempt has been made to identify them or to estimate their energy except in events where their nature was suggested by a consideration of the conservation of energy and momentum.

B. Detailed Description

A comparatively small number of hyperfragments have been reported which yield detailed information of the Λ^0 particle interaction with nuclei. For this reason it seems worthwhile to describe in some detail those events from which important information is derived.

Event 1

Event 1 was found in a plate exposed to cosmic rays and has been described in considerable detail elsewhere,[10] and therefore only a brief summary of the salient features will be given here. A photograph of the event is shown in Fig. 4. The track of the fragment is 55 microns long. The small-angle scattering and the absence of δ rays along the track near the secondary star indicate that the fragment stopped before it disintegrated. From the thin-down along the track of the hyperfragment, the charge was found to be greater than 1 and less than 6. The secondary star consists of only two tracks (tracks 1 and 2 in Fig. 4) which end in the same plate. A δ-ray count along tracks 1 and 2 shows that the charge of each particle was 2. The mass of the particle which produced track 1 was found to be 4 with 15 to 1 odds. The mass of the particle which produced track 2 was found to be either 3 or 4. The mass of both particles was determined from the intercept of the δ-ray number *vs* residual range curve.

The energy of particles 1 and 2 is 105 Mev if track 2 is assumed to have been produced by an alpha particle. If particle 2 is assumed to be a He^3 nucleus the energy is 99 Mev. Assuming that the residual momentum from particles 1 and 2 was carried away by a single neutron, the energy of the neutron is found to be 71 Mev. In this case the disintegration schemes can be written as follows:

$$_{\Lambda}Be^9 \rightarrow He^4 + He^4 + n + Q_1, \tag{1}$$

or

$$_{\Lambda}Be^8 \rightarrow He^3 + He^4 + n + Q_2. \tag{2}$$

The total kinetic energy of the reaction products is found to be, for reaction (1), $Q_1 = 176 \pm 3$ Mev; and for reaction (2), $Q_2 = 170 \pm 3$ Mev. These values can be compared with the decay energy of the free Λ^0 particle,

$$\Lambda^0 \rightarrow p + \pi^- + Q_3, \tag{3}$$

where $Q_3 = 36.9 \pm 0.22$ Mev.[16] If the π meson is absorbed, the total energy available from the decay is the rest mass of the π meson plus the decay energy minus the neutron-proton mass difference ($Q = 139.5 + 36.9 - 1.3 = 175.1 \pm 0.5$ Mev).

An energy level diagram for the decay of the hyperfragment $_{\Lambda}Be^9$ is shown in Fig. 5. Reactions involving two neutrons are possible but improbable from phase space considerations.

The close agreement of the energetics of the $_{\Lambda}Be^9$ or $_{\Lambda}Be^8$ decay with the expected value, shows that the star was caused by the nonmesonic decay of a bound Λ^0 particle. The binding energy of the Λ^0 particle in $_{\Lambda}Be^9$ or $_{\Lambda}Be^8$ was found to be -0.7 ± 3 Mev or 3.2 ± 3 Mev respectively. Although Be^8 is unstable against alpha-particle emission, $_{\Lambda}Be^8$ may be stable against $He^4 + {_{\Lambda}He^4}$ emission. An energy level diagram of Be^8 and $_{\Lambda}Be^8$ is shown in Fig. 6. It is clear that $_{\Lambda}Be^8$ will be stable against $He^4 + {_{\Lambda}He^4}$ emission if B_{Λ} (in $_{\Lambda}Be^8$) $> B_{\Lambda}$ (in $_{\Lambda}He^4$) $- 1.6$ Mev. Measurements on the binding in $_{\Lambda}He^4$ [7,15,21] give B_{Λ} (in $_{\Lambda}He^4$) $= 3.2$ Mev. Thus the stability condition for $_{\Lambda}Be^8$ becomes B_{Λ} (in $_{\Lambda}Be^8$) $> 3.2 - 1.6 \cong 1.6$ Mev. The binding energy of 3.2 ± 3 Mev obtained from the assumption that the hyperfragment was $_{\Lambda}Be^8$ is not inconsistent with this condition. It is significant to note that the binding of a Λ^0 particle is not large in a nucleus as heavy as beryllium.

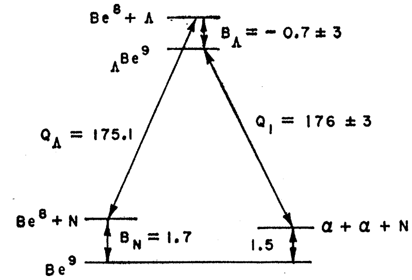

FIG. 5. Energy-level diagram of $_{\Lambda}Be^9$ and Be^9. B_{Λ} is the binding energy of a Λ^0 particle in $_{\Lambda}Be^9$. B_N is the binding energy of a neutron in Be^9.

Event 2

The hyperfragment was found in a cosmic-ray plate. The track of the fragment is 19 microns long and is deflected through a large angle near the end. Since a large-angle scattering is improbable for a particle of high velocity, it is probable that the fragment stopped before it gave rise to the secondary star. An estimate of the charge of the fragment cannot be made because the track is very short. The three tracks from the star are coplanar within the limits of measurements (± 2 degrees). A projection drawing of the event is shown in Fig. 7. Tracks 2 and 3 end in the emulsion. Track 1 leaves the emulsion after 2200 microns. Track 1 was identified as due to a proton from grain density and multiple scattering measurements. The energy of the

