pion field, χ^* its Hermitian conjugate. One obtains

$$\left(\frac{w_{\text{ch}}^{(1)}}{w_{\text{ch}}^{(2)}}\right)_{\text{vector}}=\left(\frac{w_{\text{ch}}^{(1)}}{w_{\text{ch}}^{(2)}}\right)_{\text{scalar}}\cdot\frac{4p^2}{M_0^2}\left(1-\frac{4\mu^2}{M_0^2}\right)^{-1},$$

where p is the momentum ($\sim$380 Mev/c) of the pion emerging in 1-mesonic decay. Thus the relative factor of vector to scalar ratio is $\simeq$2.7 which does not materially change the situation. In the vector case, the ratio of 0- to 2-mesonic decay is increased by a factor $(4M/M_0)(1-4\mu^2/M_0^2)^{-1}\sim 10$. Thus, like the 1-mesonic decay, the 0-mesonic mode gains in relative importance due to the stronger momentum dependence of H_g in the vector case. Also in this case, $w^{(0)}$ is still small compared to $w^{(2)}$. We have not considered cases of higher spin.

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Analysis of Properties of Secondary Particles in Nucleon-Nucleon Collisions at Very High Energy*

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Momentum measurements were made on the individual secondary particles emitted in the "outer" cone of the S-star. A lower limit to the momentum of the particles in the "inner" cone was obtained. The transformation from the laboratory system to the center-of-mass system can be made, using suitable approximations, on the assumption that all charged secondaries are pions. This leads to a symmetrical angular and energy distribution with all particles included in a forward or backward cone of half-angle 20°. The results are compared with various multiple production theories. It is possible to infer that there is no more than one pair of particles of protonic mass among the secondaries and that even the presence of one pair would violate symmetry. The number of K-mesons must also be small. The methods of analysis should be useful on other events in this energy range.

I. INTRODUCTION

THE problem of analyzing stars caused by high-energy cosmic-ray particles in nuclear emulsions is, in general, quite complicated. However, if the incident particle is a nucleon and the target is a nucleon also, the analysis is considerably simplified since it is then possible to postulate bilateral symmetry in the center-of-mass system.[1] Even in cases where the target nucleus is more complicated but where there is some evidence that the principal interaction was with a peripheral nucleon, it is possible to analyze the star on the basis of a nucleon-nucleon interaction since the energy transferred to the rest of the target nucleus may be negligibly small. It is possible to make a real distinction here between low and high energies. At low energies it is possible to have an appreciable fraction of the energy emitted from the "first nucleon-nucleon collision" in the form of secondary particles at relatively large angles. It is then quite probable that these secondaries will strike another nucleon inside the nucleus and that a nuclear cascade may develop. In the case of the very high energies ($E_p > 10^{13}$ ev), the secondaries from a given nucleon-nucleon collision are known to be very strongly collimated in the direction of motion of the incident nucleon, and hence it is quite possible that a collision with a peripheral nucleon will result in almost all the energy being transferred into a cone so narrow that it intersects no other nucleons. This is the same phenomenon which takes place in central collisions and produces the so-called "tunnel effect."[2] A simple computation shows that peripheral collisions are rather probable if the target nucleus is a light nucleus of the emulsion (C, N, O).

A number of authors have presented stars of varying degrees of complexity with primary energies of the order of 10^{13} ev or greater, generally with no possibility of analyzing the energy of the secondaries.[3]

In the present paper a careful analysis of the S star[1] is reported which illustrates the advantages of being able to measure the multiple scattering on several of the shower tracks. The S star is shown in Fig. 1. It occurred in a 200μ Ilford G-5 emulsion flown at above 90 000 feet for over 16 hours from Chicago, Illinois.

The fact that the only two secondary tracks (B and C) which are not closely collimated (within 5° in space) with the original direction of the primary track are gray tracks indicates that only an insignificant propor-

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* Supported in part by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

[1] Lord, Fainberg, and Schein, Phys. Rev. **80**, 970 (1950).

