

V. OTHER SOLUTIONS FOUND IN THE PHASE SHIFT ANALYSIS

Even in the cases where we impose the predictions of Born's approximation on the phase shifts, we find several solutions at each energy all of which fit the cross sections within the experimental error. Only one set of these has a reasonable energy dependence as shown in Figs. 1 and 2. However, if we use as a restriction only that all 3P phase shifts shall be equal (that is, no spin-orbit splitting), we find a large number of possible solutions at each energy. For the case of p -He³ scattering, those solutions which have a reasonable energy

dependence are shown in Fig. 5. The solutions which do not have a reasonable energy dependence are not shown to keep the figure readable.

Since good agreement with experiment was found by using the Born approximation, no attempt was made in the p -T case to find solutions with other types of restrictions on the δ 's.

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Particle Binding Energies and the Diffuseness of the Nuclear Boundary*

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The approximate eigenvalues for a spherical well with an exponentially diffuse boundary and with spin-orbit splittings are applied to the study of particle binding energies. If the A -value locations of the low velocity $3s$ and $4s$ maxima in the neutron cross-section surface are taken at $A=55$ and $A=150$, the general trends of experimental binding energies and experimental radii sharply restrict the degrees of diffuseness that can be allowed. It would appear that the general trends of both proton and neutron binding energies as well as their discontinuities can be accounted for if a diffuseness parameter (i.e., tail length to e^{-1} point divided by inner radius) is chosen which drifts gradually from $\delta=0.3$ for light nuclei to $\delta=0.2$ for heavy nuclei. The diffuseness parameter needed goes to somewhat smaller values ($\delta\sim 0.13$) if the critical $4s$ A -value is taken at 170. For heavy elements the diffuseness of the potential well obtained here is comparable to the diffuseness of the nuclear charge distribution obtained in recent studies.

1. INTRODUCTION

THE approximate eigenvalues for a spherical well with an exponentially diffuse boundary have been obtained in a previous study.¹ To apply these results to the investigation of particle binding energies in complex nuclei one must first determine the magnitudes of the well strength parameter ϵ_0 and the radius parameter a for various values of A . It should be obvious that for a given A , as the diffuseness parameter δ is increased, the well strength parameter ϵ_0 and the radius parameter a needed to account for specific experimental observations will grow smaller. The precise relationship $\epsilon_0(\delta, A)$ and $a(\delta, A)$ will depend upon the particular experimental observations which are taken as standard. Let us now consider a basis for arriving at these functions.

2. IDENTIFICATION OF THE WELL PARAMETERS

The particle mass used in this work is taken to be the average of the neutron and proton, i.e.,

$$m = \frac{1}{2}(m_n + m_p) = 1008.288 \text{ mMU}. \quad (1)$$

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¹ A. E. S. Green and Kiuck Lee, Phys. Rev. **99**, 772 (1955). This work will henceforth be called GLI.

Since the independent-particle model (I.P.M.) places each particle in a field of force set up by the $A-1$ other particles, the natural energy unit takes the form

$$E_0 = \hbar^2/2\mu a^2 = U_0[1 + 1/(A-1)]/a^2(\delta, A), \quad (2)$$

where

$$U_0 = 22.267 \text{ mMU} = 20.734 \text{ Mev}, \quad (3)$$

and $a(\delta, A)$ is a dimensionless distance parameter obtained by dividing a itself by 1×10^{-13} cm. The dimensionless well strength parameter is now given by

$$\epsilon_0 = (V_0/E_0)^{\frac{1}{2}} = (V_0/U_0)^{\frac{1}{2}} a(\delta, A)/[1 + 1/(A-1)]^{\frac{1}{2}}, \quad (4)$$

where the constant V_0 represents the depth of the uniform portion of the potential function. It is not unreasonable to expect V_0 to be a universal constant which measures the strength of the interaction of a single nucleon in condensed nuclear matter. Accordingly $\epsilon_0(\delta, A)$, apart from the small reduced mass effect, is expected to vary simply as $a(\delta, A)$. To proceed further, it shall be assumed that²

$$\epsilon_0(\delta, A) = f_1(\delta)A^{\frac{1}{2}} + f_2(\delta), \quad (5)$$

² It would perhaps be more desirable to let $a(\delta, A)$ be given by Eq. (5) and to let $\epsilon_0(\delta, A)$ embody the reduced mass effect. Equation (5) however, was chosen because it brings the $2s$ resonance closer to $A=11$.

an assumption which will make the nuclear radius parameter vary almost linearly with $A^{1/3}$. To identify $f_1(\delta)$ and $f_2(\delta)$, use is now made of the critical ϵ_0 values for binding the 3s and 4s states with zero energy. These critical ϵ_0 values for various δ are given in Table I of GLI. If these are assumed to correspond to the observed maxima in the low-velocity neutron cross sections at $A=55$ and $A=150^{3-5}$ one may readily solve for $f_1(\delta)$ and $f_2(\delta)$. The results of the calculation are given in the second and third columns of Table I.

