Using (B5), (B7), and (B8) we may then obtain α, a/b, and k explicitly in terms of $E_{\max}-E_R$ and $E_{\min}-E_R$. The use of this method requires, however, that the resonance energy (on the energy scale of the experiment) be known with an uncertainty which is small compared to the resonance-width, Γ. In the present experiment, however, the uncertainty in the energy scale is about 3 kev, so that this condition is not well satisfied.

In cases where $E_{\max}$ or $E_{\min}$ is not clearly defined, it is possible to use A and the width at half maximum for determining k. This width is more difficult to determine than $E_{\max}-E_{\min}$ (both experimentally and analytically) and, except for nearly symmetric resonances, this procedure is less satisfactory.

In Table I of the text, we have given corrected values of the cross sections for comparison with the observed and theoretical values for several of the $F^{19}(p,p)$ resonances. Values for the window width, α, have been determined in many of these cases and it is found that these values are always between the values calculated using Cohen's formulation[21] and those estimated as an upper limit for target surface irregularities. The values are close to the lower limit at the higher energies (1400 kev) and large scattering angles and become larger relative to the lower limit as the energy and scattering angle are decreased. Several of these values are given in Table I.

The application of these expressions to obtain relations between experimentally determined quantities may, in part, eliminate the dependence of the results on the particular form we have taken for the resolution function. In any event, the treatment gives a useful description of the qualitative features and at least a first approximation to the quantitative aspects. In addition to the application of these results to the correction of cross sections, they should be of some value in determining resonance energies and widths from elastic scattering measurements.

PHYSICAL REVIEW VOLUME 99, NUMBER 1 JULY 1, 1955

Analysis of the Elastic Scattering of Protons from F^{19}†

ELIZABETH UREY BARANGER
Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California
(Received March 21, 1955)

An analysis of the anomalies in the elastic scattering cross section of protons on F^{19} has been carried out. The assignments 1^+ for the resonances at 669, 935, and 1422 kev, and 0^+ for the resonance at 843 kev, are required by the results of the experiment. The assignment of the resonances at 873, 1348, and 1374 kev is 2^-, with 1^- not excluded by these data alone. Unique values of the partial widths are determined for these resonances and several others in this energy range. Reduced widths are given for the various particle reactions which are observed.

IN the preceding paper, Webb, Hagedorn, Fowler, and Lauritsen[1] have discussed an experiment measuring the elastic scattering of protons by F^{19}. The present paper deals with an analysis of their experimental results. The purpose of the analysis is to determine the spins, parities, and partial widths of as many as possible of the excited states of the compound nucleus Ne^{20}. Assignments of the levels examined here, except the resonances at 843 and 1422 kev, have been determined previously by several workers using methods other than elastic scattering. They were, however, unable to determine the partial widths uniquely. Their results are summarized in a review article by Ajzenberg and Lauritsen.[2] Some preliminary work on elastic scattering was done at this laboratory by Peterson *et al.*,[3] which resulted in the assignment of the 1422-kev resonance. Recently, Dearnaley[4] has examined the elastic scattering and his results as to assignment and choice of partial widths are in agreement with ours.

In analyzing proton scattering from a nucleus such as F^{19} where reactions are also possible, certain complications arise in the formulas which have led us to use simplified and not entirely accurate forms for the scattering cross section. Thus, neglecting Coulomb effects, an arbitrary scattering amplitude associated with a given J and parity could be written as $f(e^{2i\delta}-1)$, where $f\leq 1$ and δ is arbitrary. Now in the case of a single resonance we find that $f=\Gamma_p/\Gamma$, the ratio of the elastic proton width to the total width, is independent of energy and $\cot\delta$ is linear in the energy. In the case of two overlapping resonances, however, no simple energy dependence of f and δ which is consistent with resonance theory has been found. What is needed is