[2] F. C. Roesler and C. B. A. McCusker, Nuovo cimento **10**, 127 (1953).

[3] Engler, Haber-Scheim, and Winkler, Nuovo cimento **12**, 930 (1954), contains references to earlier work.

tion of the original energy was transferred to the residual nucleus. It is thus reasonable to make an analysis on the assumption of a nucleon-nucleon collision. It will be shown that the event has bilateral symmetry in the center-of-mass system which is then confirming evidence of the nucleon-nucleon character of the event.

The energy of the incident singly charged particle (assumed to be a proton) has been determined previously by a method which uses not only the information from the angular distribution but also the available information on the energy of the secondaries.[4] New measurements have been made on the multiple scattering and the energies of the secondaries have been re-evaluated. The re-evaluation of these energies does not change the previous estimate of the primary energy.

The angles and energies of the individual secondary particles are calculated in the center-of-mass system of the colliding nucleons. The details of this analysis are described in Sec. II. This is important since it is possible to draw from this analysis some conclusions about the character of the secondaries.

The main interest of the present analysis is to indicate that it would be very profitable to investigate in greater detail the mechanism of collisions at these very high energies. It is of importance at the present time to learn something more of the nature of the collision process and of the secondaries produced. The first is of fundamental interest since it may indicate whether present theoretical attacks on the problem are likely to be fruitful. In particular, it should throw light on the validity of the statistical theories of the high-energy collisions. The second is of interest since it is important to determine the role of heavy mesons and of nucleon-antinucleon pairs in high-energy collisions. If heavy mesons are really elementary particles and are strongly coupled to the nucleon then one would expect their probability of production, relative to pions, to approach approximate equality at these high energies. If these mesons are merely composite states of other fundamental particles, this conclusion would not necessarily hold.[5] Certainly, if nucleon-antinucleon pairs can be produced at all, one would expect to see them in collisions of this energy.[6] Finally these collisions should provide evidence on the interaction between free mesons; since mesons in the meson cloud around the nucleon extend far enough and since due to the Lorentz contraction of the cloud the collisions are of such short duration, some of the collisions, at least, are effectively meson-meson collisions.