It is reasonable now to regard $\epsilon(0, A)$ as the "effective" or equivalent square well strength of a spherical well with an exponentially diffuse boundary, in the sense that a square well with the well strength parameter $\epsilon_0(0, A)$ will locate the low velocity 3s and 4s maxima at the same A values as diffuse boundary wells with $\epsilon_0(\delta, A)$ given by Eq. (5). In a similar way, one might regard

$$a(0, A) = (U_0/V_0)^{1/2} [1 + 1/(A-1)]^{1/2} [f_1(0)A^{1/3} + f_2(0)] \quad (6)$$

as the equivalent square well radius for any degree of diffuseness. In view of the smallness of $1/(A-1)$ and $f_2(0)$, one might readily compare Eq. (6) with the usually accepted equation $R=r_0A^{1/3}$ and hence make the

TABLE I. Effective well strength functions.

δ	$f_1(\delta)$	$f_2(\delta)$
0.0	2.0802	-0.0567
0.1	1.8149	0.1819
0.2	1.4342	0.7497
0.3	1.2931	0.4534
0.4	1.1965	0.1966

identification:

$$r_0 \approx 2.080(U_0/V_0)^{1/2}, \quad (7)$$

where r_0 is an experimental radius constant (in units of 10^{-13} cm). Since these have ranged between 1.0 and 1.5 in recent years, it would appear best to attempt to proceed without fixing r_0 at this juncture. However, for some quantitative estimates, r_0 shall be taken as 1.3 which corresponds to $V_0=53.09$ Mev. This r_0 is a compromise between the recent Coulomb radius constants and the older reaction cross section radius constants. The V_0 values corresponding to various r_0 's are given in Table II.

3. NEUTRON BINDING ENERGIES

The magnitude of the particle energy for a given state may be expressed in the form

$$|W| = V_0(\epsilon_W^2/\epsilon_0^2). \quad (8)$$

To explore particle binding energies without first fixing V_0 it is convenient to present the eigenvalues obtained

TABLE II. Well depths for various radius constants.

r_0	V_0
1.0	89.72
1.1	71.15
1.2	62.30
1.3	53.09
1.4	45.78
1.5	39.88

in GLI by the ratios $100\epsilon_W^2/\epsilon_0^2$ as functions of ϵ_0^2 . Such a representation for the case $\delta=0.3$ is shown in Fig. 1. Clearly a specific choice of V_0 (or r_0) enables one to translate the vertical scale into an absolute energy scale.

To apply this work to the study of neutron binding energies it is necessary to relate ϵ_0 to the neutron number, which essentially means to relate N to A . In an earlier preliminary study⁶ the relationship $N=A/2$ was used. For the present more detailed investigation, an $N-A$ relationship is used which embodies the general trend of the line of beta stability. This relationship is derived from

$$D_m^e(A) = 0.4A^2/(A+200), \quad (9)$$

where $D_m^e(A)$ is the neutron excess for the minimum of the empirical mass surface valley. With this expression and Eq. (5), one may convert ϵ_0 into a function of N itself.