† Assisted by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.
[1] Webb, Hagedorn, Fowler, and Lauritsen, preceding paper [Phys. Rev. 99, 138 (1955)]. This paper is referred to as Paper A.
[2] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. 27, 77 (1955).
[3] Peterson, Barnes, Fowler, and Lauritsen, Phys. Rev. 94, 1075 (1954).
[4] G. Dearnaley, Phil. Mag. 45, 1213 (1954).

some expression which shows the influence of a broad resonance, where the parameters are almost energy independent on a superimposed narrow resonance. Lacking a better expression, we have used the form discussed by Blatt and Biedenharn,[5] i.e., $a_p(e^{2i(\delta_r+\varphi)}-1)+(1-a_p)(e^{2i\varphi}-1)$, where a_p is the ratio of the elastic proton width to the total width, δ_r is the resonant phase shift such that $\delta_r=-\cot^{-1}[(E-E_0)/\frac{1}{2}\Gamma]$, where E_0 is the resonance energy, and φ is a slowly varying phase shift chosen to describe the scattering amplitude off-resonance. This form is in error in that it assumes the reaction cross section to be zero off-resonance. Since the off-resonant cross section is not large, we feel that the use of the simplified form has introduced no error in interpretation.

The differential cross section for elastic scattering of s-wave protons and F^{19} (spin $\frac{1}{2}$) which form a compound state of a given J, which can be 0 or 1, can then be shown to be

$$\bar{\sigma}(\theta)=\sigma(\theta)/\sigma_R(\theta)=1-\frac{(2J+1)}{2k\sigma_R^{\frac{1}{2}}}\{a_p\sin(\delta_{rJ}+\varphi_J)\times\cos(\delta_{rJ}+\varphi_J+\theta_0)+(1-a_p)\sin\varphi_J\cos(\varphi_J+\theta_0)\}+\frac{(2J+1)}{4k^2\sigma_R}\{a_p^2\sin^2(\delta_{rJ}+\varphi_J)+(1-a_p)^2\sin^2\varphi_J+2a_p(1-a_p)\sin(\delta_{rJ}+\varphi_J)\sin\varphi_J\cos\delta_{rJ}\},\quad(1)$$

where k is the wave number, σ_R the Rutherford cross section, and $\theta_0=(Ze^2/\hbar v)\ln\sin^2(\theta/2)$, v being the relative velocity. This expression differs from the final expression given in Blatt and Biedenharn in that we allow the phase shifts φ_J to be arbitrary parameters depending on J as well as l, the orbital angular momentum, and have not performed a sum over l.

Far from a resonance, when $\delta_r=n\pi$, this expression simplifies considerably and was used to analyze the off-resonant cross section. Since the angular dependence of the off-resonant cross section is consistent with assuming only s-wave phase shifts, we assume the higher l-wave phase shifts to be zero. The s-wave phase shifts vary over the energy region of the experiment, since the off-resonant cross section varies. It is impossible to determine from the data unique values of the individual phase shifts $\varphi_{J=0}$ and $\varphi_{J=1}$. One can only determine the expression $\varphi_{J=0}+3\varphi_{J=1}$. This is positive at low energies and negative at the higher energies. The agreement with experiment as a function of angle can be seen from Figs. 1, 2, and 3 at 570, 770, 1320, and 1520 kev. It was also checked at 1090 kev.

Since the off-resonant phase shifts can be assumed to be zero for the higher orbital angular momentum, the differential cross section for a particular J with $l>0$ is given by a Breit-Wigner formula:

$$\bar{\sigma}(\theta)=\sigma(\theta)/\sigma_R(\theta)=1-\frac{2J+1}{k\sigma_R^{\frac{1}{2}}}a_p\sin\delta_{rJ}\cos(\theta_l+\delta_{rJ})\times P_l(\cos\theta)+\pi\frac{(2l+1)}{k^2\sigma_R}a_p^2\sin^2\delta_{rJ}\times\sum_{m,M,T,R}\alpha_R^2\alpha_T^2|Y_l^m(\theta,\varphi)|^2\times|C_{Rl}(JM;MO)C_{Tl}(JM;M-mm)|^2,\quad(2)$$