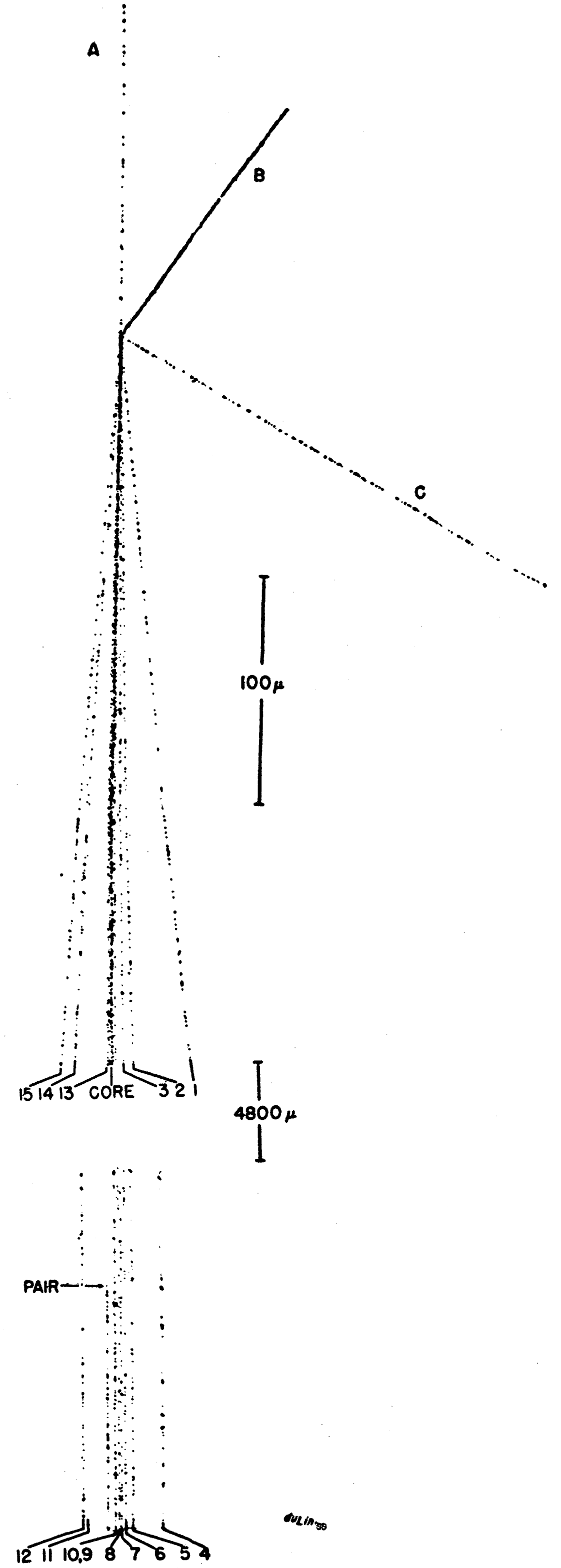

FIG. 1. Projected view of the *S* star. *A* is the incident proton, *B* and *C* are evaporation tracks. The core is shown at a distance sufficient to separate all except tracks 9 and 10. The pair is the only electron pair observed in the available distance.

II. TRANSFORMATION FROM LABORATORY TO CENTER-OF-MASS SYSTEM

The formulas for the Lorentz Transformation can be conveniently applied to the analysis of high-energy

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[4] R. G. Glasser and Marcel Schein, Phys. Rev. **90**, 218 (1953).

[5] G. Cocconi, Phys. Rev. **93**, 1107 (1954).

[6] See Proceedings of the Duke University Conference on Cosmic Rays, December, 1953 (unpublished), Session I.

stars in the following form:

$$E_i=\gamma(E_i'-\beta P_i'c\cos\theta_i'), \tag{1}$$

$$P_i\cos\theta_i=\gamma(P_i'\cos\theta_i'-\beta E_i'/c), \tag{2}$$

$$P_i\sin\theta_i=P_i'\sin\theta_i', \tag{3}$$

where E_i, P_i, and θ_i are the energy, momentum, and angle with the direction of motion, respectively, of the ith particle and primed quantities are in the laboratory (L) system of coordinates, unprimed in the center-of-mass (C) system of coordinates. As usual, c is the velocity of light, βc the relative velocity of the C and L systems, and $\gamma=(1-\beta^2)^{-\frac{1}{2}}$. Dividing (3) by (2), one gets the transformation for angles:

$$\tan\theta_i=\frac{\tan\theta_i'}{\gamma[1-\beta(E_i'/P_i'c)\sec\theta_i']}. \tag{4}$$

In the discussion of these formulas, we will find it useful to introduce the following dimensionless quantities which will generally be very small compared to one:

$$\delta_\beta=1-\beta, \tag{5}$$

$$\delta_{E_i}=(E_i'/P_i'c)-1, \tag{6}$$

$$\delta_{\theta_i}=1-\cos\theta_i'. \tag{7}$$

Introducing these quantities into Eqs. (1) and (4), the following expressions can easily be obtained:

$$E_i=\gamma E_i'[1-(1-\delta_\beta)(1-\delta_{\theta_i})/(1+\delta_{E_i})], \tag{8}$$

$$\tan\theta_i=\tan\theta_i'/\gamma[1-(1-\delta_\beta)(1+\delta_{E_i})(1-\delta_{\theta_i})^{-1}. \tag{9}$$

Under the assumption that one can neglect δ_β, δ_{θ_i}, and δ_{E_i} and all products of these compared to one, (8) and (9) reduce to:

$$E_i=\gamma E_i'(\delta_\beta+\delta_{\theta_i}+\delta_{E_i}), \tag{10}$$

$$\tan\theta_i=\tan\theta_i'/[\gamma(\delta_\beta-\delta_{E_i}-\delta_{\theta_i})]. \tag{11}$$

These formulas are in a convenient form to derive the energy and angle in the C system from the measurements in the L system for the high-energy collisions we are treating.

