Reaction $p+p\rightarrow\pi^++d$ with Polarized Protons*

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The reaction $p+p\rightarrow\pi^++d$ is analyzed. Using the statistical operator methods, we derive: (i) the production cross section for s-, p-, and d-wave mesons for the case of a polarized proton beam; (ii) the polarization of the deuteron for both polarized and unpolarized incident protons. With the restriction to s- and p-waves, the deuteron polarization gives new information about the production amplitudes; for an unpolarized proton beam for the p-wave production only, for a polarized one for s-waves as well. We discuss: (i) the possibilities of measuring the deuteron polarization; (ii) the relation of the phases of the production amplitudes to the pp-scattering phases.

1. INTRODUCTION

THE object of this paper is to discuss the reaction $p+p\rightarrow\pi^++d$, using polarized incident protons. This process was first analyzed by Marshak and Messiah,[1] who restricted themselves to s- and p-wave pions, and has recently been discussed by Rosenfeld,[2] Wolfenstein,[3] and Gell-Mann and Watson.[4] It has been studied experimentally by Crawford and Stevenson[5] and by Fields *et al.*[6]

We shall give a general expression for the S-matrix for the reaction considered and hence derive the cross section for s-, p-, and d-wave pions. Restricting ourselves to s- and p-waves, we shall consider the polarization of the recoil deuteron which gives new information about the production amplitudes. In fact, together with the data already available, a knowledge of the deuteron polarization would determine these amplitudes completely. This is of particular interest as, near threshold, these amplitudes are simply related to certain proton-proton scattering phases. The information gained from the deuteron polarization assumes rather different forms according to whether one uses an unpolarized or a polarized incident proton beam. In the former case it concerns only p-wave production, in the latter both s- and p-waves. The possibilities of measuring the deuteron polarization, which would be very difficult, are discussed.

2. GENERAL THEORY

To calculate the cross section and the deuteron polarization we shall use the S-matrix formalism first developed in this connection by Dalitz[7] and by Wolfenstein and Ashkin.[8] In this formalism the differential cross section for the reaction

$$p+p\rightarrow\pi^++d \tag{1}$$

is given, in terms of its scattering matrix S, by

$$d\sigma/d\Omega=\mathrm{Tr}(SU_iS^\dagger), \tag{2}$$

where[9]

$$U_i=\tfrac{1}{4}(1+\mathbf{p}\cdot\boldsymbol{\sigma}^{(1)}). \tag{3}$$

U_i is the statistical operator describing the polarization of the incident proton beam, $\boldsymbol{\sigma}^{(1)}$ its Pauli spin operator and $\mathbf{p}$ its polarization vector, given by

$$\mathbf{p}=\langle\boldsymbol{\sigma}^{(1)}\rangle=\mathrm{Tr}(\boldsymbol{\sigma}^{(1)}U_i). \tag{4}$$

The polarization of the deuteron, having spin 1, is a much more complicated quantity than that of a proton. Its specification requires not only the expectation values of the deuteron spin $\mathbf{S}$ but also of the second rank tensor $T_2{}^m$ which can be formed from the spin vector. We define

$$T_1{}^{\pm1}=\mp\tfrac{1}{2}\sqrt{3}(S_x\pm iS_y),\quad T_1{}^0=\sqrt{\tfrac{3}{2}}S_z, \tag{5a}$$

and

$$\begin{aligned}T_2{}^{\pm2}&=\tfrac{1}{2}\sqrt{3}\{(S_x{}^2-S_y{}^2)\pm i(S_xS_y+S_yS_x)\},\\ T_2{}^{\pm1}&=\mp\tfrac{1}{2}\sqrt{3}\{(S_xS_z+S_zS_x)\pm i(S_zS_y+S_yS_z)\},\\ T_2{}^0&=(1/\sqrt{2})\{3S_z{}^2-2\}.\end{aligned} \tag{5b}$$

If U_f is the statistical operator of the final state, given by

$$U_f=SU_iS^\dagger, \tag{6}$$

then the polarization of the deuteron is specified by

$$P_j{}^m=\langle T_j{}^m\rangle=\mathrm{Tr}(U_fT_j{}^m)/\mathrm{Tr}U_f\quad(m\leqslant|j|,\ j=1,2), \tag{7}$$

where the denominator $\mathrm{Tr}U_f$ ensures correct normalization. It is simply the differential cross section for the scattering angles considered, as follows from (2) and (6).