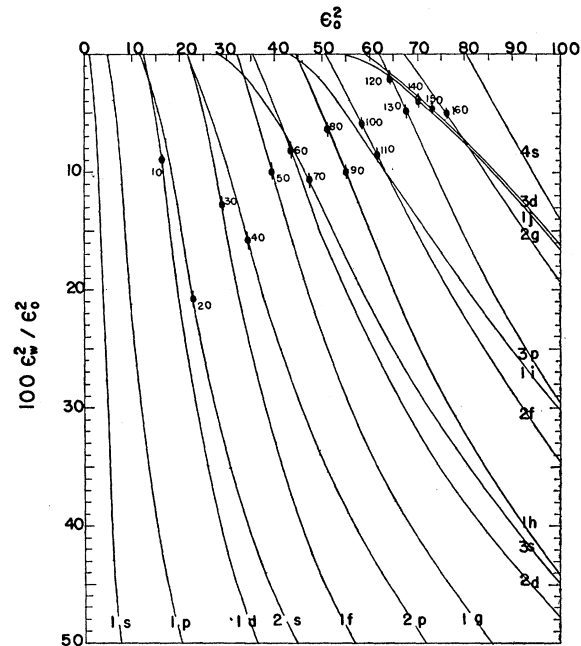


FIG. 1. Energy eigenvalues for various states in units of $(V_0/100)$ as functions of ϵ_0^2 under the assumption of $\delta=0.3$. The points represent the approximate location of the outermost neutron in beta stable nuclides with $N=10, 20$, etc.

³ K. W. Ford and D. Bohm, Phys. Rev. **79**, 745 (1950).

⁴ R. K. Adair, Phys. Rev. **94**, 737 (1954).

⁵ Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

⁶ The results of this study were reported at the meeting of the Southeastern Section of the American Physical Society in Gainesville on April 8, 1955.

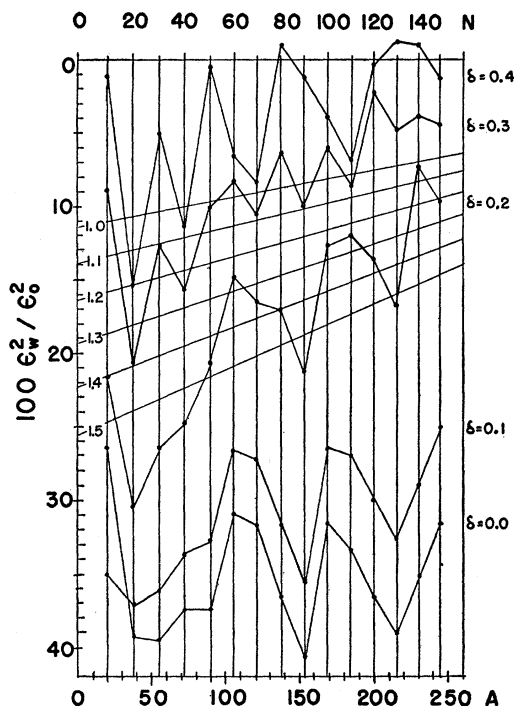


FIG. 2. Neutron binding energies for $\delta=0, 0.1, 0.2, 0.3$, and 0.4 . The straight lines labeled by the corresponding values of r_0 represent Eq. (10).

To investigate the general trends of theoretical neutron binding energies, the following sequence of calculations was made: (1) The ϵ_0^2 values for N values in steps of 10 from 10 to 150 and for $\delta=0, 0.1, 0.2, 0.3$, and 0.4 were calculated. (2) For each combination the array of energy levels then is found by means of a vertical line through the ϵ_0^2 value on the appropriate graph (see Fig. 1). The N particles are then placed in the lowest states permitted by the Pauli exclusion principle. The $100\epsilon_w^2/\epsilon_0^2$ values for the outermost particles obtained in this way are shown in Fig. 1 for $\delta=0.3$. The results for all δ 's, presented as functions of N or A , are shown in Fig. 2.

The general trends of experimental neutron binding energies for beta stable nuclides are fairly well summarized by the expression:

$$B_n = 10.235 - 0.01862A \text{ (Mev)}, \quad (10)$$

which is derived from the empirical mass equation.⁷ This expression is also represented on Fig. 2 for each r_0 and V_0 listed in Table II. An examination of this diagram reveals that agreement between the general trends of experiment and theory is secured only within the diffuseness parameter range $\delta=0.25 \pm \sim 0.05$. Within this limited range the required diffuseness parameter depends upon r_0 .

⁷ A. E. S. Green, *Nuclear Physics* (McGraw Hill Book Company, Inc., New York, 1955).