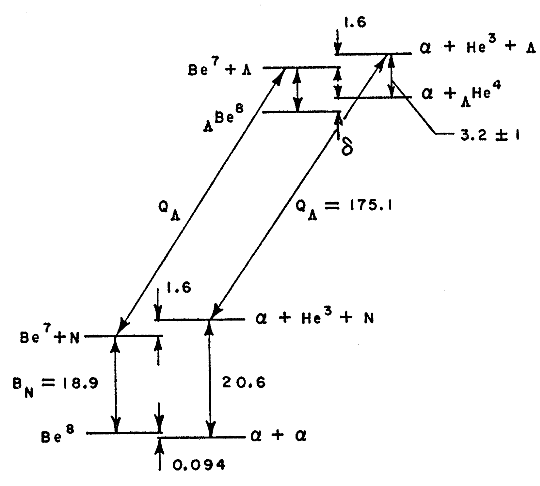

FIG. 6. Energy level diagram of $_{\Lambda}Be^8$ and Be^8. If the binding energy of the Λ^0 particle in $_{\Lambda}Be^8$ (B_{Λ}) is greater than $3.2 - 1.6$, a $_{\Lambda}Be^8$ nucleus will be stable against emission of $He^4 + {_{\Lambda}He^4}$.

[21] Baldo, Belliboni, Ceccarelli, Grilli, Sechi, Vitale, and Zorn (private communication, Padua, 1954).

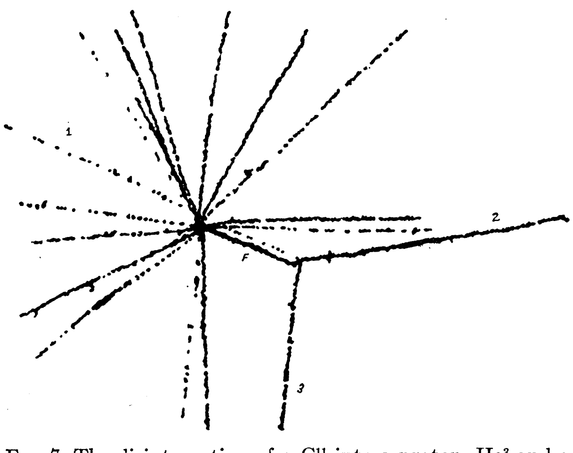

FIG. 7. The disintegration of a C^{11} into a proton, He^3 and a Li^7 is shown above (event 2).

proton was determined from the grain density and was found to be 103±11 Mev. Tracks 2 and 3 could not be identified directly from their characteristics, however the Z of the particle was estimated to be greater than one in both cases.

The coplanarity of the three tracks strongly suggests that only these particles were involved in the disintegration of the hyperfragment. If a neutron was emitted perpendicular to the plane of the three tracks, its energy would be at most 0.1 Mev. Since the energy release is high it seems very improbable that such a slow neutron was emitted. The component of the momentum perpendicular to the proton should be zero; therefore the assigned mass and charge of particles 2 and 3 must be such as to satisfy this requirement. For a given assignment, the energy can be found very accurately from the range. If track 2 is assumed to be due to Li^7 ($E=17.5$ Mev) and track 3 to be due to He^3 ($E=26$ Mev), the momentum balance in the direction perpendicular to the proton track is 2±10 Mev/c which is very small in comparison to the large value for the momentum of the Li and He particles (∼400 Mev/c). No other choice of stable nuclei for particles 2 and 3 satisfies the requirements of momentum balance. For example, if particle 3 is assumed to have a mass of 4 instead of 3, the perpendicular momentum is found to be 60 Mev/c, which could easily have been detected because the momentum unbalance arising from measurements of angles and ranges is estimated to be at most 15 Mev/c. A further check on the assignment of particles 2 and 3 can be made from a comparison of the momentum of the proton, obtained from grain counting, with the parallel momentum component of particles 2 and 3. The parallel component of particles 2 and 3 is 452±14 Mev/c which is to be compared with the value of the momentum of the proton which is 450±34 Mev/c. The energy of the proton can be determined more accurately from the momentum balance than from grain counting. It is 104±6 Mev. The total energy from the star is then $(104+17.5+26)=147.5\pm6$ Mev. The energy-level diagram for ${}_{\Lambda}C^{11}$ is shown in Fig. 8.

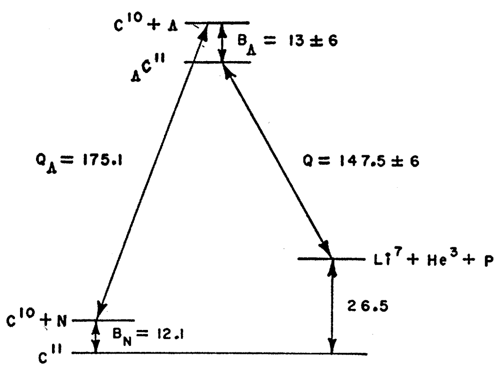

FIG. 8. Energy-level diagram of ${}_{\Lambda}C^{11}$ and C^{11} and the disintegration products.

The close agreement of the energy from the fragment disintegration with the expected value from ${}_{\Lambda}C^{11}$ confirms the assumption that the fragment contained a bound Λ^0 particle. The binding energy of the Λ^0 particle was found to be 13.0±6 Mev.

The visible energy release of 147±6 Mev suggests that the secondary star might have been due to the absorption of a negative π meson which was trapped in an orbit about the fragment and was absorbed after the fragment stopped.[1] Such a reaction was investigated by making various assumptions for the identity of the fragment and the particles from its disintegration. In all cases the observed kinetic energy of the particles, plus the energy needed to account for the difference in the mass of the initial and final nuclear products, was found to be significantly greater than the rest energy of a π meson. Therefore this interpretation can be excluded.