The S-matrix can generally be written

$$S=\textstyle\sum_{if}\Psi_f\mu_{fi}\Psi_i{}^\dagger,$$

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[1] R. E. Marshak and A. M. L. Messiah, Nuovo cimento **11**, 337 (1954).
[2] A. H. Rosenfeld, Phys. Rev. **96**, 139 (1954).
[3] L. Wolfenstein, Phys. Rev. **98**, 766 (1955).
[4] M. Gell-Mann and K. M. Watson, Ann. Rev. Nuc. Sci. **4**, 219 (1954).
[5] F. S. Crawford and M. L. Stevenson, Phys. Rev. **95**, 112 (1954).
[6] Fields, Fox, Kane, Stallwood, and Sutton, Phys. Rev. **96**, 812 (1954).
[7] R. H. Dalitz, Proc. Phys. Soc. (London) **A65**, 175 (1952).
[8] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).
[9] One easily sees that this is still the correct definition of U_i for two identical fermions since only antisymmetrized states are allowed.

where Ψ_i and Ψ_f are the asymptotic forms of complete sets of wave functions describing the initial and final states of the system and the μ_{fi} are suitably normalized. We choose these states as eigenstates of the initial and final relative orbital angular momenta l and l', of the total angular momentum J and its z-component M. Since we are only interested in the angular dependence, we can write for the initial states

$$\Psi_i=C_{0M}{}^{lSJ}Y_l{}^0(0,0)\psi_M, \qquad (8)$$

where we have taken the z-axis, $\theta=0$, for the direction of the incident proton beam, (θ,ϕ) being the scattering angles of the meson in the c.m. system. The $C_{m_1m_2}{}^{l_1l_2l}$ are Clebsch-Gordan coefficients, and the ψ_m are the four spin states of the pp-system ($M=-1$, 0, $+1$ for the triplet states, and a different label, say $M=0'$, for the singlet state). Because of the choice of axes, no summation occurs in (8). For the final states we have similarly

$$\Psi_f=\sum_m C_{M-mm}{}^{l'1J}Y_{l'}{}^{M-m}(\theta,\phi)\phi_m, \qquad (9)$$

where ϕ_m ($m=-1$, 0, 1) represents the three deuteron spin states. The matrix element μ_{fi} corresponding to the transition from state (8) to (9) we shall label $\mu_{Jl'}{}^l$.

Angular momentum and parity selection rules allow these transitions:

$$pp\text{-singlet},\quad l=\text{even},\ J=l;\rightarrow l'=l\pm1,$$

$$pp\text{-triplet},\quad l=\text{odd}\begin{cases}J=l;\rightarrow l'=l\pm1,\\ J=l\pm1;\rightarrow l'=l\pm1.\end{cases}$$

Instead of the matrix elements $\mu_{Jl'}{}^l$, we found it convenient to use the production amplitudes

$$a_{Jl'}{}^l=\left(\frac{2l+1}{4\pi}\right)^{\frac{1}{2}}\mu_{Jl'}{}^l. \qquad (10)$$

Combining these results, the S-matrix becomes

$$S=\sum_{m,M}\phi_m\{\sum_l\sum_J\sum_{l'=l\pm1}a_{Jl'}{}^lC_{M-mm}{}^{l'1J}\times C_{0M}{}^{lSJ}Y_{l'}{}^{M-m}(\theta,\phi)\}\psi_M{}^\dagger, \qquad (11)$$

where $J=l$ if l is even, and $J=l-1$, l, $l+1$ if l is odd.

We construct S by a second method, particularly useful if only low angular momenta are involved.

Let χ_i, χ_f be unit vectors in the directions of the initial and final relative momenta, and let $\boldsymbol{\psi}$ and $\boldsymbol{\phi}$ be the vectors ψ_M, ϕ_m defined above. S must be bilinear in $\boldsymbol{\phi}$ and $\boldsymbol{\psi}^\dagger$ or $\psi_{0'}{}^\dagger$ (depending on whether the initial pp-state is triplet or singlet); it depends on χ_i and χ_f and must transform like a pseudoscalar. With the restriction to s- and p-wave pions, simple invariance considerations show that S must be of the form

$$S=\frac{1}{(4\pi)^{\frac{1}{2}}}\left\{a_0(\chi_f\cdot\boldsymbol{\phi})\psi_{0'}{}^\dagger+\frac{i}{\sqrt{2}}a_1\boldsymbol{\phi}\cdot[\chi_i\times\boldsymbol{\psi}^\dagger]-\frac{a_2}{\sqrt{2}}[3t(\chi_i\cdot\boldsymbol{\phi})-(\chi_f\cdot\boldsymbol{\phi})]\psi_{0'}{}^\dagger\right\}, \qquad (12)$$

where $t=\cos\theta=(\chi_i\cdot\chi_f)$. We shall explain the generalization of S for the case of higher angular momenta for singlet pp-states. The extension to triplet states will then be apparent. From (12), one sees that the singlet part S^{sing} has the form

$$S^{\text{sing}}=A(t)(\chi_i\cdot\boldsymbol{\phi})\psi_{0'}{}^\dagger+B(t)(\chi_f\cdot\boldsymbol{\phi})\psi_{0'}{}^\dagger. \qquad (13)$$