4. PROTON BINDING ENERGIES

The Coulomb repulsion, of course, acts upon the individual protons and hence appreciably complicates the calculations for proton binding energies. Fortunately an approximate procedure has been developed⁸ for correcting for the Coulomb effect which permits the utilization of the eigenvalues determined in GLI. To do so, the single-particle Coulomb potential energy in the uniform part of the well is approximated by a constant equal to its rms value. In the exponential outer region the Coulomb potential energy is approximated by a constant plus an exponentially decaying potential with the same decay length as the nuclear potential. The outer potential is chosen to match the Coulomb potential at $r=1.1a+\delta a$. With these prescriptions it is found, after straightforward but tedious calculation, that the individual-particle Coulomb energy may be expressed by

$$V_c = (Z-1)U_c[\alpha(\delta)+\beta(\delta)]/a(\delta,A), \quad r \leq a \\ = (Z-1)U_c[\alpha(\delta)e^{(\alpha-r)/\delta a}+\beta(\delta)]/a(\delta,a), \quad r \geq a, \quad (11)$$

where $U_c=0.86392$ Mev and $\alpha(\delta)$ and $\beta(\delta)$ are as given in Table III.

To apply the eigenvalues in GLI, one essentially first goes to a composite well strength value and looks up the corresponding eigenvalues which represent the energies measured to the top of the Coulomb barrier. To get the actual energy levels, one then subtracts away the height of the barrier. Because of the $a(\delta,A)$ term, the Coulomb correction is dependent upon the choice of r_0 . The results of energy calculations for $\delta=0.2$ and $\delta=0.3$ each for $r_0=1.1, 1.3$, and 1.5 are shown by the broken lines in Fig. 3. It should be clear from this figure that the proton levels are rather insensitive to the radius constant assumed for the Coulomb correction. The straight lines on this diagram correspond to the expression:

$$B_p = 9.453 - 0.01862A \text{ (Mev)}, \quad (12)$$

which also is derived from the empirical mass surface and which represents the general trends of proton binding energies fairly well. Again it appears that the

TABLE III. Coulomb correction functions.^a

δ	$\alpha(\delta)$	$\beta(\delta)$
0.0	0.242	0.758
0.1	0.280	0.654
0.2	0.286	0.562
0.3	0.267	0.493
0.4	0.240	0.437
0.5	0.213	0.392

^a The values of α and β given here have been determined so that the *IPM* Coulomb potential energy approximates one half the classical Coulomb energy of a single proton in the electrostatic field of the other $Z-1$ protons. These other protons are assumed to be distributed according to a density function which is proportional to the nuclear potential function.

⁸ Kiuck Lee and A. E. S. Green (unpublished).

general trends of experimental particle binding energies are predicted when $\delta = 0.25 \pm \sim 0.05$.

5. EFFECT OF SPIN-ORBIT SPLITTINGS

In view of the successes of the Mayer,⁹ Haxel-Jensen-Suess¹⁰ strong spin-orbit coupling model, it would appear essential to incorporate a phenomenological spin-orbit energy into a study of particle binding energies. For initial studies, one might proceed by assuming a simple spin-orbit energy given by

$$W_{so} = -\beta \mathbf{l} \cdot \mathbf{s} = \frac{1}{2} \beta (l+1), \quad i = l - \frac{1}{2} \\ -\frac{1}{2} \beta l, \quad i = l + \frac{1}{2}. \quad (13)$$

The shifts in $100\epsilon_W^2/\epsilon_0^2$ values caused by this perturbation have been computed for a β such that

$$100\beta/V_0 = 1. \quad (14)$$

For $V_0 = 53$ Mev (i.e., $r_0 = 1.3$), this corresponds to $\beta = 0.53$ Mev which is very close to a spin-orbit coupling constant used successfully by Malenka.¹¹ The neutron binding energies for $\delta = 0.2$ and $\delta = 0.3$ for every second particle arrived at by calculations similar to those described in Sec. 3 are shown in Fig. 4. The proton binding energies for $\delta = 0.2$ and $\delta = 0.3$ and $r_0 = 1.3$ are shown in Fig. 5. Also shown are empirical particle binding energy curves based upon $r_0 = 1.3$ which incorporate approximate shell corrections which have been inferred pre-