Event 3

The primary star was produced by a 3-Bev proton from the Cosmotron accelerator and was found in a 1000-micron glass-backed plate. The range of the hyperfragment is only 8 microns and therefore no information can be obtained from its track. The secondary star has three prongs. A projection drawing of the event is shown in Fig. 9. Track 1 stops in the emulsion after 75.4 microns and appears to have been produced by an alpha particle. Track 3 is 262 microns long and the absence of δ rays along it implies that the charge of the particle was one. Multiple-scattering measurements, using the constant sagitta method,[22,23] indicate that it was a proton. The probability that the track was produced by a deuteron is about 23 percent, whereas the probability of it being a triton is small (about 9 percent). Track 2 leaves the emulsion after 2300 microns and was followed into the adjacent plate. Multiple scattering and

[22] Biswas, George, and Peters, Proc. Indian Acad. Sci. **38**, 418 (1953).

[23] Dilworth, Goldsack, and Hirschberg, Nuovo cimento **11**, 113 (1953).

grain counting identify the particle as a proton. The grain density gives a value of 98.5±8 Mev for its energy.

The vector addition gives for the momentum of these three charged particles, 318±7 Mev/c (assuming track 3 to be a proton), clearly indicating that one or more neutral particles were involved in the disintegration. Of course it is impossible to determine the number of neutrons involved, but it seems worthwhile to assume that only one neutron was emitted and see if the energetics are consistent with the decay energy of a bound Λ^0 particle. The total charge from the secondary star is 4 therefore we assume that the fragment was an isotope of beryllium. If we assume that track 3 was produced by a proton, the reaction can be written as follows:

$$ {}_{\Lambda}\mathrm{Be}^7 \rightarrow \mathrm{He}^4 + p + p + n + Q_4. \tag{4}$$

Q_4 was found to be 169.7±8 Mev. From the energy diagram for this reaction, Fig. 10, the binding energy of

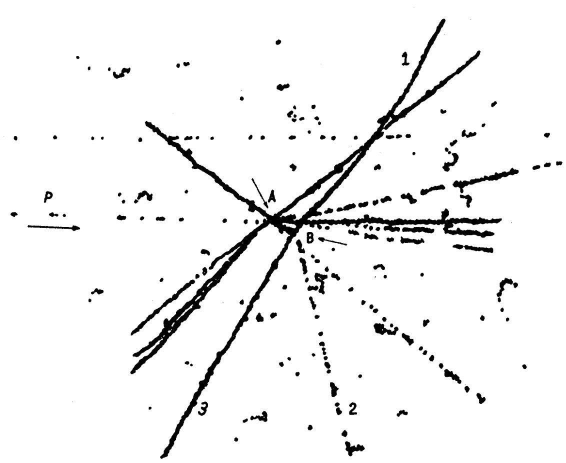

FIG. 9. A drawing of two connected stars. The primary star is indicated by the arrow A; the secondary star by arrow B. A fast proton is ejected from the secondary star (track 2). The initiating 3-Bev proton track is indicated by the arrow P.

the Λ^0 particle is found to be 5.9±8 Mev.

The following alternative reactions have been considered:

$$ {}_{\Lambda}\mathrm{Be}^8 \rightarrow \mathrm{He}^4 + p + d + n + Q_5, \tag{5}$$

$$ {}_{\Lambda}\mathrm{Be}^7 \rightarrow \mathrm{He}^3 + p + d + n + Q_6. \tag{6}$$

Q_5 was found to be 179.2±8 Mev and Q_6 was found to be 203±8 Mev. The binding of the Λ^0 particle is then −10.6±8 and −45±8 Mev respectively. The negative values for the binding energy excludes reaction 6 and probably reaction 5.

The close agreement of the Q value from reaction 4 with the expected value suggests that this interpretation may be correct. It should be borne in mind that two neutrons might have been involved and that the above agreement may be fortuitous.

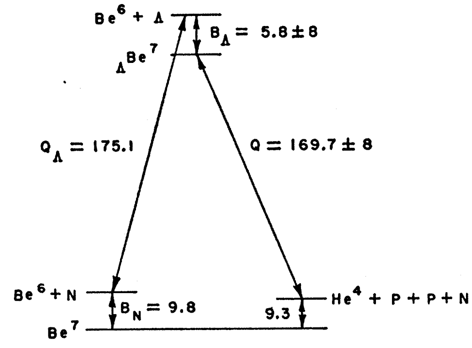

FIG. 10. The appropriate energy levels for the decay of $_{\Lambda}\mathrm{Be}^7$.

Event 4

The initial star was produced by a 3-Bev proton. The connecting track is 77 microns long and exhibits a measurable amount of multiple scattering near the end. There are no measurable δ rays along the track. The small-angle scattering indicates that the particle was traveling slowly and that it probably stopped before producing the secondary star. The scattering along this track is consistent with a He^3 or a He^4 nucleus but the statistics are so poor that other nuclei of charge 1 or 2 cannot be excluded.

The secondary star consists of two tracks which are 226 and 6 microns long. A projection drawing of the event is shown in Fig. 11. The multiple scattering of track 2 is indicative of a proton mass although a deuteron is possible but less likely. Track 1 is too short to be identified directly. The secondary star could not have been produced by a scattered proton because the short recoil track is in the backward direction.

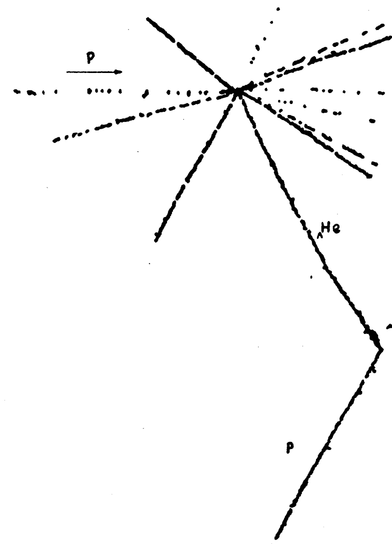

FIG. 11. A small 2-prong secondary star which was produced by a particle from a 3-Bev proton star. The incoming proton track is indicated by the arrow P. The short track from the secondary star is indicated by the arrow.