If $S_{l,l+1}{}^{\text{sing}}$ denotes that part of S^{sing} corresponding to a transition from orbital angular momentum l to $l+1$ ($l=$even, since we have a singlet state, and $J=l$), then $S_{l,l+1}{}^{\text{sing}}$ must be a spherical harmonic of order l in χ_i, of order $l+1$ in χ_f, irrespective of how the other variables are fixed. Hence if we put $\psi_{0'}{}^\dagger=1$, $\boldsymbol{\phi}=(0,0,1)$, and $\chi_i=(0,0,1)$ or $\chi_f=(0,0,1)$, the result will be a function of the z-component of χ_i or χ_f respectively, and, as explained, has to be a Legendre polynomial of order l or $l+1$ respectively, i.e.,

$$A(t)+tB(t)=\alpha P_{l+1}(t),\quad tA(t)+B(t)=\beta P_l(t), \qquad (14)$$

where α and β are constants. Putting $t=1$, we obtain $\alpha=\beta$, and we can write, without loss of generality, $\alpha=\beta=l+1$. Equations (14) have the solution

$$A(t)=-P_l{}'(t),\quad B(t)=P_{l+1}{}'(t), \qquad (15)$$

where the prime denotes differentiation with respect to t. It follows that, apart from a constant,

$$S_{l,l+1}{}^{\text{sing}}\propto P_{l+1}{}'(t)(\chi_f\cdot\boldsymbol{\phi})\psi_{0'}{}^\dagger-P_l{}'(t)(\chi_i\cdot\boldsymbol{\phi})\psi_{0'}{}^\dagger, \qquad (16a)$$

and similarly one obtains

$$S_{l+2,l+1}{}^{\text{sing}}\propto P_{l+1}{}'(t)(\chi_f\cdot\boldsymbol{\phi})\psi_{0'}{}^\dagger-P_{l+2}{}'(t)(\chi_i\cdot\boldsymbol{\phi})\psi_{0'}{}^\dagger. \qquad (16b)$$

For the triplet pp-states, the same procedure leads to the formula

$$\begin{aligned}S_{ll'}{}^{\text{trip}}=\lambda[&P_l{}''(t)\{\chi_i\cdot[\chi_f\times\boldsymbol{\phi}](\chi_i\cdot\boldsymbol{\psi}^\dagger)\\&+(\chi_i\cdot\boldsymbol{\phi})\chi_i\cdot[\chi_f\times\boldsymbol{\phi}^\dagger]\}\\&-P_{l'}{}''(t)\{\chi_i\cdot[\chi_f\times\boldsymbol{\phi}](\chi_f\cdot\boldsymbol{\psi}^\dagger)\\&+(\chi_f\cdot\boldsymbol{\phi})\chi_i\cdot[\chi_f\times\boldsymbol{\psi}^\dagger]\}]\\&+\mu[P_l{}'(t)\chi_i\cdot[\boldsymbol{\phi}\times\boldsymbol{\psi}^\dagger]\\&-P_{l'}{}'(t)\chi_f\cdot[\boldsymbol{\phi}\times\boldsymbol{\psi}^\dagger]],\end{aligned} \qquad (16c)$$

where l is now odd and $l'=l-1$, l or $l+1$. λ and μ are to be determined by fixing J. This can be done by observing that $S_{ll'}{}^{\text{trip}}$ arises by contraction of two irreducible tensors of rank J, constructed from $\chi_i\boldsymbol{\psi}^\dagger$ and $\chi_f\boldsymbol{\phi}$. Suppose we have $l'=l-1$, $J=l$. If we replace

TABLE I. Coefficient μ in Eq. (16c) for different transitions ($\lambda=1$). The initial orbital angular momentum is l (odd).

Final orbital angular momentum l'	Total angular momentum J	μ
$l-1$	$l-1$	$1-l$
$l-1$	l	$-l-1$
$l+1$	$l+1$	$l+2$
$l+1$	l	$+l$

ϕ by χ_f, the resulting tensor will be an odd function of degree l in χ_f; but we know it must be a tensor of rank $J=l$, hence it must vanish. This condition gives us a relation between λ and μ. The same method enables us to determine S in all other cases as well. The result is shown in Table I, where we have taken $\lambda=1$.

3. CROSS SECTION FOR *s*-, *p*-, AND *d*-WAVE PIONS

For final meson states with $l'\leq 2$, there are 7 possible transitions, shown in Table II. The first column specifies the pp-system; the second specifies the π^+d system, the affix 3 referring to the deuteron spin. We have simplified the general notation introduced in Sec. 2 and label the amplitudes (10), a_0, a_1, $\cdots a_6$, as shown in the last column of Table II. If we consider only s- and p-wave mesons, only the amplitudes a_0, a_1 and a_2 differ from zero. They were so ordered that in this case the suffix gives the J-value of the transition considered, i.e., we can write a_J, $J=0$, 1, 2. Of the 7 transitions, the first two arise from pp-singlet states, the remaining five from pp-triplet states.