The assumptions that these quantities, δ_β, δ_{θ_i}, δ_{E_i}, are small compared to one will be briefly discussed. If δ_β is small compared to one, this means that the relative velocity of the C and L systems is nearly equal to that of light and hence corresponds to a very high-energy primary. To the same order of accuracy as is used in neglecting δ_β compared to one, we can derive from (3) and the definition of γ:

$$\delta_\beta\approx 1/2\gamma^2. \tag{12}$$

The smallness of δ_{θ_i} means that the laboratory angle is itself very small, and hence we can write from (7):

$$\delta_{\theta_i}\approx\theta'^2/2. \tag{13}$$

If δ_{E_i} [in (6)] is small compared to one, this means that the laboratory energy of the secondaries is high. In this case we can write:

$$\delta_{E_i}\approx\tfrac{1}{2}(M_ic^2/E_i')^2. \tag{14}$$

The conditions that δ_β, δ_{θ_i}, δ_{E_i}, be small compared to one are well fulfilled in some high-energy stars as will be shown in the next section.

III. ANALYSIS OF THE *S* STAR

The energy of the primary proton of the *S* star has previously been determined by the quartile method.[4] This method is valuable for stars where some multiple-scattering data is available, since it makes use of this information which can greatly affect the energy determination. In the determination of the primary energy, it was assumed that all the secondaries are pions. The energy estimate would not be affected appreciably if a few of the secondaries had heavier mass. The energy was shown to be $(2^{+1.45}_{-0.85})\times 10^{13}$ ev.

The secondary tracks of the *S* star have been measured on the precision scattering stage in this laboratory.

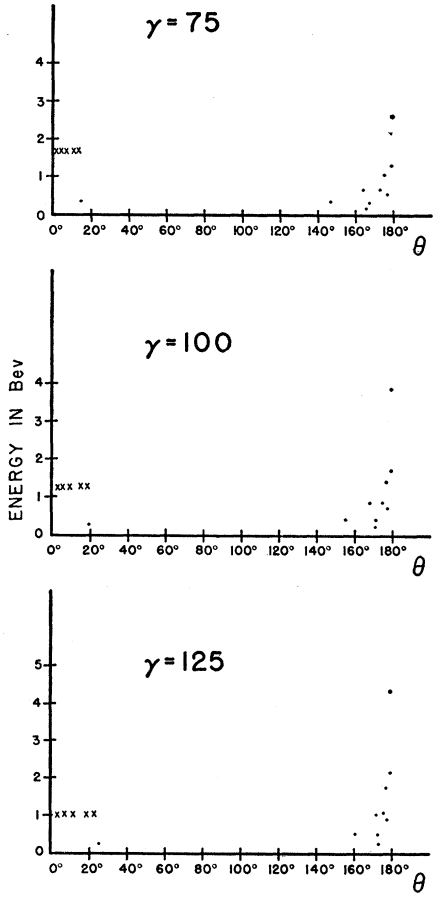

FIG. 2. Energy *vs* angle correlation for three values of γ. These correspond to the most probable value and values one standard deviation away. Points marked with an × represent particles for which only a lower limit to the energy can be obtained.

TABLE I. Transformation from L-system to C-system for three values of γ. $\gamma=100$ is the most probable value. Auxiliary quantities have been included also for $\gamma=100$ to illustrate their magnitude.