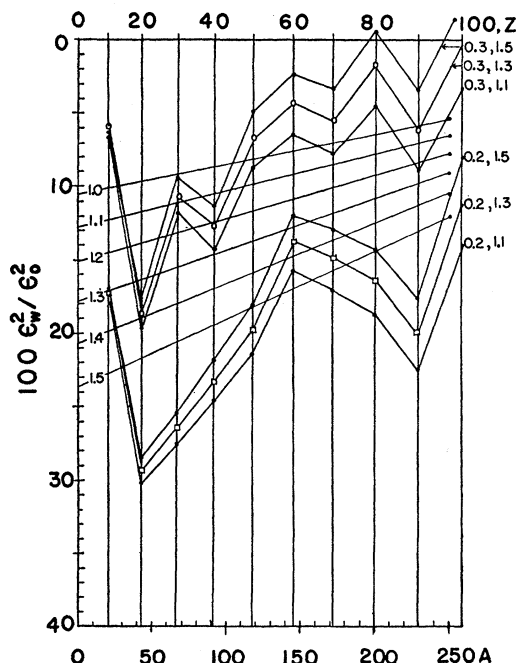


FIG. 3. Proton binding energies for $\delta = 0.2$ and 0.3 and when $r_0 = 1.1, 1.3$, and 1.5 are used in the Coulomb corrections. The straight lines represent Eq. (12).

⁹ M. G. Mayer, Phys. Rev. **78**, 16 (1950).

¹⁰ Haxel, Jensen, and Suess, Z. Physik **128**, 295 (1950).

¹¹ B. J. Malenka, Phys. Rev. **86**, 68 (1952).

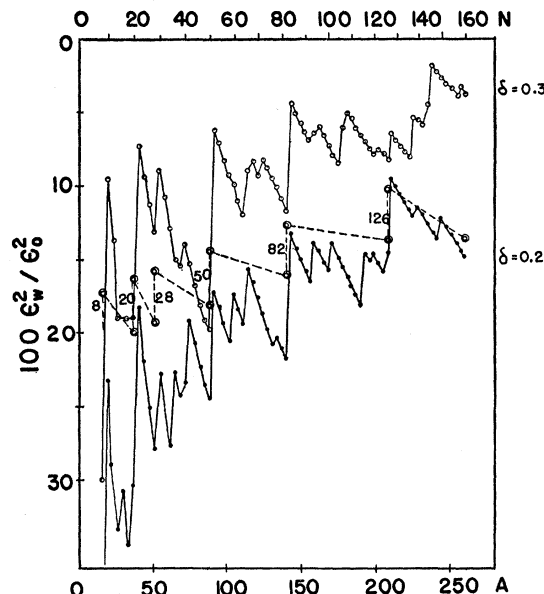


FIG. 4. Neutron binding energies when phenomenological spin-orbit splittings are included. The dashed line represents an empirical expression discussed in references 7 and 12 and which is based upon $r_0 = 1.3$.

viously from the experimental data.^{7,12} In both the proton and neutron cases, the empirical curve drifts gradually from the $\delta = 0.3$ points for light nuclei towards the $\delta = 0.2$ points for heavy nuclei. It would appear, therefore, that the best sets of theoretical particle binding energies would be those inferred by interpolation between these cases. If one envisages these intermediate cases one is led to a theoretical set of particle binding energies which (a) reproduces the over-all general trends of the experimental binding energies, (b) generates the major magic numbers, (c) produces energy discontinuities of reasonable magnitudes, (d) is approximately consistent with the empirical line of beta stability.

6. DISCUSSION AND CONCLUSION

It is perhaps appropriate first to call attention to several limitations of the current study. These limitations which are all suggestive of further lines of work are as follows:

(a) The simple spin-orbit energy with constant parameter should not be taken too literally. In the absence of a firm theoretical basis for the nuclear spin-orbit energy, one might at least proceed on the basis of a Thomas-like expression. The evaluation of the relative separation constants however is then a tedious chore which has yet to be completed. Changes in relative separations will influence appreciably the fine details of particle binding energies between major shells.

(b) There are some uncertainties as to the validity of the use of 55 and 150 for the A -value locations of the

¹² A. E. S. Green and D. F. Edwards, Phys. Rev. **91**, 46 (1953).