Using Eqs. (11) or (16), and (2), we obtain for the differential cross section for reaction (1), allowing s-, p-, and d-wave mesons, and assuming the incident proton beam polarized in the y-direction [i.e., $\mathbf{p}=(0,p,0)$]:

$$\frac{d\sigma(\theta,\phi)}{d\Omega}=\frac{1}{32\pi}\{\gamma_0+\gamma_2\cos^2\theta+\gamma_4\cos^4\theta-p\sin\theta\cos\phi[(\lambda_0+\lambda_2\cos^2\theta)+(\lambda_1\cos\theta+\lambda_3\cos^3\theta)]\},\quad(17)$$

where the coefficients γ_0, $\cdots$, λ_0, $\cdots$ are defined in terms of the amplitudes a_m as follows. Let

$$a_m=r_me^{i\alpha_m},\quad(18)$$

$$\omega_{mn}=r_mr_n\cos(\alpha_m-\alpha_n),\quad \Omega_{mn}=r_mr_n\sin(\alpha_m-\alpha_n).\quad(19)$$

Then,

$$\gamma_0=(1/28)[56r_0^2+56r_1^2+28r_2^2+70r_3^2+70r_4^2+20r_5^2+35r_6^2+56\sqrt{2}\omega_{02}-28\sqrt{2}\omega_{13}-(28\sqrt{10})\omega_{14}-(8\sqrt{35})\omega_{15}-56\sqrt{2}\omega_{16}-(28\sqrt{5})\omega_{34}-(4\sqrt{70})\omega_{35}+70\omega_{36}+(20\sqrt{14})\omega_{45}+(14\sqrt{5})\omega_{46}+(2\sqrt{70})\omega_{56}],\quad(20a)$$

$$\gamma_2=(1/14)[42r_2^2-21r_3^2+35r_4^2+60r_5^2+21r_6^2-84\sqrt{2}\omega_{02}+42\sqrt{2}\omega_{13}+(42\sqrt{10})\omega_{14}+(12\sqrt{35})\omega_{15}+84\sqrt{2}\omega_{16}+(42\sqrt{5})\omega_{34}+(6\sqrt{70})\omega_{35}-294\omega_{36}-(90\sqrt{14})\omega_{45}-(126\sqrt{5})\omega_{46}-(18\sqrt{70})\omega_{56}],\quad(20b)$$

$$\gamma_4=(1/28)[-100r_5^2+35r_6^2+630\omega_{36}+(200\sqrt{14})\omega_{45}+(350\sqrt{5})\omega_{46}+(50\sqrt{70})\omega_{56}],\quad(20c)$$

$$\lambda_0=\tfrac{1}{2}[4\sqrt{2}\Omega_{01}-4\Omega_{12}-8\Omega_{03}-6\Omega_{06}-4\sqrt{2}\Omega_{23}-3\sqrt{2}\Omega_{26}],\quad(21a)$$

$$\lambda_2=(1/14)^{\frac{1}{2}}[(15\sqrt{14})\Omega_{06}+(18\sqrt{7})\Omega_{23}+(6\sqrt{35})\Omega_{24}+(6\sqrt{10})\Omega_{25}-(9\sqrt{7})\Omega_{26}],\quad(21b)$$

$$\lambda_1=(5/14)^{\frac{1}{2}}[(4\sqrt{7})\Omega_{14}-6\sqrt{2}\Omega_{15}+(2\sqrt{14})\Omega_{34}-6\Omega_{35}+(10\sqrt{5})\Omega_{45}+(\sqrt{14})\Omega_{46}-3\Omega_{56}],\quad(22a)$$

$$\lambda_3=(5/14)^{\frac{1}{2}}[-(20\sqrt{5})\Omega_{45}-(5\sqrt{14})\Omega_{46}+15\Omega_{56}].\quad(22b)$$

The angular distribution (17) is of the form to be expected from general considerations and one can easily extend it to include higher partial waves. The asymmetry due to the polarization consists of two parts. The first is an interference effect between s- and p- or between d- and p-waves; it arises from interference of a singlet and a triplet pp-state. The second part results from interference of s- and d-wave mesons; it arises from two triplet states of the pp-system. This second part is asymmetric about $\theta=90°$, so d-waves should show up as such an asymmetry. Although the s-wave amplitude itself is small, this asymmetry might be a more sensitive test of the presence of d-waves than deviations from a $(\gamma_0+\gamma_2\cos^2\theta)$-dependence of the unpolarized cross section. With the restriction to s- and p-waves, our result agrees with that of Marshak and Messiah,[1] and our parameters are easily related to those used by Rosenfeld[2] and by Gell-Mann and Watson.[4] In particular,

$$A=\gamma_0/\gamma_2,\quad Q=\lambda_0/\gamma_0,\quad \alpha\eta=\tfrac{1}{4}r_1^2,\quad \beta\eta^3=\tfrac{1}{4}(r_0^2+r_2^2),$$
$$X=\frac{2r_0^2+r_2^2+2\sqrt{2}\omega_{02}}{3(r_2^2-2\sqrt{2}\omega_{02})}.\quad(23)$$

4. DEUTERON POLARIZATION

We now restrict ourselves to s- and p-wave mesons. The reaction is then completely specified in terms of

TABLE II. Transitions involved for s-, p-, and d-wave pions.