where $Y_l^m(\theta,\varphi)$ is a spherical harmonic; $P_l(\cos\theta)$ is a Legendre polynomial; $C_{jj'}(JM;mm')$ are Clebsch-Gordan coefficients; R and T are channel spin quantum numbers; α_R is the channel spin ratio such that $\sum_R\alpha_R^2=1$; and

$$\theta_l=\theta_0+2\sum_{n=1}^{l}\tan^{-1}(Ze^2/\hbar vn).$$

We assume that only the lowest value of l for a given J need be included because of the rapid increase in the barrier penetration factor with l. We must add to (2) the two off-resonant s-wave contributions discussed above and also an interference term. The contribution of this interference term to $\bar{\sigma}$, which is valid either in

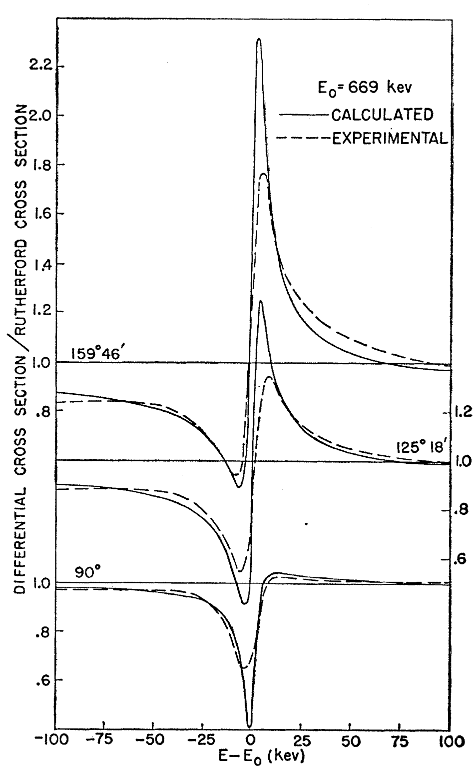

FIG. 1. The ratio of the differential cross section for the elastic scattering of protons by F^{19} to the Rutherford cross section for several center-of-mass angles as a function of the bombarding proton energy minus the resonance energy of 669 kev.

[5] J. M. Blatt and L. C. Biedenharn, Revs. Modern Phys. 24, 258 (1952).

the vicinity of an s-resonance or when no s-resonance is near, is

$$\frac{(2J+1)}{2k^2\sigma_R}P_l(\cos\theta)\alpha_{J'}{}^2 a_{pJ}\{a_{pJ'}\sin(\delta_{rJ'}+\varphi_{J'})\times\sin\delta_{rJ}\cos(\theta_l-\theta_0+\delta_{rJ}-\delta_{rJ'}-\varphi_{J'})+(1-a_{pJ'})\sin\varphi_{J'}\sin\delta_{rJ}\cos(\theta_l-\theta_0+\delta_{rJ}-\varphi_{J'})\},\quad(3)$$

where J' is the angular momentum of the s-wave resonance and J that of the other resonance. This interference term is proportional to $P_l(\cos\theta)$. There is obviously no interference between two s-waves.

We can determine the orbital angular momentum, l, by observing $\bar{\sigma}-1$ at angles θ for which $P_l(\cos\theta)=0$. When the orbital angular momentum is not zero, we have assumed φ_J to be zero, and therefore it is clear from Eq. (2) that $\bar{\sigma}-1$ should be symmetric around E_0 and positive. At this angle, the effect of the nonresonant s-wave phase shifts given in Eqs. (1) and (3) is merely to produce a constant background. Thus if $\bar{\sigma}-1$ is not symmetric and positive for any θ such that $P_l(\cos\theta)=0$, we assume that $l=0$. We thus assign $l=0$ to the resonances at 669, 843, 935, and 1422 kev. If $\bar{\sigma}-1$ is symmetric and positive at the angles where $P_l(\cos\theta)=0$ and not at other angles, we shall assume that l is given by this value. It might happen that $\bar{\sigma}-1$ could be accidentally symmetric and positive for certain values of θ_l, so we must be sure that $l\neq0$ for such a resonance. If the interference term in Eq. (2) is usually much larger than the purely resonant term, $\bar{\sigma}-1$ will essentially vanish at this angle. The resonance anomalies at 873, 1346, and 1372 kev vanish at 90°, and are not symmetric and positive at 125°, The anomalies at 1346 and 1372 kev are not symmetric and positive at 136° which is close to the angle where $P_3(\cos\theta)$ vanishes. (873 kev was not measured at this angle.) Thus at 1374 and 1348 kev, the data rules out $l=2$ and 3 and indicates $l=1$. At 873 kev, the data rule out $l=2$ and indicate $l=1$. Dearnaley[4] measured this energy region at 140.8° and his data rule out $l=3$. We examined $l=0$ for these three resonances and can also eliminate this possibility.

