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Elastic Scattering of Protons by He^3 and T†

R. M. Frank and J. L. Gammel
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico

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A phase shift analysis is made of the p-He^3 and p-T elastic scattering data using theoretical predictions wherever possible to reduce the number of parameters. It is pleasing that the resulting phase shifts all have their expected signs and reasonable magnitudes and satisfy relations connecting the p-T and p-He^3 phase shifts. The analysis was also made without using the theoretical conditions, and we present all the solutions we have found. However, we reject these other solutions. The phase shifts which we accept as correct are discussed in terms of resonances.

I. INTRODUCTION

PHASE shift analyses of the p-He^3 and p-T elastic scattering data have been made before.[1,2] The present work is new in that theoretical predictions are applied wherever reasonable to reduce the number of parameters which occur in the phase shift analysis.

Phase shifts (δ's) for values of the angular momentum $l \geqslant 2$ are assumed to be zero. This assumption is probably safe in the energy range (1–3.5 Mev) of the experiments analyzed here. The effects of inelastic scattering are neglected. This is certainly safe for p-He^3, but the threshold for the $T(p,n)He^3$ reaction (1 Mev) is not high enough to justify its neglect in the p-T case. These same assumptions were made in references 1 and 2.

The problem was coded for an IBM 701 computer. The code allows for the possibility of spin-orbit splitting. However, the data can be fitted without spin-orbit splitting, and even, in the case of p-He^3, without spin dependence, as Lowen pointed out. Therefore, efforts to study a possible spin-orbit splitting were abandoned.

Born's approximation makes qualitative predictions about the phase shifts. It says, for example, that in the p-He^3 case there should be little spin dependence, just as Lowen observed. It says that in the p-T case, the δ's for the two spin states (singlet and triplet) should be equal in magnitude and opposite in sign.

We find that it is possible to impose these predictions of Born's approximation on the δ's in the p-He^3 case for $l=0$ and $l=1$ and in the p-T case for $l=1$. These are auxiliary conditions on the phase shift analysis which reduce the number of parameters used in fitting the data. It is very satisfying that when this is done, the resulting δ's all have the sign predicted by Born's approximation. It is also pleasing that relations which result from Born's approximation and connect the p-T and p-He^3 phase shifts are satisfied by the P phase shifts and very roughly by the S phase shifts.

The analysis was also made without the auxiliary conditions suggested by Born's approximation, and we present all of the solutions we have found. However, we reject these other solutions. The δ's which we accept as correct are discussed in terms of resonances.

II. BORN'S APPROXIMATION

Born's approximation is

$$k^2\sigma = \left| \frac{k}{4\pi} J \right|^2, \tag{1}$$

$$J = \frac{2mM}{(m+M)\hbar^2} \int \mathbf{S}(\psi_f)\mathbf{S}(V\psi_i)d\tau. \tag{2}$$

Here $m=1$ and $M=3$ (the masses of the proton and He^3 or T, respectively). The wave number k is given by

$$k^2 = \frac{2m}{\hbar^2}\left(\frac{M}{m+M}\right)^2 E_{\text{lab}} = 2.71 E_{\text{lab}} \times 10^{24}\ \text{cm}^{-2}, \tag{3}$$

where E_{lab} is the energy of the proton in the laboratory system. ψ_i and ψ_f are the wave functions for the initial and final states, and V is the sum of the potentials between the proton and the particles in He^3 or T. $\mathbf{S}$ is an operator which makes $\mathbf{S}\psi$ antisymmetrical in the coordinates of any pair of neutrons or protons of which ψ is a function. The integral stands for a sum over the spins of all the particles involved as well as an integral over all of their coordinates.

Both He^3 and T have spin $S=\frac{1}{2}$. Their spin functions are

$$\begin{aligned} \chi_a^+ &= \alpha_1(\alpha_2\beta_3 - \beta_2\alpha_3)/\sqrt{2}, \\ \chi_s^+ &= (\alpha_1\alpha_2\beta_3 + \alpha_1\beta_2\alpha_3 - 2\beta_1\alpha_2\alpha_3)/\sqrt{6}, \end{aligned} \tag{4}$$

where the superscript $+$ means $S_z=+\frac{1}{2}$, and the subscripts a and s mean that χ is antisymmetrical or symmetrical in particles 2 and 3 (the two protons in He^3 or the two neutrons in T), respectively. We assume that the spatial parts of the He^3 or T ground state wave functions are symmetrical in all three particles, and therefore we must use χ_a because of Pauli's principle.