					$\gamma=100$					$\gamma=75$		$\gamma=125$	
Track	E' (Bev)	$\tan\theta'$	δ_E ($\times10^{-6}$)	$\delta\theta$ ($\times10^{-6}$)	$\delta\beta+\delta\theta+\delta_E$ ($\times10^{-6}$)	$\delta\beta-\delta\theta-\delta_E$ ($\times10^{-6}$)	E (Bev)	$\tan\theta$	θ (deg)	E (Bev)	θ (deg)	E (Bev)	θ (deg)
1	0.30	0.1733					3.9	−0.0117	179.3	2.6	179.1	4.3	179.4
2	1.5	0.0517	4360	1340	5750	−5650	0.86	−0.0916	174.8	0.65	173.0	1.07	175.8
3	6.0	0.0268	272	359	681	−581	0.41	−0.4613	155.2	0.32	146.6	0.50	160.3
4	5.5	0.00047	324	11	385	−285	0.21	−0.1649	170.6	0.17	165.7	0.25	172.9
5	>250	0.00013	0.16	0.8	51.0	49.0	1.28	0.2653	14.9	1.69	11.1	1.03	18.5
6	>250	0.00008	0.16	0.3	50.5	49.5	1.26	0.1616	9.2	1.68	6.9	1.02	11.5
7	>250	0.0016	0.16	1.3	51.5	48.5	1.29	0.3299	18.3	1.70	13.7	1.05	22.8
8	50	0.0016	3.9	1.3	55.2	44.8	0.28	0.3571	19.6	0.35	14.3	0.23	25.5
9	>250	0.0005	0.16	0.13	50.3	49.7	1.26	0.1006	5.7	1.67	4.3	1.01	7.2
10	>250	0.0002	0.16	0.04	50.2	49.8	1.26	0.0398	2.3	1.67	1.7	1.01	2.9
11	3.0	0.0200	1090	200	1340	−1240	0.40	−0.1614	170.8	0.31	167.5	0.50	172.7
12	1.5	0.0212	4360	225	4640	−4540	0.70	−0.0467	177.3	0.53	176.4	0.87	177.9
13	0.6	0.0397	28 000	788	28 800	−28 700	1.71	−0.0138	179.2	1.30	178.9	2.16	179.4
14	3.0	0.0590	1090	1740	2880	−2780	0.86	−0.2122	168.0	0.66	164.0	1.07	172.0
15	1.0	0.0908	9800	4120	14 000	−13 900	1.40	−0.0653	176.3	1.05	175.0	1.75	177.0

The $p\beta$ of the tracks in the outer cone (Fig. 1, tracks 1–4, 11–15) was measured directly by using the sagitta method of multiple scattering. The multiple scattering was definitely above noise in the largest cell length used on each track. The energies quoted should be accurate within a factor of two.

For the fast tracks of the inner cone (tracks 5–10), a track-to-track measurement of the multiple scattering was made. This method is the only one which can be used on these very fast tracks since it eliminates the small emulsion distortions and stage motion which contribute to the noise. This showed definite scattering on one of the tracks (track 8). But the other five all showed no scattering sagitta as large as 250 A, which was the limit of detection for the track position, in the 8000μ observable track length. This corresponds to a $p\beta$ value of over 250 Bev/c for these tracks. They were treated in the rest of the computations as though this lower limit were a true, measured value.

Since scattering measurements are thus available on all but the fastest tracks, it is possible to apply the analysis outlined in the preceding section to this event. For the purpose of this computation, all particles have been assumed to be pions. The calculations have been carried through for values of $\gamma=100(E_p=2\times10^{13}$ ev), $\gamma=125$ ($E_p=3.1\times10^{13}$ ev), and $\gamma=75$ ($E_p=1.1\times10^{13}$ ev). Table I gives the results of this calculation for $\gamma=100$. The results of the energy-angle correlation in the center-of-mass system for all three values of γ are shown in Fig. 2.

It should be noted that the points marked with crosses in Fig. 2 and for which it is possible to obtain only a lower limit to the energy by scattering measurements are relatively insensitive to the actual energy determination. That is, a change in the L-system energy does not affect the C-system angle, as can be seen from Table I. This is in distinction to the situation for the slow particles in the outer core where a change in measured energy affects both the C-system angle and energy.

Two facts show up very clearly in Fig. 2. One is that the shower particles are clearly divided into two cones of half-angle approximately 20°. The fact that the forward cone contains six charged particles and the backward cone nine, is presumably due merely to statistical fluctuations.[7] The other fact is that the points plotted as crosses (for which only a lower limit to the energy was measured) corresponding to fast particles in the forward cone are probably very nearly at that lower limit, since if one assumed the actual energy were much higher the symmetry would be completely destroyed, although one would expect symmetry in such a nucleon-nucleon collision. This would seem to be supported by the fact that the one track in the forward cone whose energy is measured has an energy of the same order of magnitude as this lower limit.