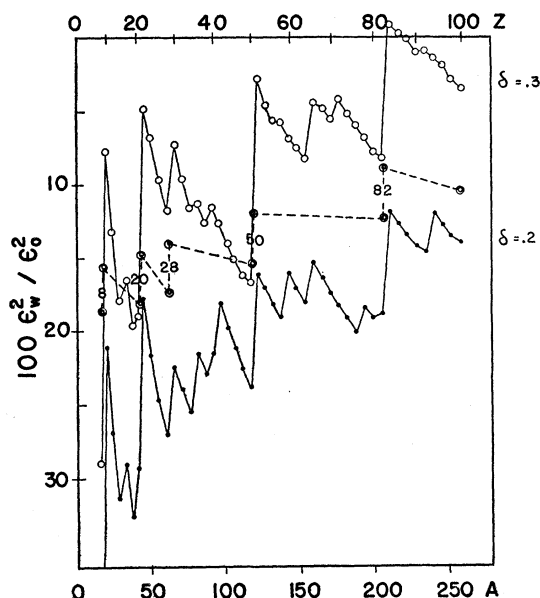


FIG. 5. Proton binding energies when phenomenological spin-orbit splittings are included. The dashed lines represent an empirical expression discussed in references 7 and 12.

third and fourth s -wave maxima. In particular, measurements of Γ/D suggest that the $4s$ maximum occurs at $A \sim 170$.¹³ The use of this standard with a given δ does not appreciably alter the predicted particle binding energies of light and middle weight nuclides but does significantly lower those for heavy weight nuclides. Accordingly, with the new standard the diffuseness parameter falls off more rapidly with A . When 170 is chosen as the critical A -value, the effective well strength $\epsilon(0, A)$ and the effective radius parameter $a(0, A)$ both have large constant terms. Agreement with Emmerich's radius function,

$$R = 1.26A^{1/3} + 0.7, \quad (15)$$

is secured if V_0 is taken as 42 Mev. With these parameter assignments, the diffuseness parameter would have to migrate to about 0.13 for heavy elements to yield particle binding energies of the correct order of magnitude.

This sensitivity of the results to the function $\epsilon_0(\delta, A)$ points to the need for very careful consideration of experimental data which might be used to fix this function. In a preliminary study, use was made of the simple explicit function:

$$\epsilon_0(\delta, A) = (2.08 - 2\delta)A^{1/3}, \quad (16)$$

which was arrived at in an effort to standardize upon the $2s$ as well as the $3s$ and $4s$ maxima. While this function is not very much different from that used here, it led to a somewhat higher average estimate for the diffuseness parameter (i.e., $\delta \sim 0.3$).

(c) For expediency, in this study use has been made

¹³ W. S. Emmerich, Phys. Rev. **98**, 1148(A) (1955), and private communication.

of empirical summaries of the experimental data rather than the experimental data themselves. In doing so, it must be emphasized that the empirical expressions only indicate the gross tendencies of the experimental data. Nevertheless, it might be pointed out that the simple analytic shell correction upon which the dotted lines in Figs. 4 and 5 are based accounts directly for the six relevant conclusions on nuclear structure reached by Way and Wood¹⁴ from a detailed study of beta systematics.

(d) Configuration interactions as well as two-body and many-body forces which are not embodied in the central field undoubtedly influence the fine details of particle binding energies in a complicated manner.

(e) In view of the great sensitivity of the outermost nuclear levels to small changes in the diffuseness parameter, one might expect also a rather large sensitivity to aspherical deviations in the shape of the potential.

Despite these limitations, it is thought that the present investigation has served a useful function in exposing the great influence of the diffuseness of the nuclear boundary upon particle binding energies and indicating the approximate extent of diffuseness necessary to account for particle binding energies. Clearly the square well must be rejected because it leads to particle binding energies which are much too large. A well with a diffuseness parameter greater than $\delta = 0.4$ must be rejected because it leads to particle binding energies which are too small. The cut-off harmonic oscillator potential is such a well.

While there are still many uncertainties, it is believed that the restriction of the diffuseness parameter to the approximate range $\delta = 0.25 \pm \sim 0.05$ is relatively insensitive to the factors discussed above. If, for example, forces not embodied in the central field change the binding energies of the outermost particles by as much as 1 or 2 Mev, only a small change in our diffuseness parameter estimate would serve to make the net energy again agree with experiment.

This study affords a rather simple explanation for the progressive increase of particle binding energies as major shells are filled. Rather than reflecting the effect of interparticle interactions, this behavior may primarily reflect the great sensitivity of the outermost energy levels to the small changes in ϵ_0^2 associated with the progressive addition of particles. The fact that sub-shell effects are not very apparent in the binding energy data might be accounted for by appropriate spacing of the energy levels within a shell. This spacing would be expected to be dependent upon fine details of nuclear forces.