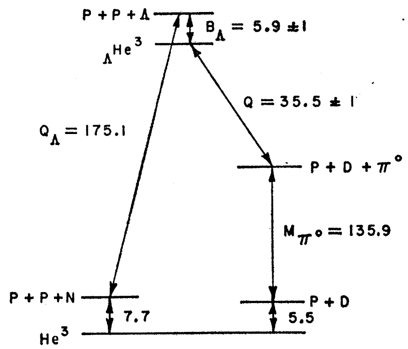

FIG. 12. Energy level diagram for the mesonic decay of $_\Lambda He^3$. The drawing is not to scale. The $_\Lambda He^4$ diagram is very similar.

The small amount of kinetic energy from the secondary star suggests that if a Λ^0 particle were bound in the fragment, most of the energy was carried away by a π^0 meson, corresponding to the free decay of the Λ^0 particle.

$$\Lambda^0 \rightarrow n + \pi^0 + Q_7. \quad (7)$$

In fact, the energetics strongly support this hypothesis. If track 1 is assumed to be due to a deuteron and 2 a proton, the residual momentum is found to be 93±1.6 Mev/c which gives 29±1 Mev for the energy of the π^0 meson and a total energy release of 35.5±1 Mev for the reaction

$$_\Lambda He^3 \rightarrow p + d + \pi^0 + Q_8. \quad (8)$$

The binding energy of the Λ^0 particle is found to be 5.9±1 Mev. The energy-level diagram is shown in Fig. 12.

An alternative decay scheme is also possible. If track 1 is assumed to have been produced by a triton, the residual momentum is found to be 94±1.6 Mev/c and the energy of the π^0 meson, 29.6±1 Mev:

$$_\Lambda He^4 \rightarrow t + p + \pi^0 + Q_9. \quad (9)$$

Then for this reaction the total kinetic energy is found to be 36.1 Mev and the binding energy of the Λ^0 particle is found to be 3.9±1 Mev. This value for the binding energy is in agreement with that found from the charged mesonic decays of $_\Lambda He^4$ reported by others.[7,15,21]

Of course it is possible that the residual momentum from the two charged particles was carried away by two or more fast neutrons instead of a π^0 meson and that the reasonable value for the binding energy of the Λ^0 particle is fortuitous. Such an argument cannot be excluded. However it would seem to be somewhat improbable, from phase space considerations, because the neutrons would have nearly all of the energy (~170 Mev) while the energy of the charged particles is small (~6 Mev).

The binding energy of the Λ^0 particle in $_\Lambda He^4$ is found to be −40 Mev if tracks 1 and 2 are both assumed to be due to deuterons, and therefore we conclude that this assignment is incorrect.

Event 5

A photograph of event 5 is shown in Fig. 13. The range of the connecting track is 193 microns. From the thin-down, and from δ rays near the primary star, the charge of the fragment was found to be either 3 or 4. There is an appreciable amount of small-angle scattering along the track of the fragment near the secondary star, which suggests that it stopped before disintegrating. The δ rays along the two longer tracks from the secondary star show that they were produced by particles of charge one. From conservation of charge, the short recoil track must have been produced by an isotope of hydrogen or helium. For all consistent combinations of mass assignments for particles one and two, and for charge and mass assignments of particle three, the residual momentum is greater than would be expected from a bound Λ^0 particle which decayed by π^0-meson emission, and less than would be expected if only one neutron were ejected. Thus it seems probable that two or more neutrons were emitted.

Although the decay energy of the fragment cannot be evaluated in this event, it represents a very good example of a hyperfragment which lived for a time greater than 10^{-11} second and subsequently disintegrated.

Event 6

In this event the three-pronged secondary star is connected to the 3-Bev proton primary star by a short saturated track of 5 microns in length. The secondary star consists of a lightly ionizing particle, probably a

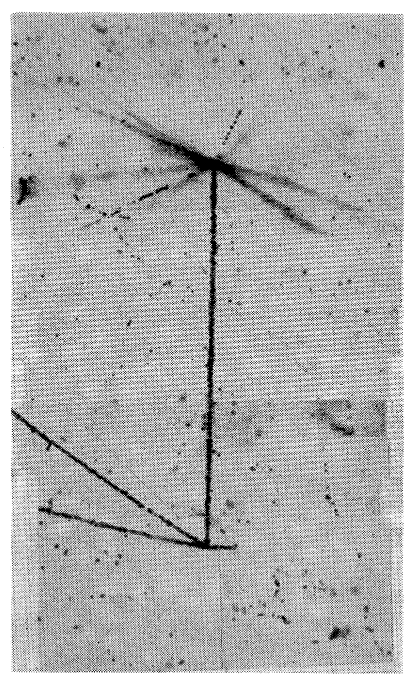

FIG. 13. The disintegration from rest of a hyperfragment. The total disintegration energy from the secondary star cannot be evaluated. The event was found in a cosmic-ray plate.

proton of 120±12 Mev (although a K-meson is not inconsistent with the grain density and scattering data); a short recoil track of 3.2 microns; and a third track which leaves the emulsion after 840 microns and is ascribed to a proton of 25±10 Mev. The identity of the recoil cannot be established, however for various assignments, the residual momentum of the three charged particles is not appreciably different. If it is assumed that the residual momentum was given to two neutrons, the minimum kinetic energy from the secondary star is found to be 166 Mev. The total energy from the star is significantly higher than the rest energy of a π meson and of the right order of magnitude for a bound Λ^0 particle.

Event 7

The primary star was produced by a 1.5-Bev negative π meson. The connecting track is very short, only 5 microns, and therefore the only conclusion that can be drawn is that the particle was probably heavily ionizing.