The average energy of the secondaries in the backward cone is 0.84 Bev, 1.2 Bev, or 1.4 Bev for $\gamma=75$, 100, or 125, respectively. Since symmetry between forward and backward cones should prevail, this can be considered as the best estimate of the average energy of all the secondaries. We observe fifteen charged secondaries; if we add one-half of this number for the neutral secondaries, the total energy carried away by the secondaries is about 25 Bev in the C-system. This should be compared with the total available energy in the C-system for the most probable value of γ ($\gamma=100$) which is 200 Bev. On the basis of this comparison we conclude that only about one eighth of the available energy is transferred to the secondaries in this collision. It should be emphasized that this conclusion is based on the assumption that the average energy of the neutral particles is the same as the energy of the charged

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[7] In the original publication (reference 1), track number 4 was placed in the forward cone since its energy was estimated to be much higher than the value now believed to be correct.

pions as derived above, which in turn depends on the assumption of symmetry with respect to the plane perpendicular to the direction of motion of the primary.

These conclusions depend also on the identity of the charged particles which were assumed to be pions. If the assumption as to the nature of some of the particles were changed by assuming that they have mass M different from M_π, the principal effect would be to change δ_{E_i} by the factor $M^2/M_\pi{}^2$ as can be seen from (14), since the L-system energy is high enough that the measurement by multiple scattering can be considered a measurement of E'. If one assumes that some of the particles are of K-meson mass (about 1000 m_e) this factor is 13.3, and if one assumes particles of nucleonic mass it is 45. We will consider the effects with the estimate that $\gamma=100$. The assumption that some of the particles are of K-meson mass would not affect the previous computations for the high energy forward particles (No. 5–7, 9–10 of Fig. 1) by more than five percent, but would for instance mean that if particle No. 8 were a K-meson instead of going at 20° with an energy of 0.3 Bev in the C-system, it would have been going at 100° with an energy of 0.5 Bev, an angle at which no pions are found according to the previous analysis. For the particles in the outer cone, this assumption would mean that they will be going almost straight backward in the C-system and with greatly increased energies. For example, we find that if particle No. 4 had K-meson mass it would be located in Fig. 2 at 179.5° instead of 170.0° with an energy of 2.4 Bev instead of 0.21 Bev, while all the others would have much higher energies.

In case one assumes some of the particles to be of nucleonic mass, the same qualitative features are observed but greatly intensified; there would even be a small effect on the high-energy particles in the core, while the effect on the particles in the backward cone would be extremely large, giving energies up to 45 times that already estimated and all angles essentially backward (180°). Thus the assumption of more than one or two of the secondary particles being of either K-meson or nucleon mass would appreciably disturb the symmetry, since the forward cone is affected only slightly while the backward cone is completely distorted. The only theoretical possibility of restoring this symmetry approximately would be by making the radical assumption that essentially all of the secondaries were of a higher mass and as a consequence that the primary energy was appreciably lower. There is considerable evidence that pions are produced in these high-energy collisions since they can sometimes be identified and since the large number of electron pairs can best be explained by neutral pions. Therefore, this would not seem to be a reasonable assumption.

IV. DISCUSSION

Under the assumption that all the charged particles are pions, the energy and angular distributions in the C-system show the following characteristic features: (1) symmetry under reflection in the plane perpendicular to the primary direction; (2) all the charged particles are included in two cones, one backward and one forward, of half-angle approximately 20°; (3) the average energy of the charged particles is between 0.8 and 1.4 Bev; (4) the energies of the individual particles are not much different from the average.