Then ψ_i (for example) is formed as follows:

$$\begin{aligned} &\psi_i = \alpha_4\chi_a^+\phi(123)f(\mathbf{k},4), \quad S=1, \quad S_z=1, \\ &\psi_i = (1/\sqrt{2})(\alpha_4\chi_a^- - \beta_4\chi_a^+)\phi(123)f(\mathbf{k},4), \\ &\qquad\qquad S=0, \quad S_z=0, \end{aligned} \tag{5}$$

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[1] H. R. Lowen, Phys. Rev. **96**, 826(A) (1954).

[2] McIntosh, Gluckstern, and Sack, Phys. Rev. **88**, 752 (1952).

where particle 4 is the incident proton, ϕ is the ground state wave function of He^3 or T, and f is a plane wave function of $\mathbf{k}$ and the radius vector locating particle 4 with respect to the center of gravity of particles 1, 2, and 3. In the p-T case

$$\mathbf{S}=(1-P_{14}), \tag{6}$$

and in the p-He^3 case

$$\mathbf{S}=(1-P_{24}-P_{34}), \tag{7}$$

where P_{14} (for example) interchanges all of the coordinates (including the spins) of particles 1 and 4. The $\mathbf{S}$'s should not be normalized in any way; at large distances $\mathbf{S}\psi$ will approach a plane wave for particle 4 times the ground-state wave function for particles 1, 2, and 3 as it should.

The potential between a neutron i and a proton j is

$$\begin{aligned} V(ij)=U(ij)\Bigg[&{}^3V_{np}{}^{+}\left(\frac{1+P_{ij}{}^{x}}{2}\right)\left(\frac{1+P_{ij}{}^{\sigma}}{2}\right)\\ &+{}^1V_{np}{}^{+}\left(\frac{1+P_{ij}{}^{x}}{2}\right)\left(\frac{1-P_{ij}{}^{\sigma}}{2}\right)\\ &+{}^3V_{np}{}^{-}\left(\frac{1-P_{ij}{}^{x}}{2}\right)\left(\frac{1+P_{ij}{}^{\sigma}}{2}\right)\\ &+{}^1V_{np}{}^{-}\left(\frac{1-P_{ij}{}^{x}}{2}\right)\left(\frac{1-P_{ij}{}^{\sigma}}{2}\right)\Bigg], \end{aligned} \tag{8}$$

where $P_{ij}{}^{x}$ and $P_{ij}{}^{\sigma}$ exchange the space and spin coordinates of particles i and j. $U(ij)$ is a potential of some shape, and its depth is fixed by the requirement that ${}^3V_{np}{}^{+}=1$; that is, $U(r)$ is the potential for the ground state of the deuteron. When i and j are identical, V has the same form except the triplet even parity and singlet odd parity forces are not present because of Pauli's principle.

It is necessary to insert Eqs. (4)–(8) in Eq. (2) and perform the spin sums. This tedious work can be simplified somewhat by using the Hermitean character of the $\mathbf{S}$ and the relations

$$\begin{aligned} \mathbf{S}^2&=2\mathbf{S}\quad \text{for } p\text{-He}^3,\\ \mathbf{S}^2&=3\mathbf{S}\quad \text{for } p\text{-T}. \end{aligned} \tag{9}$$

We find for the J's the following expressions:

$$J=\alpha J_1+\beta J_2+\gamma J_3, \tag{10}$$

where

$$\begin{aligned} J_1&=-\frac{3}{2}\frac{M}{\hbar^2}\int\phi(123)f^*(\mathbf{k}_f,4)U(14)\phi(123)f(\mathbf{k}_i,4)d\tau,\\ J_2&=-\frac{3}{2}\frac{M}{\hbar^2}\int\phi(134)f^*(\mathbf{k}_f,2)U(14)\phi(123)f(\mathbf{k}_i,4)d\tau,\\ J_3&=-\frac{3}{2}\frac{M}{\hbar^2}\int\phi(123)f^*(\mathbf{k}_f,4)U(14)\phi(234)f(\mathbf{k}_i,1)d\tau, \end{aligned} \tag{11}$$

where the integrals are now over the spatial coordinates. $\mathbf{k}_f$ and $\mathbf{k}_i$ are the final and initial wave number vectors of the scattered proton [their magnitudes are, of course, equal for elastic scattering and given by Eq. (3); the angle of scattering is the angle between them]. The results of the spin sums are contained in α, β, and γ. For p-T, $S=1$,

$$\begin{aligned} \alpha&=2{}^3V_{pp}{}^{-}+\tfrac{3}{2}{}^3V_{np}{}^{+}+\tfrac{3}{2}{}^3V_{np}{}^{-}+\tfrac{1}{2}{}^1V_{np}{}^{+}+\tfrac{1}{2}{}^1V_{np}{}^{-},\\ \beta&=-3{}^3V_{np}{}^{+}-{}^1V_{np}{}^{+},\\ \gamma&=-2{}^3V_{pp}{}^{-}+\tfrac{3}{2}{}^3V_{np}{}^{+}-\tfrac{3}{2}{}^3V_{np}{}^{-}+\tfrac{1}{2}{}^1V_{np}{}^{+}-\tfrac{1}{2}{}^1V_{np}{}^{-}. \end{aligned} \tag{12}$$