The secondary star has 6 prongs. The large number of prongs from the secondary star strongly suggests that the star was not produced by a stopping π meson. The probability that a stopped negative π meson will produce a 6-prong star in an emulsion is very small ($\sim$1/500). The total energy release from the disintegration cannot be evaluated.

Events 8, 9, 10, and 11

In these four events the connecting tracks have a range from 15 to 45 microns. Even though the range is short, the tracks appear to have been produced by nuclear fragments rather than by stopping π mesons. The visible energy is low (20 to 40 Mev). The large residual momentum excludes the possibility that only a π^0 meson carried away this momentum and if it is assumed that only one neutron was ejected, the total energy is considerably less than the expected value from a bound Λ^0 particle. These events can be interpreted as the nonmesonic decay of a Λ^0 hyperfragment with the emission of more than one neutron.

Events 12–26

In events 12 through 26 (Table II) no definite conclusion can be drawn from any one of the events. In each of these cases, the connecting track is too short to be identified. However, in each case the track is saturated indicating that the secondary star was produced by a slow particle. It is possible that some of these events may have been produced by stopped negative particles such as π mesons or K-mesons; however, in none of these cases are there "Auger" electrons associated with the secondary star. It was previously found that 27 percent of stopped negative μ mesons have associated low-energy electrons.[24] The percentage of electrons from

[24] W. F. Fry, Nuovo cimento **10**, 490 (1953).

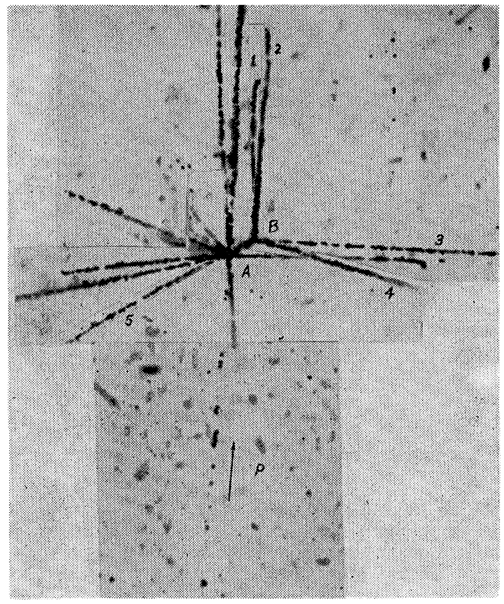

FIG. 14. A photograph of event 14. The primary star A was produced by a 3-Bev proton. The secondary star B has 5 associated tracks. The total energy from the secondary star cannot be evaluated.

stopped π mesons is about the same.[25] The absence of any such electrons is taken to be an indication that at least a portion of these secondary stars was not caused by stopped negative π mesons.

A photograph of event 14 is shown in Fig. 14.

ENERGETIC DISINTEGRATIONS

Event 27

This event has been described in detail elsewhere[26] and only the salient features will be given here. The connected stars were found in a plate exposed to cosmic rays. The primary star is of the type (22+9p). The track of the nuclear fragment is 192 microns long. The thin-down along the connecting track as well as the absence of δ rays and the increase of the multiple scattering near the secondary star are strong evidence that the nuclear particle stopped before it disintegrated. From the thin-down and from the δ rays near the parent star, the charge of the fragment is found to be greater than 2e and definitely less than 5e. The secondary star has four prongs. The characteristics of these tracks are summarized in Table IV. The identity of track 4 cannot be established. If track 4 is ascribed to a π meson, the minimum kinetic energy (assuming the residual momentum was given to two neutrons) is found to be 92 Mev; if track 4 is assumed to be due to a K-meson, the kinetic energy becomes 210 Mev; if track 4 is assumed to be due to a proton, the minimum kinetic energy is

[25] Cosyns, Dilworth, Occhialini, and Schoenberg, Proc. Phil. Soc. (London) **A62**, 801 (1949); A. Bonetti and G. Tomasini, Nuovo cimento **8**, 693 (1951); W. F. Fry, Phys. Rev. **83**, 594 (1951).

[26] W. F. Fry and M. S. Swami, Phys. Rev. **96**, 809 (1954).

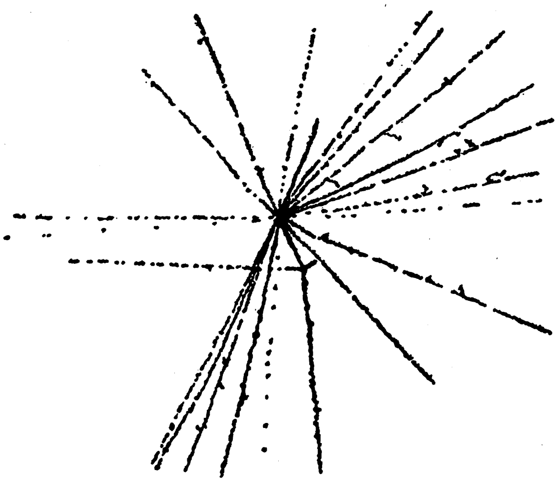

FIG. 15. A drawing of event 28. The connecting track is 10.5 microns long and quite straight. The secondary star is to the right of the large primary star and consists of three outgoing particles; a proton of 185±20 Mev, a short recoil and a proton of greater than 20 Mev.

found to be 328 Mev. Since the charge of the fragment is found to be less than $5e$, the particle which produced track 4 must have been negative in order to conserve charge, therefore the proton assumption is not valid. In any case, the energy released from the disintegration of the fragment is significantly greater than the available energy from a Λ^0 hyperfragment.