${}^1S_0\rightarrow{}^3P_0$	a_0
${}^1D_2\rightarrow{}^3P_2$	a_2
${}^3P_1\rightarrow{}^3S_1$	a_1
${}^3P_1\rightarrow{}^3D_1$	a_3
${}^3P_2\rightarrow{}^3D_2$	a_4
${}^3F_2\rightarrow{}^3D_2$	a_5
${}^3F_3\rightarrow{}^3D_3$	a_6

the 5 parameters r_0, r_1, r_2 and two of the phases α_0, α_1 and α_2. At present, experiments only furnish us with four data at any given energy: A, Q, α, and β, Eqs. (23). A and Q are obtained at any energy from the angular distribution of the unpolarized cross section and the asymmetry when using polarized protons. α and β are obtained by assuming the energy-dependence of the s- and p-wave contributions to the total cross section, as given in (23), η being the pion momentum in the center-of-mass system. As Rosenfeld[2] shows, this energy dependence gives good agreement with experiment, but, except near threshold, there is no reason why it should be generally valid.

Following Rosenfeld[2] and Gell-Mann and Watson,[4] we introduce the following ratios of production amplitudes

$$\delta_0=|\delta_0|e^{i\gamma_0}=a_0/a_2,\quad \delta_1=|\delta_1|e^{i\gamma_1}=a_1/a_2. \tag{24}$$

As fifth parameter to specify the reaction completely, we can then introduce the phase angle ω_0 given by

$$\delta_0=-(1/\sqrt{2})(1+3X)+(3/\sqrt{2})(X^2+X)^{\frac{1}{2}}e^{i\omega_0}. \tag{25}$$

The determination of the parameter ω_0 is of considerable interest, as this would completely specify the parameters entering the reaction. Moreover, near threshold these parameters are simply related to the scattering phases of the pp-states involved (see Sec. 5 below). There are some theoretical arguments[10] to suggest that $|\delta_0|$ is small which would imply that ω_0 is small.[11]

ω_0 must be determined from a measurement of the deuteron polarization. From Eqs. (5) to (7) one obtains the polarization of a deuteron emitted in the direction θ, ϕ (i.e., the meson is now emitted in the direction $\pi-\theta$, $\phi+\pi$),

$$P_j^m(\theta,\phi)=\{V_j^m(\theta,\phi)+pW_j^m(\theta,\phi)\}/\{d\sigma(\pi-\theta,\phi+\pi)/d\Omega\}, \tag{26}$$

where the incident proton beam again has polarization $\mathbf{p}=(0,p,0)$ and

$$V_1^{\pm1}=-i\frac{\sqrt{3}}{2}\left[\frac{-3\sqrt{2}}{16\pi}\Omega_{02}\right]\cos\theta\,\sin\theta e^{\pm i\phi},$$
$$V_1^0=0, \tag{27a}$$

$$W_1^{\pm1}=i\frac{\sqrt{3}}{2}\left[\frac{1}{16\pi}(-\sqrt{2}\omega_{01}+2\omega_{12})\right]\cos\theta,$$
$$W_1^0=-\sqrt{\tfrac{3}{2}}\left[\frac{1}{16\pi}(\sqrt{2}\omega_{01}+\omega_{12})\right]\sin\theta\,\sin\phi, \tag{28a}$$

$$V_2^{\pm2}=\frac{\sqrt{3}}{2}\left[-\frac{1}{32\pi}(2r_0^2+r_2^2+2\sqrt{2}\omega_{02})\right]\sin^2\theta e^{\pm2i\phi},$$
$$V_2^{\pm1}=\mp\frac{\sqrt{3}}{2}\left[\frac{1}{16\pi}(-2r_0^2+2r_2^2+\sqrt{2}\omega_{02})\right]\cos\theta\,\sin\theta e^{\pm i\phi},$$
$$V_2^0=\frac{1}{\sqrt{2}}\left[-2\frac{d\sigma(\pi-\theta,\phi+\pi)}{d\Omega}+\frac{3}{16\pi}\{r_1^2+\tfrac{1}{2}(2r_0^2+r_2^2+2\sqrt{2}\omega_{02})\}\sin^2\theta\right], \tag{27b}$$

$$W_2^{\pm2}=-\frac{\sqrt{3}}{2}\left[\frac{1}{16\pi}(\sqrt{2}\Omega_{01}-\Omega_{12})\right]\sin\theta e^{\pm i\phi},$$
$$W_2^{\pm1}=\pm\frac{\sqrt{3}}{2}\left[\frac{1}{16\pi}(\sqrt{2}\Omega_{01}+2\Omega_{12})\right]\cos\theta, \tag{28b}$$
$$W_2^0=\frac{1}{\sqrt{2}}\left[\frac{3}{16\pi}(\sqrt{2}\Omega_{01}-\Omega_{12})\right]\sin\theta\,\cos\phi.$$