Theories of multiple meson production have been presented by Heisenberg,[8] Wataghin,[9] Oppenheimer *et al.*,[10,11] Fermi,[12] Landau,[13] and Bhabha.[14] A comparison of the experimental facts presented in this paper can be made with the theories of Fermi, Heisenberg, and Landau. All these theories predict symmetry in the C-system. All of them predict the "two-cone effect," although for different reasons. One of the essential differences is that Fermi's theories would predict a much higher average energy (about 10 Bev) in the C-system for a primary energy of 2×10^4 Bev since the available energy is about 200 Bev and the number of particles is about 25. On the other hand, Heisenberg's calculations on the S-star predict an average energy of about 1 Bev in the C-system in agreement with the observed values in the backward cone if the elasticity parameter (which he calls γ in his work) is about 8 (a value which would also be in agreement with our multiplicity on his theory). The two-cone effect in Landau's theory is explained by the evaporation of particles from a comparatively flat surface. He gets an average angular spread of about 15° which fits well with our observations. The typical pion energy in his theory is also in general agreement with what we find, although he predicts the existence of some particles with extremely high energies, which however we have not observed.

If heavy mesons exist among the secondaries, then the average energy in the C-system would be somewhat higher. On the assumption of approximate equipartition of energy, one can say that only those particles which show up as low-energy pions in Fig. 2 could be heavy mesons since, for example, those which appear to be above 1 Bev considered as pions would turn out to have energies above 5 Bev considered as K-mesons and would thus have energies well above those of the pions. Since, as stated previously, the angles in the forward cone are relatively insensitive to mass while the angles in the backward cone are very sensitive, we

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[8] W. Heisenberg, Z. Physik **101**, 533 (1936); **113**, 61 (1939); **126**, 569 (1949); Naturwiss. **39**, 69 (1952).

[9] G. Wataghin, Phys. Rev. **63**, 137 (1943).

[10] Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. **73**, 127 (1948).

[11] H. W. Lewis, Revs. Modern Phys. **24**, 24 (1952), contains also references to other theoretical work.

[12] E. Fermi, Progr. Theoret. Phys. (Japan) **5**, 570 (1950); Phys. Rev. **81**, 683 (1951); *Elementary Particles* (Yale University Press, New Haven, 1951), p. 84.

[13] L. D. Landau, Akad. Nauk. S.S.S.R. (Ser. Fiz.) **17**, 51 (1953). The authors are indebted to Dr. M. Hammermesh of Argonne National Laboratory for his translation of this article.

[14] H. J. Bhabha, Proc. Roy. Soc. (London) **A219**, 293 (1953).

cannot rule out *K*-mesons in the forward cone; however, we can safely assume that not more than one or two of the particles in the backward cone could be *K*-mesons.

For proton-antiproton pairs or hyperons, the same argument holds, only with much greater strength. If any of the particles in the backward cone is of protonic mass, then it is at a radically different energy (about 45 times higher) than all the pions and in addition is emitted essentially straight backward in the *C*-system. It is thus probable that none of these secondaries are of nucleonic mass on an assumption of approximate equipartition. Even if one does not assume equipartition, if more than one of the charged secondaries in each of the two cones were of protonic mass the total energy of these particles would already be the total available energy in the *C*-system, which would then leave no energy for the charged pions or for the neutral particles, and is thus not possible. This makes us believe that if nucleon-antinucleon pairs or hyperons are produced in collisions similar to the *S*-star, their number must be at most about 10 percent of all charged particles. If such particles are produced at these energies, they certainly are not in equilibrium with the pions and statistical theories based on equilibrium cannot be applied.

V. CONCLUSIONS

(1) Direct momentum measurements on the *S* star have yielded the energy and angular distributions in the *C*-system, assuming all particles to be pions.

(2) The bilateral symmetry appears to be well verified for the angular distribution and is consistent with the energy distribution.

(3) The average energy of the pions in the *C*-system is close to 1 Bev.

(4) All the observed particles are emitted in the *C*-system within two cones of half-angle 20°.

(5) The number of particles of protonic mass created in the collision is at most two and most likely none, on the assumption of equilibrium.

(6) The results appear consistent with either Heisenberg's or Landau's theory.

(7) In this collision, if all secondaries are pions, the visible energy is small compared to the available energy in the *C*-system.