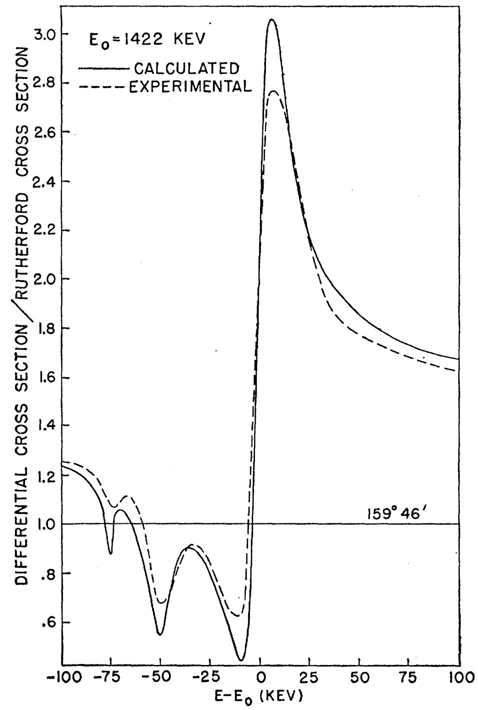

FIG. 3. The ratio of the differential cross section for the elastic scattering of protons by F^{19} to the Rutherford cross section for center-of-mass angle 159°46′ as a function of the bombarding proton energy minus 1422 kev.

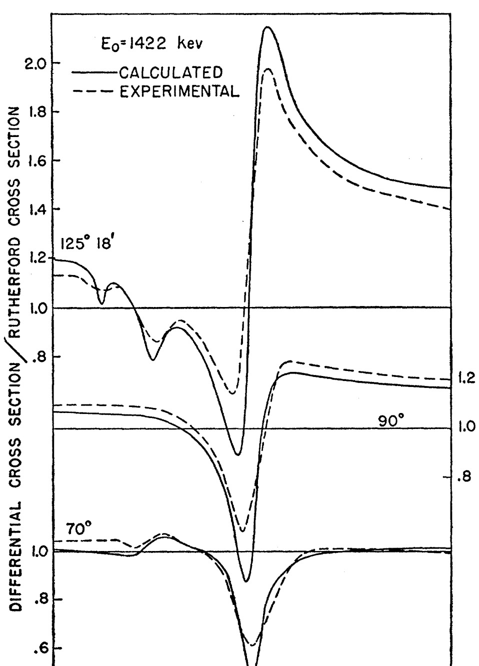

FIG. 2. The ratio of the differential cross section for the elastic scattering of protons by F^{19} to the Rutherford cross section for several center-of-mass angles as a function of the bombarding proton energy minus the resonance energy of 1422 kev.