Event 28

A projection drawing of event 28 is shown in Fig. 15. The primary star was produced by a 3-Bev proton. The connecting track is 10.5 microns long, quite straight and saturated. These facts suggest that the particle was a nuclear fragment with a low velocity at the time it produced the secondary star. The secondary star consists of three outgoing tracks. Track 1 (Fig. 15) is only 2 microns long; track 2 leaves the air surface and was followed through the adjacent plate. Track 3 leaves the glass side of the emulsion after 1000 microns. The characteristics of the tracks are given in Table V. If the recoil is assumed to be a carbon nucleus and track 3 a proton, the residual momentum of the three particles is about 330 Mev/c which gives an energy of about 60 Mev for a single neutron. If two neutrons were involved their energy is greater than 30 Mev. The minimum kinetic energy from the secondary star is then $(185-20)+1+30=216$ Mev. The choice of the recoil does not significantly affect the total energy. The disintegration energy is clearly too large for a Λ^0 hyperfragment, and is possibly consistent with the energy release in event 27.

Event 29

In this event, found in a cosmic-ray plate, a K-meson was ejected from the secondary star. Since the event seemed to be rather unusual, it was published previously,[27] but a brief summary is given here for the sake of completeness.

Unfortunately, the connecting track is only 9 microns long. From the appearance of this track we conclude that the particle had a low velocity, but it is impossible to decide whether it was produced by a nuclear fragment, a stopping negative hyperon, or a negative K-meson. The secondary star has three outgoing particles: a recoil, a proton of 10 Mev (a deuteron or triton cannot be excluded), and a K-meson of 43 Mev. The three tracks are not coplanar, indicating that one or more neutral particles were also ejected. The residual momentum of the three particles is approximately 200 Mev/c. A single neutron of this momentum would have an energy of 20 Mev.

The track of the K-meson was followed through four pellicles and into a fifth pellicle where it stopped. The mass of the particle was measured in each plate by several methods and in each plate the mass was found to be that of a K-meson. By using the constant-sagitta method[22,23] along all of the track, the mass of the particle was found to be $1170\pm200\ m_e$. There are no associated particles at the end of the K-meson track; therefore we assume it was negative.

The total energy from the secondary star, including the rest energy of the K-meson, is greater than $496+54=550$ Mev. There appear to be four possible explanations for the event: the secondary star may have been produced (1) by the nuclear absorption of a stopped negative "heavy" hyperon; (2) by the absorption of a stopped negative "heavy" K-meson; or (3) by the disintegration of a nuclear fragment which contained a bound "heavy" hyperon; or (4) by the disintegration of a fragment which contained a bound "heavy" K-meson. If assumption (1) or (3) is correct, the mass of the hyperon must have been greater than $(938+550)/0.511=2910\ m_e$. If assumption (2) or (4) is correct, the mass of the "heavy" K-meson was greater than $550/0.511=1075\ m_e$.

It is possible that the event was caused by a hyperon of the same type as the hyperon which decayed into a K-meson, previously reported by Eisenberg.[28]

TABLE IV. Characteristics of tracks from the secondary star.

Track	Range in microns	Ionization	Identity	Energy in Mev
1	62	Black	p, d, t	2.7 if p
2	109	Black	α	14.0
3	2000[a]	Black	p, d, t	$20<E<26$ if p
4	300[b]	Gray	π, K, or p	45 ± 18 if π 160 ± 65 if K 300 ± 125 if p

[a] The track left the emulsion after a range of 2000 microns. The tota range is estimated to be less than 2500 microns.
[b] The track dips steeply and leaves the emulsion after 300 microns. The energy was determined from blob density along the track. The blob density is 1.5 ± 0.12 times the minimum. The track is too straight for an electron with the observed ionization.

[27] Fry, Schneps, and Swami, Phys. Rev. **97**, 1189 (1955).
[28] Y. Eisenberg, Phys. Rev. **96**, 541 (1954).

IV. SUMMARY AND DISCUSSION

From the experimental data, the following general conclusions can be drawn about the production and decay of hyperfragments. Of the 29 disintegrations, a π meson was ejected in only 2 cases, and a K-meson in one event. Since it is not clear that the K-mesonic event was due to a fragment disintegration, it will be discussed separately. The remaining 26 disintegrations involve only nuclear particles. The two π-mesonic decays occurred from fragments with a charge of 2 and 3. The charge of the fragment, in the nonmesonic cases, was found to be greater than 2 in all cases. Only one of the four fragments with charge 3 decayed mesonically. The complete absence of the mesonic decay of fragments with $Z>3$ (22 events) is in agreement with the predictions of Cheston and Primakoff[19] that the decay in a heavy fragment could be "stimulated" with the result that the π meson would not be emitted.

The average charge of the fragments is found to be between 4 and 5. There may be some bias in the charge distribution because the range of very heavy fragments may be very short. Hence the secondary star may practically coincide with the primary star and therefore not be observed. On the other hand, light hyperfragments will in general have a considerable range and therefore may leave the emulsion before disintegrating. This effect would seem to be small, however, because the average energy of the observed hyperfragments is quite low (see Fig. 3). The charge and energy distributions indicate that the Λ^0 particles are preferentially captured in slow, moderately heavy fragments.

In four cases the total disintegration energy could be evaluated. The close agreement of the total energy with the decay energy of a free Λ^0 particle is strong evidence that these events were due to the disintegration of a Λ^0 particle which was bound in the fragment. The binding energy of the Λ^0 particle could be found from the kinetic energy of the decay products, the known decay energy of the Λ^0 particle and nuclear mass data. The data on the binding energy in four nuclei are given in Table VI. Because a Λ^0 particle is different from a nucleon, in a nucleus it may not be subject to the Pauli exclusion principle. Hence it is not clear that the binding energy of the Λ^0 particle should be compared to the binding energy of the last neutron in a similar stable nucleus. However the data show that the binding energy of a Λ^0 particle is not greatly different than that of the last neutron and also that the binding energy is not large in a nucleus as heavy as carbon.

TABLE V. Characteristics of energetic decay star.

