Deuteron Stripping Processes at High Energies

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The processes which lead to the formation of stripped particle beams in encounters of high-energy deuterons with heavy nuclei are discussed. It is shown that a significant role is played by a previously unnoticed dissociation process arising from diffractive effects in which, from a classical standpoint, the particles suffer no collisions.

AMONG the first observations made with particles accelerated to high energies was the discovery that a beam of deuterons striking a target gives rise to narrowly collimated beams of neutrons and protons of approximately half the incident kinetic energy.[1] The origin of these beams has been very simply explained by Serber,[2] as lying in collisions in which one of the two particles of the deuteron strikes a target nucleus while the other misses and retains its forward momentum. In the present note we shall indicate than an additional contribution to these beams of comparable magnitude comes from processes in which neither of the two particles of the deuteron suffers any collision, in the classical sense.

As Serber has shown, the simple properties of the stripping effect depend on the looseness of the deuterons's binding. In particular the internal motion in the deuteron is negligible during the time it requires for the deuteron to travel a distance equal to a nuclear diameter. Hence it is meaningful to speak of the instantaneous positions of the nucleons during encounters with nuclei. To illustrate what takes place in these encounters we employ, for the present, the simplified model introduced by Serber to estimate the stripping cross section. Neglecting the effects of nuclear transparency, we imagine all of the nucleons entering nuclei to suffer collisions which remove the possibility of their interfering further with the incident beam. We may then picture a target nucleus as a black sphere whose radius is larger than that of the deuteron or, more simply still, as a black disk lying in a plane perpendicular to the direction of incidence.

The stripping processes considered by Serber are those in which the instantaneous positions of the neutron and proton at the moment of collision are such that one strikes the disk and is absorbed by it, while the other clears the edge and continues. It may easily be seen, however, that breakup of the deuteron is not limited to cases in which particles strike the nucleus. It takes place with considerable likelihood when neither of the particles is intercepted. The reason is to be found in the localization the two-particle wave packet suffers as it passes the edge of the disk. The internal wave function of the deuteron, which is spherically symmetric [Fig. 1(a)] before passing the edge of the disk suffers a certain truncation by virtue of its passage. This is illustrated in Fig. 1(b) where it is evident that neither of the particles is to be found in the lower region of the sphere, which lies in the shadow of the disk's edge.

Let us assume that both particles have cleared the disk's edge and examine their wave function an instant later. Then, since the particles remain symmetrically placed about their center of mass, which suffers negligible deflection in so short a time, it follows that neither of them can be located in the oppositely defined upper cap of the sphere either.[3] This reduction in volume of the two-particle wave packet leads to an increase of the internal kinetic energy, and a certain probability of dissociation. More precisely phrased, the final state of the two-particle system is no longer simply the ground state of the deuteron, but contains an admixture of excited states, all of which are unbound and lead to disintegration. In the two-body problem, absorptive dissociation implies the existence of a certain amount of free dissociation much as absorption implies the existence of shadow scattering in single-particle diffraction problems.

To secure a quantitative estimate of the effect, we

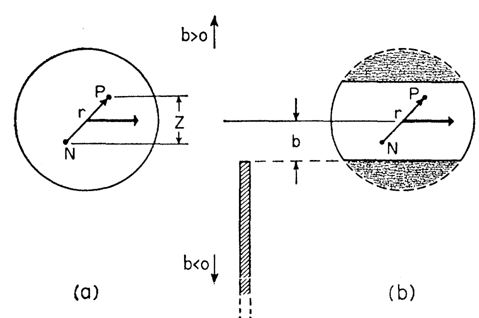

FIG. 1. Schematic representation of the initial and final states of high-energy deuterons grazing the edge of a nucleus, represented by an absorbing screen. The initial spherically symmetric internal wave function is shown figuratively as the sphere (a). The change which takes place in the two-particle wave function due to the encounter with the screen is illustrated at a later instant in (b). If neither of the particles collides with the screen, the final wave function must exclude them from the shaded regions of the sphere (b).

[1] Helmholtz, McMillan, and Sewell, Phys. Rev. 72, 1003 (1947).
[2] R. Serber, Phys. Rev. 72, 1008 (1947). The effect was in fact, predicted on the basis explained.
[3] A particle may be found in the upper shaded region of the sphere [Fig. 1(b)] only if the other one has been absorbed by the screen.