The ω_{mn} and Ω_{mn} are defined in Eq. (19).

To consider the vector part of the polarization, we introduce the expectation value of the spin

$$\langle\mathbf{S}\rangle=\{\mathbf{v}+p\mathbf{w}\}/\{d\sigma/d\Omega\}, \tag{29}$$

where $\mathbf{v}$ and $\mathbf{w}$ are simply related to V_1^m and W_1^m, as follows from Eqs. (5a) and (7).

The vector $\mathbf{v}$, due to unpolarized protons, is perpendicular to the plane of scattering. Its angular dependence is given by $\sin2\theta/(A+\cos^2\theta)$. It vanishes for $\theta=0$; for $A\sim0.29$, it attains its maximum value for $\theta=65°$. From (25) and (27a) we see that $\mathbf{v}$ is proportional to $\sin\omega_0$. The polarization $\mathbf{v}$ agrees with the result of Watson and Richman[12] if one allows for the fact that they only considered p-wave mesons. It should be noted that this contribution to the polarization depends only on p-wave mesons, and so only involves $(\alpha_0-\alpha_2)$.

With polarized incident protons, there is a second contribution $\mathbf{w}$ to the vector polarization. In the forward direction, $\theta=0$, only the y-component of $\mathbf{w}$ differs from zero and moreover has a maximum. For other angles, $\mathbf{w}$ has a more complicated behavior; in particular, it also has a nonzero z-component. $\mathbf{w}$ involves both $(\alpha_0-\alpha_2)$ and $(\alpha_1-\alpha_2)$, i.e., it also yields information about the s-wave pions.

In Fig. 1, we show the polarization $\langle\mathbf{S}\rangle$ as a function of ω_0. Curve (A) gives the polarization from an unpolarized proton beam [i.e., $v_y/(d\sigma/d\Omega)$] in the direction where this polarization is a maximum. Curves (B) and (C) show the contribution to the polarization due to the polarized part of the incident beam [i.e., $w_y/(d\sigma/d\Omega)$] in the forward direction $\theta=0$. The proton energy is 315 Mev. For the parameters (23) we have taken (see

[10] K. A. Brueckner and K. M. Watson, Phys. Rev. **86**, 923 (1952); and Aitken, Mahmoud, Henley, Ruderman, and Watson, Phys. Rev. **93**, 1349 (1954).

[11] For the value of X deduced from experiment, $(1/\sqrt{2})(1+3X)>(3/\sqrt{2})(X^2+X)^{\frac{1}{2}}$.

[12] K. M. Watson and C. Richman, Phys. Rev. **83**, 1256 (1951).

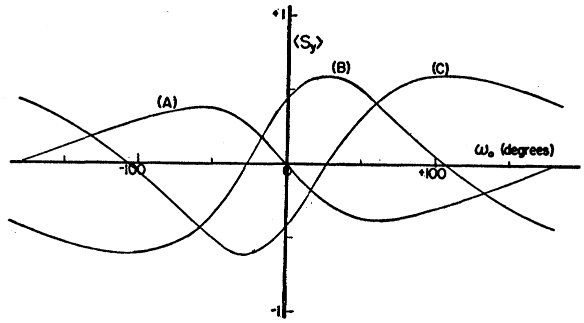

FIG. 1. y-component of the deuteron polarization $\langle \mathbf{S}\rangle$ *versus* ω_0. Curve (A): polarization produced by an unpolarized proton beam emitted in the direction of maximum polarization. Curves (B) and (C): polarization produced by the polarized part of a completely polarized beam $[\mathbf{p}=(0,1,0)]$, the deuteron being emitted in the forward direction. The two curves (B) and (C) correspond to the two possible values of τ_1. The other components of $\langle \mathbf{S}\rangle$ vanish for the cases considered.

reference 2) $\alpha=0.14$ mb, $\beta=1.0$ mb. For X we took the more recent value[5] $X=0.082$, instead of $X=0.1$ suggested by Rosenfeld,[2] and for Q we took the value 0.39 of Crawford *et al.*[5] The two curves (B) and (C) are the result of the fact that the polarization Q does not determine $\tau_1=(\alpha_1-\alpha_2)$, Eq. (24), unambiguously but only through $\sin[\arg(\delta_0+\sqrt{\tfrac{1}{2}})-\tau_1]$.