Next, a_p can be determined in terms of J from the reaction cross sections since

$$\sigma_r=(2J+1)\frac{\pi}{k^2}\frac{\Gamma_p\Gamma_r}{\Gamma^2}=(2J+1)\frac{\pi}{k^2}(1-a_p)a_p,\quad(4)$$

where σ_r is the sum of all reaction cross sections for protons on F^{19} and Γ_r the corresponding width. Relation (4) yields two possible values of a_p, whose sum is one. The particle reactions which occur are α-particle emission leading to states of O^{16} and inelastic proton emission leading to low states in F^{19}. We use the notation $\alpha_{0,\pi,1,2,3}$ and $p_{1,2}$ of Barnes.[6] Values of σ_r were obtained from the data of Barnes[6] for inelastic scattering and from Chao *et al.*[7] and Freeman[8] for the production of α particles. The ratio of the different reaction cross sections is given by the ratio of the widths in Table I. These widths were taken from the paper by Barnes.

It is usually possible to determine J and a unique value of a_p from a comparison with the experiment of $\bar{\sigma}_{\max}-\bar{\sigma}_{\min}$, the maximum value of $\bar{\sigma}$ minus the minimum value. This function equals $[A^2(\theta)+B^2(\theta)]^{\frac{1}{2}}$, where $A(\theta)$ is the coefficient of $\sin^2(\delta_{rJ}+\varphi_J)$ and $B(\theta)$ the coefficient of $\sin(\delta_{rJ}+\varphi_J)\cos(\delta_{rJ}+\varphi_J)$, if one assumes that the resonance is narrow enough so that $A(\theta)$ and $B(\theta)$ are constant over the resonance. It depends mainly on l, J, and a_p, since the contribution of the fairly small nonresonant phase shifts is a small correction.

Webb *et al.*, observed resonance at incident proton energies of 669, 843, 873, 935, 1346, 1372, and 1422 kev. The analysis of each is discussed below.

669 kev.—Values of $\bar{\sigma}_{\max}-\bar{\sigma}_{\min}$ are given in Table I, Paper A, for a 1^+ assignment with $a_p=0.976$. They are independent of the nonresonant phase shifts. The smaller value of a_p would be obviously impossible. $J=0$ can be eliminated since it gives values $\frac{1}{3}$ those for $J=1$. Plots of the complete resonance curves are shown in Fig. 1, where $\varphi_{J=0}$ was taken to be zero, and $\varphi_{J=1}$ to be 0.05 radian.

843 kev.—Table I of Paper A gives $\bar{\sigma}_{\max}-\bar{\sigma}_{\min}$ for a 0^+ assignment with $a_p=0.996$, where this is again independent of the nonresonant phase shifts. The smaller value of a_p and also an assignment 1^+ are impossible. The values $\bar{\sigma}_{\max}$ and $\bar{\sigma}_{\min}$ were calculated by assuming $\varphi_{J=1}=0$ and are independent of $\varphi_{J=0}$. This assignment of 0^+ agrees with the fact that α-particles leaving O^{16} in its ground state and first excited state (0^+) have been observed.

935 kev.—σ_r is too large for Eq. (4) to be satisfied with $J=0$. $\bar{\sigma}_{\max}-\bar{\sigma}_{\min}$ is given in Table I, Paper A, with $J=1$, $a_p=0.17$ and with a nonresonant phase shift of $\varphi_{J=1}=0$ radian. If $\varphi_{J=1}=-0.19$, which is the same as the value used at 1422 kev, $\bar{\sigma}_{\max}-\bar{\sigma}_{\min}$ becomes 0.18, 0.31 and 0.41 at 90°, 125°, 160° respectively. If one subtracts from the experimental values of $\bar{\sigma}_{\max}$ and $\bar{\sigma}_{\min}$ rough experimental values of the background, the results agree quite well with the calculated ones.

1422 kev.—By using the same arguments that were used for the 669-kev resonance, this can be shown to have a 1^+ assignment with $a_p=0.85$. Plots of the complete resonance curve from 1320 kev to 1520 kev are shown in Figs. 2 and 3. The nonresonant phase shifts were chosen to be $\varphi_{J=1}=-0.19$ radian and $\varphi_{J=0}=0.09$ radian. These values of the phase shifts are not unique and no attempt was made to adjust them to give the best fit of the data.