neglect the curvature of the edge of the target nucleus so that the disk is replaced by a semi-infinite black screen. Let b be the impact parameter of the center of mass of a deuteron relative to the edge of the screen. Then the incident beam of deuterons will be distributed uniformly over positive and negative values of b. Let the internal wave function of the deuteron in its ground state be $\varphi_0(\mathbf{r})$, where $\mathbf{r}$ is the relative separation of the neutron and proton, $\mathbf{r}_p-\mathbf{r}_n$. For positive impact parameters the probability, denoted by $J(b)$, that either of the particles strikes the screen is just the probability that one of them, say the proton, lies in either of the two spherical caps noted earlier. That is, we have

$$J(b)=\int_{|z|>2b}|\varphi_0(\mathbf{r})|^2 d\mathbf{r}, \tag{1}$$

where z is the component of $\mathbf{r}$ perpendicular to the edge of the screen. An analogous argument for negative impact parameters shows that the probability that one of the particles misses the screen while the other is absorbed is just $J(|b|)$. These processes, which we may speak of as absorptive stripping, evidently have a total cross section per unit length of $2\int_0^\infty J(b)db$. Since such collisions produce either stripped neutrons or protons with equal likelihood, the cross section per unit length for producing either one is just $\int_0^\infty J(b)db$. The free dissociation process which we discuss next liberates neutrons and protons simultaneously in the stripped beam, and should therefore be separately detectable by coincidence techniques.

Encounters in which neither particle is absorbed by the screen may take place only for positive impact parameters. When they do occur, the internal wave function suffers the truncation mentioned earlier, so that its final form, which we represent as $\varphi_t(\mathbf{r})$, is

$$\varphi_t(\mathbf{r})=\begin{cases}\varphi_0(\mathbf{r}), & |z|<2b\\ 0, & |z|>2b.\end{cases} \tag{2}$$

The function φ_t defined in this way is evidently unnormalized. The normalization integral $\int|\varphi_t|^2 d\mathbf{r}$ is just $1-J(b)$, so that we may define the normalized final wave function as

$$\varphi_t'(\mathbf{r})=[1-J(b)]^{-\frac{1}{2}}\varphi_t(\mathbf{r}). \tag{3}$$

Now the probability that the final two-particle system remains bound is just the absolute value, squared, of the $\varphi_0(\mathbf{r})$ component contained in $\varphi_t'(\mathbf{r})$.

$$\begin{aligned}\int\varphi_0^*(\mathbf{r})\varphi_t'(\mathbf{r})d\mathbf{r}&=[1-J(b)]^{-\frac{1}{2}}\int\varphi_0^*(\mathbf{r})\varphi_t(\mathbf{r})d\mathbf{r}\\ &=[1-J(b)]^{-\frac{1}{2}}\int|\varphi_t(\mathbf{r})|^2 d\mathbf{r}\\ &=[1-J(b)]^{\frac{1}{2}}.\end{aligned} \tag{4}$$

The probability that the system remains a deuteron is evidently $1-J(b)$, and the dissociation probability is therefore just $J(b)$. When the dissociation probability is multiplied by the probability $1-J(b)$, that both particles pass the screen, as assumed, we have the probability of free dissociation at any impact parameter, $b>0$. The cross section per unit length for free dissociation is thus

$$\sigma_{\text{f.d.}}=\int_0^\infty J(b)[1-J(b)]db. \tag{5}$$

The cross section per unit length for the production of either type of particle by absorptive stripping is, once again,

$$\sigma_{\text{a.s.}}=\int_0^\infty J(b)db \tag{6}$$

Serber has shown that this is just $\frac{1}{4}\langle r\rangle_d$, where $\langle r\rangle_d$ is the mean value of the neutron-proton separation in the deuteron.[4] Since the free dissociation probability, however, has no classical analog, no such compact evaluation is available for it. It must be evaluated for an explicit representation of the deuteron wave function. To secure a simple estimate we neglect range corrections, employing a wave function of the form

$$\varphi_0(\mathbf{r})=(\alpha/2\pi)^{\frac{1}{2}}e^{-\alpha r}/r, \tag{7}$$

for which the absorptive stripping is $\sigma_{\text{a.s.}}=\frac{1}{4}\langle r\rangle_d=1/8\alpha$. With this choice of wave function, we have

$$J(b)=e^{-4\alpha b}-4\alpha\beta\int_{4\alpha b}^\infty(e^{-x}/x)dx, \tag{8}$$

which, inserted in (5) and integrated, yields

$$\begin{aligned}\sigma_{\text{f.d.}}&=(1/8\alpha)[(4/3)\log 2-(1/3)].\\ &=0.59\sigma_{\text{a.s.}}\end{aligned} \tag{9}$$

The free-dissociation effect therefore represents a 59% correction to the cross section for absorptive stripping of either neutrons or protons.

To estimate the actual nuclear stripping cross section when either neutrons or protons are detected, the sum of the above cross sections per unit length is multiplied by $2\pi R$, the nuclear circumference, so that $\sigma_s=1.59\times(\pi/4)R\langle r\rangle_d$. The observed stripping cross sections[5] are, however, still larger and show a somewhat stronger increase with mass number than those predicted in this way. There is reason to believe that the difference is due at least in part to the transparency of the nucleus neglected here. A more complete theory of stripping based on the optical model of the nucleus has been devised for study of this problem, and will be reported upon when the calculations are completed.

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[4] This is evident on reversing the orders of integration in (1) and (6).

[5] Millburn, Birnbaum, Crandall, and Schecter, Phys. Rev. **95**, 1268 (1954).