Track	Range in microns	Ionization	Identity	Energy in Mev
1	2	Black	Recoil	0.5 if α; 1.0 if B
2	2000	Black	p, d, t	>20 if p; >28 if d
3[a]	1000	Gray	k or p	98 ± 12 if K 185 ± 20 if p

[a] The energy was estimated from the grain density. Multiple-scattering measurements are consistent with a proton or a K-meson but are inconsistent with a π meson.

TABLE VI. Summary of binding energy data.

Nucleus	Event	Binding of Λ^0 particle in Mev	Binding of last neutron in stable nucleus
$_\Lambda He^3$	4	5.9 ± 1	7.7
or $_\Lambda He^4$		3.9 ± 1	20.6
$_\Lambda Be^7$	3	5.9 ± 8	9.8
$_\Lambda Be^8$	1	3.7 ± 3	18.9
or $_\Lambda Be^9$		-0.7 ± 3	1.7
$_\Lambda C^{11}$	2	13 ± 6	12.1

Two events were found where the energy of the disintegration products is significantly greater than the available energy from a bound Λ^0 particle. In one of these events (event 27), the connecting track is long enough to establish, without question, that the secondary star was produced by the disintegration of a moderately heavy hyperfragment. The energy release shows that an unstable particle whose available energy is greater than the decay energy of a Λ^0 particle can be bound in a nucleus and be stable for a time greater than 10^{-11} sec. It is possible that this disintegration was due to a bound charged hyperon or possibly a bound K-meson. The disintegration energy available from a charged hyperon Ξ^+ which decays by the reaction $\Xi^+\rightarrow p+\pi^0$ is $116+136=252$ Mev.[29] The total energy available from the disintegration of the charged cascade hyperon Ξ^-, which decays by the reaction $\Xi^-\rightarrow\Lambda^0+\pi^-$, is $175+139+66=380$ Mev.[30] The available energy from a K-meson is just its rest energy which is 496 Mev.[31] Events of this nature may provide a crucial test for theories of unstable particles.[32,33]

The second energetic decay (event 28) may also be due to the same process as the previous event. Since the connecting track is short, it is not possible to exclude the nuclear capture of a slow negative hyperon or K-meson.

Since the connecting track in event 29 is short, it is not possible to decide whether the particle which pro-

[29] Genoa-Milan and Padua groups, Report of the Padua Conference (unpublished); W. F. Fry and M. S. Swani, Phys. Rev. **96**, 235 (1954).
[30] E. W. Cowan, Phys. Rev. **94**, 161 (1954).
[31] Thompson, Buskirk, Etter, Karzmark, and Rediker, Phys. Rev. **90**, 1122 (1953).
[32] M. Gell-Mann, Phys. Rev. **92**, 833 (1953). According to the isotopic spin assignments of Gell-Mann, the Ξ^+ has $T_z=1$. This hyperon should not exist in a nucleus for a time comparable to the lifetime of observed hyperfragments because the reaction $\Xi^++n\rightarrow\Lambda^0+p$ should occur rapidly. Similarly the cascade hyperon, Ξ^-, should rapidly undergo the reaction $\Xi^-+n\rightarrow\Lambda^0+\Lambda^0$. If a θ meson could be bound in a nucleus, then from isotopic spin considerations ($T_z=-\frac{1}{2}$) it could be stable for a period comparable to the stopping time of a fragment in emulsion.
[33] M. Gell-Mann and A. Pais, Proceedings of the Glasgow conference, 1954 (to be published).

duced the secondary star was bound in a nuclear fragment or whether it stopped in the emulsion and was captured by a nucleus. Nevertheless, in either case it is necessary to postulate that the secondary star was produced by a "heavy" hyperon or a "heavy" K-meson.

It now seems clear that a Λ^0 particle can be bound in a nucleus and furthermore that at least one other unstable particle can also be bound. It is hoped that additional energetic events, which are more favorable, will be found, so that the particle or particles which are responsible can be identified.

Although there are data on the binding energy of a Λ^0 particle in a few nuclei, knowledge of the variation of the binding energy with nuclear charge and mass is inadequate. Such information would be valuable in an attempt to understand the interaction of the Λ^0 particle with nuclei.

The authors are very grateful to the Cosmotron group, and especially to Dr. R. K. Adair, for making the facilities of the Cosmotron available. Their interest and help are appreciated. The cosmic-ray exposures were made for us by Major David G. Simons, United States Air Force, whose efforts have made this portion of the program possible. Many discussions with Professor R. G. Sachs have been stimulating and helpful. J. Slowey, J. Behrendt, and R. Benson assisted in the scanning.

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Wave Equation for Spin 0 in Hamiltonian Form

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It is shown that the Hamiltonian form for spin 1 particles also describes spin 0 particles if the 5-dimensional representation of the Duffin-Kemmer algebra is used.

I. INTRODUCTION

SCHRÖDINGER[1] has recently shown that the field-free equations for a spin 1 (Proca) particle can be written in the Hamiltonian form

$$i\partial\Psi/\partial t=\mathcal{H}\Psi, \tag{1}$$

where

$$\mathcal{H}=\boldsymbol{\beta}\cdot\mathbf{P}+\kappa\beta_4. \tag{2}$$

Here the matrices β_1, β_2, β_3, β_4 form a ten-dimensional representation of the Duffin-Kemmer algebra.[2] The wave function Ψ is subject to the initial condition

$$(\mathcal{H}\beta_4-\kappa)\Psi=0. \tag{3}$$

As a consequence of Eq. (1) the condition (3) is maintained for all time.