Considering the tensor polarization next, one finds that only the components $V_2^{\pm1}$ and $W_2^{\pm1}$ lead to new information; i.e., the particular combinations of the reaction amplitudes which occur in the other components can be expressed in terms of the parameters X, γ_2, and λ_0, defined above.

To measure the deuteron polarization is clearly difficult. The deuteron has a threshold energy of 141 Mev and hence is emitted nearly in the forward direction, so that in any experiment one necessarily integrates over a wide cone in the c.m. system. The cross section for the reaction is small, particularly at low energies, and so the intensities may not be adequate for the additional scattering experiment required to analyze the deuteron polarization.

To analyze this deuteron scattering, we introduce a new coordinate system with the direction of the initial momentum of the deuteron in the laboratory system as polar axis. (Quantities referred to this coordinate system will be distinguished by primes.) From general invariance considerations it can be shown[13] that the azimuthal asymmetry of the scattered deuterons is given by

$$\begin{aligned}I=I_0&+K_0L_2^{0\prime}\\&+K_1\{\cos\beta'(L_2^{1\prime}+\rho M_1^{1\prime})+\sin\beta'(-\rho L_1^{1\prime}+M_2^{1\prime})\}\\&+K_2\{\cos2\beta'(L_2^{2\prime})+\sin2\beta'(M_2^{2\prime})\},\end{aligned}\tag{30}$$

where

$$\langle T_j^m\rangle'=L_j^{m\prime}+iM_j^{m\prime},\tag{31}$$

and β' is the azimuthal angle of the scattered deuteron in the primed coordinate system. I_0 is the intensity for an unpolarized deuteron beam. The K's and ρ are unknown functions of the scattering angle α' of the deuteron and of the magnitudes of its initial and final momenta. A calibration, by means of a double scattering experiment, of the target used to analyze the deuteron beam will give the K's. To obtain ρ, the sensitivity of the deuteron analyzer to the two types of polarization must be known. From (30) it follows that we cannot separate $\langle T_2^1\rangle'$ and $\langle T_1^1\rangle'$ by a double scattering experiment. The only unambiguous azimuthal asymmetry results from the terms in $\cos2\beta'$ and $\sin2\beta'$.

One easily obtains the $\langle T_j^m\rangle'$ from the $\langle T_j^m\rangle$ for a general rotation of axes. If ξ is the ratio of the deuteron velocity in the proton-proton c.m. system to the velocity of the center of mass, and Θ, Φ are the angles of emission of the deuteron in the laboratory system of the pp collision, then $\Phi=\phi$ and

$$\cos\Theta=1+O(\xi^2),\quad \sin\Theta=\xi\sin\theta+O(\xi^2),\tag{32}$$

where we used the fact that ξ is small even for quite high proton energies ($\xi\sim0.04$ for 315-Mev protons and $\xi\sim0.15$ for 800-Mev protons). For the component of interest, $\langle T_2^2\rangle$, one then finds

$$\begin{aligned}L_2^{2\prime}&=\{L_2^2\cos2\phi+M_2^2\sin2\phi\}\\&\quad+\xi\sin\theta\{L_2^1\cos\phi+M_2^1\sin\phi\}+O(\xi^2),\\M_2^{2\prime}&=\{-L_2^2\sin2\phi+M_2^2\cos2\phi\}\\&\quad+\xi\sin\theta\{-L_2^1\sin\phi+M_2^1\cos\phi\}+O(\xi^2).\end{aligned}\tag{33}$$

Hence the determination of ω_0 requires experiments sufficiently accurate to measure the terms proportional to $\xi\sin\theta$ in (33) (since $V_2^{\pm2}$ and $W_2^{\pm2}$ depend only on $X\gamma_2$ and λ_0).

We briefly consider the experimental possibilities of measuring ω_0 on the assumption that we have an analyzer for the deuteron scattering whose sensitivity to the vector and tensor types of polarization we know, i.e., ρ is known, so that we can deduce $\langle \mathbf{S}\rangle$ from a double scattering experiment.

Using unpolarized protons requires a double scattering experiment. This would determine ω_0 and the remaining parameters (apart from the ambiguity mentioned above) if we assume the energy-dependence of the reaction amplitudes, Eq. (23). The simple angular distribution of the polarization and the fact that it is proportional to $\sin\omega_0$ may allow a comparatively simple experiment and analysis. For example, an experiment might be possible where "no azimuthal asymmetry in the second scattering" could be interpreted as $\omega_0=0$ within the experimental error.