This study also suggests that a coupling between individual particle motion and the collective motion of the outer nucleons might occur through the intermediary of changes in the diffuseness parameter. Thus, if the collective interactions near closed shells cause

¹⁴ K. Way and M. Wood, Phys. Rev. **94**, 119 (1954).

contraction of the region of diffuseness, the independent-particle model levels will separate. This is as experimental observations suggest. Such a coupling is analogous to that which occurs in the Bohr collective model of the nucleus through the intermediary of aspherical surface distortions.¹⁵

If one wishes to compare the diffuseness parameters arrived at here with the diffuseness corresponding to other shapes, the 0.9 to 0.1 fall-off distance for a fixed distance to the 0.5 point might serve as a reasonable basis for comparison. Let us suppose, for a heavy element, that the distance to the 0.5 point is $R=6.7$ (in units of 10^{-13} cm). Then $\delta=0.2$ corresponds to a fall-off distance of 2.6. This is only slightly more diffuse than the experimentally determined fall-off length (2.2) for the nuclear charge distribution in lead,¹⁶ but appreci-

ably less diffuse than Swiatecki's recent theoretical determination (3.1→4.3) based upon surface energies.¹⁷

In closing, it might be remarked that practically all of the evidence relating to nuclear shell structure comes from phenomena originating in the outermost regions of the nucleus. In view of the sensitivity of particle binding energies to the diffuseness parameter, one might well hope that many of the quantitative difficulties with the independent particle model might be removed by the use of nuclear potentials with appropriate degrees of diffuseness.

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¹⁷ W. J. Swiatecki, *Phys. Rev.* **98**, 204 (1955).

¹⁵ A. Bohr and B. R. Mottelson, *Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd.* **27**, (16) (1953).

¹⁶ Hill, Freeman, and Ford, *Bull. Am. Phys. Soc.* **30**, No. 3, 49 (1955).

Neutron-Deficient Activities of Terbium

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A survey was made of the neutron-deficient activities of terbium produced by proton bombardment of enriched isotopes of gadolinium. Half-lives and mass assignments are made for Tb¹⁵⁶ and Tb¹⁵⁴, and the limits of half-lives for Tb¹⁵⁸, Tb¹⁵⁷, and Tb¹⁵⁵ are defined. Data were obtained on the gamma-ray spectrum of Tb¹⁵⁶.

A SURVEY has been completed of the neutron-deficient activities of terbium which were produced by proton bombardment of enriched stable isotopes of gadolinium. The stable gadolinium isotopes of masses 152 through 158, in the form of oxides, were bombarded with protons in the Oak Ridge National Laboratory 86-inch cyclotron. After bombardment, ion exchange methods¹ were used to separate the products. The energy of the incident proton beam was controlled with appropriate aluminum absorbers. In separate runs, beam energies of 9.5, 14.0, and 22.4 Mev were used in order to insure that the reaction was chiefly (p,n) , $(p,2n)$, or $(p,3n)$. Following separation, the terbium fraction was investigated for nuclear properties, such as half-lives and radiations, by the use of absorption and decay data, scintillation spectrometers, and alpha and beta counters. Assignment of mass numbers is based on excitation functions as determined experimentally and on enrichment factors of the stable gadolinium isotopes, Table I. Limits for the half-lives of some mass

numbers are based on the relative production of the 5.2-day activity assigned to Tb¹⁵⁶. By assuming equal counting efficiencies and with corrections for length of bombardment, decay, chemical yields, and mass analysis of the stable isotopes, it is possible to calculate the minimum half-life of the longer lived activities. Because of these assumptions, these half-lives may be in error by as much as an order of magnitude. By using shorter bombardment times (2 or 3 minutes) and without attempting any separations, upper limits for half-lives are assigned to several mass numbers. This upper limit was controlled by the time required to get the

TABLE I. Analyses of enriched gadolinium isotopes.

Isotope	Isotopic analyses (%)						
	152	154	155	156	157	158	160
152	14.96	9.75	27.26	19.32	10.08	11.67	6.97
154	0.32	33.17	38.57	15.92	5.49	4.50	2.05
155	0.46	1.23	72.28	17.72	4.60	2.86	0.81
156	...	0.25	4.34	80.22	10.02	4.30	0.86
157	0.04	0.11	1.23	7.31	69.68	19.90	1.74
158	...	<i>t</i>	0.30	0.84	3.15	92.87	2.84

¹ B. H. Kettle and G. E. Boyd, *J. Am. Chem. Soc.* **69**, 2800 (1947).