It is well known[2] that the field-free equations for both spin-one and spin-zero particles can be written in the form

$$(\beta_\mu\partial_\mu+\kappa)\Psi=0. \tag{4}$$

The only difference is that for spin one we need to use a ten-dimensional representation and for spin zero a five-dimensional representation of the algebra defined by the relations

$$\beta_\mu\beta_\nu\beta_\rho+\beta_\rho\beta_\nu\beta_\mu=\delta_{\mu\nu}\beta_\rho+\delta_{\rho\nu}\beta_\mu. \tag{5}$$

This suggests that, similarly, Eqs. (1) and (2) also describe spin-zero (Klein-Gordon) particles with the appropriate replacement of the β matrices.

[1] E. Schrödinger, Proc. Roy. Soc. (London) **A229**, 39 (1955).
[2] N. Kemmer, Proc. Roy. Soc. (London) **A173**, 91 (1939).

II. THE HAMILTONIAN FORM

If we describe a spin-zero particle by a scalar function ϕ and a four-vector ϕ_μ, the field-free equations are[3]

$$\partial\phi/\partial x_\mu=\kappa\phi_\mu,\quad \partial\phi_\mu/\partial x_\mu=\kappa\phi. \tag{6}$$

On splitting ϕ_μ into a space vector $\boldsymbol{\phi}$ and a space scalar ψ ($\phi_4=i\psi$), these equations become

$$\nabla\phi=\kappa\boldsymbol{\phi}, \tag{7}$$

$$\partial\phi/\partial t=-\kappa\psi, \tag{8}$$

$$\partial\psi/\partial t=-\nabla\cdot\boldsymbol{\phi}+\kappa\phi. \tag{9}$$

Differentiating (7) with respect to time and eliminating $\partial\phi/\partial t$ using (8) gives

$$\partial\boldsymbol{\phi}/\partial t=-\nabla\psi. \tag{7a}$$

If (7a), (8), and (9) are required to hold for all times (7) need only be required to be true initially. It will then automatically be satisfied at later times. Hence the field equations can be written, on multiplying by i, as

$$i\partial\boldsymbol{\phi}/\partial t=\mathbf{P}\psi,\quad i\partial\phi/\partial t=-i\kappa\psi,$$
$$i\partial\psi/\partial t=\mathbf{P}\cdot\boldsymbol{\phi}+i\kappa\phi, \tag{8}$$

where

$$\mathbf{P}=-i\nabla. \tag{9}$$

The admissible solutions are those which satisfy (7) initially.

Combining the quantities $\boldsymbol{\phi}$, ϕ, and ψ into a single

[3] $x_4=it$. $\hbar$ and c are taken to be unity.

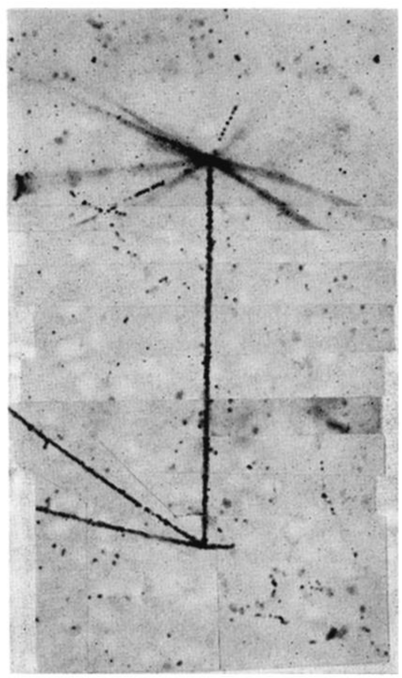

FIG. 13. The disintegration from rest of a hyperfragment. The total disintegration energy from the secondary star cannot be evaluated. The event was found in a cosmic-ray plate.

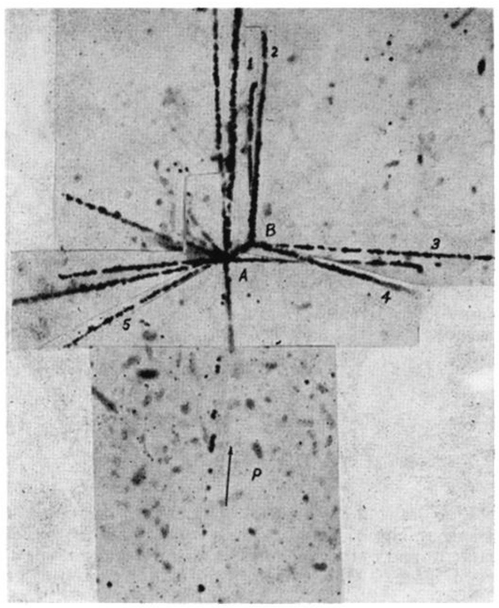

FIG. 14. A photograph of event 14. The primary star A was produced by a 3-Bev proton. The secondary star B has 5 associated tracks. The total energy from the secondary star cannot be evaluated.

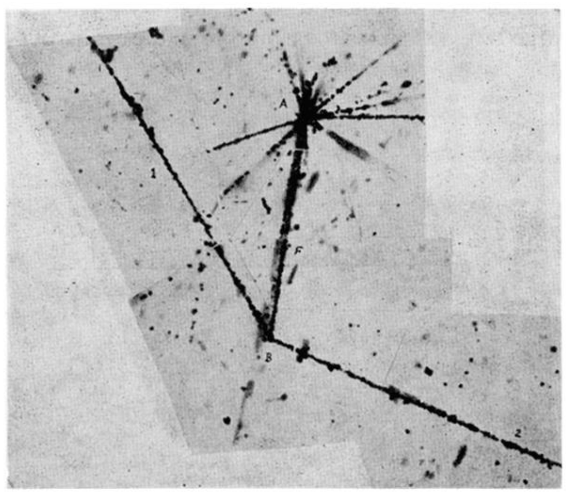

Fig. 4. Photograph of event 1. The secondary star B has two outgoing particles, tracks 1 and 2, which stop in the emulsion.