1372 kev.—σ_r is too large for J to be zero. The assignment $l=0$, $J=1$ is eliminated because $\bar{\sigma}_{\max}-\bar{\sigma}_{\min}$ would be observable at 90°. If l is one, J can thus be 1 or 2. The assignment 1^- cannot be eliminated because it is possible to choose channel spin ratios so that $\bar{\sigma}$ for 1^- and 2^- become approximately the same for the smaller value of a_p. However, the assignment 1^- seems unlikely since the level does not disintegrate to the ground state or the first excited state of O^{16}. It should be noted that this is not the same level as the somewhat broader level at 1367 kev which does so disintegrate.[9] The values of $\bar{\sigma}_{\max}-\bar{\sigma}_{\min}$ and of each term separately are given in Table I, Paper A, for a 2^- assignment with $a_p=0.17$ and include the effects of the strong 1^+ resonance at 1422 kev. They were calculated by using $\varphi_{J=1}=-0.19$ radian, $\varphi_{J=0}=0.09$ radian. Plots of the complete resonance curves are shown in Figs. 2 and 3.

1346 kev.—This resonance is believed to have an assignment 2^- and this is consistent with the elastic scattering if a_p has its smaller value of 0.067. For the assignments 1^+ and 0^+, which latter is possible only if the measured σ_r is lowered 20 percent, the calculated anomaly at 90° is about 0.1. This would most probably have been observed, particularly since this region was studied very carefully. The assignment 0^- is not possible since α-particle emission is observed. We cannot rule out 1^-.

873 kev.—This level is definitely known to have the assignment 2^- and this is seen to be consistent with the elastic scattering results if a_p is taken to have its smaller value of 0.21. Interference effects with the nearby resonance at 935 kev or with nonresonant amplitudes were not included but would be less than 20 percent. If the experimental nonresonant background is roughly subtracted from the individual values of $\bar{\sigma}_{\max}$ and $\bar{\sigma}_{\min}$ the results agree with the calculated values. Using the same arguments as for the 1372 kev resonance, we can rule out 0^+ and 1^+, but cannot eliminate on elastic scattering data alone the assignment 1^-.

There are many known resonances with unknown l and J in this energy region which produce essentially no resonance anomaly in elastic scattering. These have been tabulated by Barnes[6] in his Table II. For these resonances a_p should have its smaller value. A minimum value for $\bar{\sigma}_{\max}-\bar{\sigma}_{\min}$ with $a_p=1$ is $[(2J+1)/2k\sigma_R^{\frac{1}{2}}]|P_l(\cos\theta)\cos\theta_l|$. This quantity for $l\leq 4$ at 159° or 125° for the various resonances was found to be larger than 0.3 except when $l=1$, $J=0$, which is impossible when α particles are observed. Since the larger value of a_p for these resonances is very close to 1, a_p must have its smaller value.

The reduced widths, γ^2, are given in Table I for the various resonances studied. The assignments of the

[6] C. A. Barnes, Phys. Rev. **97**, 1226 (1955). The author is indebted to Dr. Barnes for communicating his results prior to publication and for discussions concerning the partial widths of these reactions.

[7] Chao, Tollestrup, Fowler, and Lauritsen, Phys. Rev. **79**, 108 (1950).

[8] J. M. Freeman, Phil. Mag. **41**, (1950).

[9] E. B. Paul and R. L. Clarke (private communication).

TABLE I. Partial widths and reduced widths of certain $F^{19}+p$ resonances.[a]