Using polarized protons, one requires a triple scattering experiment. The resultant loss of intensity makes this a much more difficult experiment and rough estimates suggest that one would at any rate have to go to higher energies, say 600 Mev, to obtain adequate intensities. This experiment has the advantage that a

[13] W. Lakin, Phys. Rev. 98, 139 (1955).

sufficiently detailed analysis of the deuteron beam would give information about both $\alpha_0-\alpha_2$ and $\alpha_1-\alpha_2$ and consequently would allow one (possibly in conjunction with the first type of experiment) to determine the reaction amplitudes completely without assuming their energy dependence. However, the more complicated angular dependence might make it difficult to obtain sufficiently detailed information.

One may question whether an analysis in terms of s- and p-waves is still adequate at the high energies considered above, particularly since interference effects, such as we are discussing, are very sensitive to small admixtures of states. But the fact that d-waves do not seem to play an important role in pion-nucleon scattering even at quite high energies, together with the explanation, in terms of the $p_{\frac{3}{2}}$ resonant state, of recent experiments[14] on pion production in nucleon-nucleon collisions up to energies of 1 Bev, suggest that such an analysis may be adequate. On the other hand, the determination of the reaction amplitudes would be of particular interest near threshold because of their relation to the pp-scattering phases.

5. RELATION TO THE pp-SCATTERING PHASES

Watson[15] has related the phases of the amplitudes for photomeson production to the pion-nucleon scattering phases. Using the same method, we have related the meson production amplitudes in reaction (1) to the pp-scattering phase shifts. One obtains

$$\begin{aligned}\tau_0&=\alpha({}^1S_0)-\alpha({}^1D_2)+n_1\pi,\\(\tau_1-\pi/2)&=\alpha({}^3P_1)-\alpha({}^1D_2)+n_2\pi,\end{aligned}\tag{34}$$

where n_1 and n_2 are integers and $\alpha({}^{2S+1}L_J)$ is the pp-scattering phase in the state ${}^{2S+1}L_J$ at the energy of the pp-system considered. These relations are also stated by Gell-Mann and Watson.[4] In deriving them, the meson-nucleon interaction is taken as sufficiently weak to be treated only in first-order perturbation theory. This is justifiable for low meson energies for which all the meson-nucleon scattering phases, including α_{33}, are small. In spite of this uncertainty as to their range of validity, relations (34) should be very useful in determining the pp scattering phases in the 300–400-Mev region, once the meson production parameters τ_0 and τ_1 are known. We illustrate this in Fig. 2, where we have plotted τ_0 and $\tau_1-\pi/2$ *versus* ω_0 for a proton energy of 315 Mev, using the same data as were used for Fig. 1. One sees that a knowledge of ω_0 would quite strongly restrict the values of the three pp-phases concerned, particularly if ω_0 really turns out to be small. The pp-phases are not sufficiently well known for the procedure to be reversed. Klein[16] has analyzed the pp-data in the energy range 170 Mev–330 Mev in terms of s- and p-waves, using the differential cross section, including small angle scattering, and the polarization. For positive polarization, as determined by Marshall and Marshall[17] at low energies, Klein obtains several sets of phase-shifts but none of these agree particularly well with Fig. 2. This is perhaps not surprising, since Klein cannot fit the 330-Mev polarization data in terms of s- and p-waves, as has already been pointed out by Fried.[18]

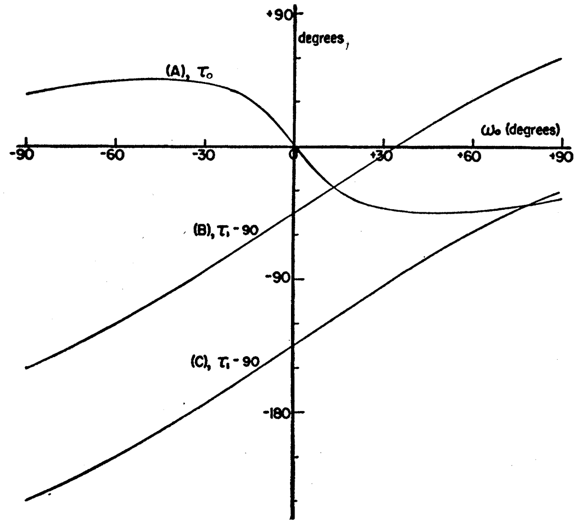

FIG. 2. τ_0 and $\tau_1-90°$ (which are related to the pp scattering phases by Eq. (34)) as a function of ω_0. Curve (A) gives τ_0; curves (B) and (C) give the two possible values of τ_1 which were used in the corresponding curves (B) and (C) of Fig. 1.

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[14] S. J. Lindenbaum and L. C. L. Yuan, 1955 Rochester Conference on High Energy Nuclear Physics (to be published).
[15] K. M. Watson, Phys. Rev. **95**, 228 (1954).
[16] C. A. Klein (to be published).
[17] L. Marshall and J. Marshall, Nature (London) **174**, 1184 (1954).
[18] B. D. Fried, Phys. Rev. **95**, 851 (1954).