E_{res} (kev)	Ne^{20*} J,π	Γ	Γ_p/Γ	Γ_{p0} / γ_{p0}^2	Γ_{p1} / γ_{p1}^2	Γ_{p2} / γ_{p2}^2	$\Gamma_{\alpha1}$ / $\gamma_{\alpha1}^2$	$\Gamma_{\alpha2}$ / $\gamma_{\alpha2}^2$	$\Gamma_{\alpha3}$ / $\gamma_{\alpha3}^2$	Γ_{π} / γ_{π}^2	$\Gamma_{\alpha0}$ / $\gamma_{\alpha0}^2$
843	0^+	23	0.996	23	~0.05	<0.01				~0.005	~0.033
				270	~3.6	<23				~0.0042	~0.0033
340	1^+	2.9	0.016	0.045	<0.0005	<0.0001	2.8	0.016	0.075	x	x
				94	<360	$<10^6$	110	6.1	42	x	x
669	1^+	7.5	0.976	7.3	0.046	<0.0005	0.110	0.0004	0.025	x	x
				240	15	<10	1.5	0.027	1.6	x	x
935	1^+	8.0	0.17	1.4	3.0	<0.020	2.8	0.100	0.78	x	x
				11	125	<20	19	2.0	14	x	x
1422	1^+	14.6	0.85	12.4	2.2	≤0.035		Total <0.04		x	x
				23	14	≤ 2.8		Total <0.2		x	x
598	2^-	37	0.0012	0.043	<0.5	<0.5	37	<0.1	<0.1	x	x
				9.5	<700	<3600	180	<3	<30	x	x
873	2^-	5.2	0.21	1.1	<0.002	0.57	2.4	0.85	0.30	x	x
				36	<1.7	67	6.2	8.5	20	x	x
1346	2^-	4.5	0.067	0.3	0.3	0.6	1.8	0.45	1.05	x	x
				1.7	23	6.1	2.1	1.2	12	x	x
1372	2^-	15	0.17	2.5	0.7	1.4	9.1	0.84	0.52	x	x
				13	48	14	10	2.0	5.0	x	x

[a] The first row for each resonance is the partial width, the second row the reduced widths, all widths being given in kev. The partial width were obtained from Table II of Barnes (reference 6). An x indicates the emission is forbidden by J and π considerations.

340- and 598-kev resonances, which could not be determined by this experiment, have been determined previously.[2] γ^2 is defined as $\Gamma(2kR)^{-1}(F_l^2+G_l^2)$, where F_l and G_l are Coulomb wave functions evaluated at R, l being the lowest possible orbital angular momentum, and R is the interaction radius which is taken to be 1.41 $(A_1^{1/3}+A_2^{1/3})\times10^{-13}$ cm, where A_1 and A_2 are the mass numbers of the two particles involved. The partial widths Γ have been given by Barnes[6] in his Table II and are repeated in our Table I. The penetration factors for the protons were obtained from the calculations of Christy and Latter.[10] For the α particles, calculations were based on the tabulations of Coulomb wave functions by Breit and his collaborators.[11] There is evidence[12,13] that the α particles are emitted with two orbital angular momenta. This would introduce a correction of about 10 percent. The single-particle reduced width, $\hbar^2/\mu R^2$, where μ is the reduced mass of the system, is 1660 kev for the protons and 390 kev for the α particles. Most of the reduced widths of the α particles are rather low, except for the large α_1 widths for the 340- and 598-kev resonances which are 28 percent and 46 percent respectively of the single-particle widths. These are much larger than usual α-particle widths. For the protons the widths at 669 and 843 kev are large, being 16 percent of the single-particle width. The reduced α_π and α_0 widths for the resonance at 843 kev are 1.1×10^{-3} percent and 0.85 $\times10^{-3}$ percent respectively of the single-particle width. As previously suggested by Barnes,[6] these low values indicate that this is a state of isotopic spin 1 of Ne^{20}. The upper limit of the reduced α widths at 1422 kev, which is 0.20 kev, is low and this may also indicate a $T=1$ state. The penetration factors, $(2kR)^{-1}(F_l^2+G_l^2)$, partially explain the branching ratios of the inelastic scattering.

The author would like to thank T. S. Webb and W. A. Fowler for many discussions concerning the interpretation of their results and to thank R. F. Christy for discussions concerning the method of analysis.

[10] R. F. Christy and R. Latter, Revs. Modern Phys. 20, 185 (1948).
[11] Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. 23, 147 (1951).
[12] J. Seed and A. P. French, Phys. Rev. 88, 1007 (1952).
[13] Peterson, Fowler, and Lauritsen, Phys. Rev. 96, 1250 (1954).