For p-T, $S=0$,

$$\begin{aligned} \alpha&=2{}^1V_{pp}{}^{+}+\tfrac{3}{2}{}^3V_{np}{}^{+}+\tfrac{3}{2}{}^3V_{np}{}^{-}+\tfrac{1}{2}{}^1V_{np}{}^{+}+\tfrac{1}{2}{}^1V_{np}{}^{-},\\ \beta&=3{}^3V_{np}{}^{+}+{}^1V_{np}{}^{+},\\ \gamma&=2{}^1V_{pp}{}^{+}+\tfrac{3}{2}{}^3V_{np}{}^{+}-\tfrac{3}{2}{}^3V_{np}{}^{-}+\tfrac{1}{2}{}^1V_{np}{}^{+}-\tfrac{1}{2}{}^1V_{np}{}^{-}. \end{aligned} \tag{13}$$

For p-He^3, $S=1$,

$$\begin{aligned} \alpha&=\tfrac{3}{2}{}^3V_{np}{}^{+}+\tfrac{3}{2}{}^3V_{np}{}^{-}+\tfrac{3}{2}{}^1V_{pp}{}^{+}+\tfrac{9}{2}{}^3V_{pp}{}^{-},\\ \beta&=-3{}^3V_{np}{}^{+}-3{}^1V_{pp}{}^{+},\\ \gamma&=\tfrac{3}{2}{}^3V_{np}{}^{+}-\tfrac{3}{2}{}^3V_{np}{}^{-}+\tfrac{3}{2}{}^1V_{pp}{}^{+}-\tfrac{9}{2}{}^3V_{pp}{}^{-}. \end{aligned} \tag{14}$$

For p-He^3, $S=0$,

$$\begin{aligned} \alpha&=\tfrac{3}{2}{}^1V_{np}{}^{+}+\tfrac{3}{2}{}^1V_{np}{}^{-}+\tfrac{3}{2}{}^1V_{pp}{}^{+}+\tfrac{9}{2}{}^3V_{pp}{}^{-},\\ \beta&=-3{}^1V_{np}{}^{+}-3{}^1V_{pp}{}^{+},\\ \gamma&=\tfrac{3}{2}{}^1V_{np}{}^{+}-\tfrac{3}{2}{}^1V_{np}{}^{-}+\tfrac{3}{2}{}^1V_{pp}{}^{+}-\tfrac{9}{2}{}^3V_{pp}{}^{-}. \end{aligned} \tag{15}$$

We are not going to calculate the integrals in Eq. (11); we are only going to use what can be read from Eqs. (12)–(15).

An immediate consequence of these equations is that if the odd-parity forces are weak, then no matter what the values of J_1, J_2, and J_3 (they may even be calculated with a better approximation for $f(\mathbf{k}_i,4)$ than a plane wave) the p-He^3 phase shifts for the different spin states are related as follows:

$$\frac{\delta\text{-triplet}}{\delta\text{-singlet}}=\frac{{}^3V_{np}{}^{+}+{}^1V_{pp}{}^{+}}{{}^1V_{np}{}^{+}+{}^1V_{pp}{}^{+}}=1.2, \tag{16}$$

where the numerical value is calculated assuming that the potentials are charge-independent and that the singlet potentials are weaker than the triplet potentials by a well-known factor (which is slightly dependent on the shape of $U(r)$; we use the one for a Yukawa potential); that is, assuming

$${}^1V_{nn}{}^{+}={}^1V_{pp}{}^{+}=0.69{}^3V_{np}{}^{+}. \tag{17}$$

We suppose that J_2 is larger than J_1 and J_3 for $l\geqslant 1$. It is the only integral in which the potential overlaps one of the wave functions ϕ. We suppose that J_2 is positive for even l and negative for odd l. All this is in accord

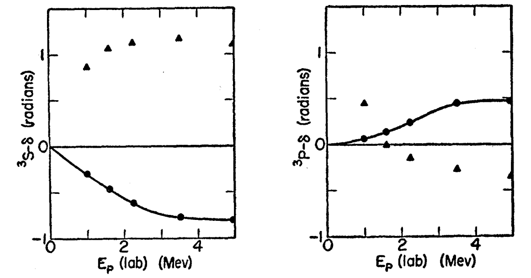

FIG. 1. The ^3S and ^3P phase shifts found for p-He^3 scattering when the ^1S and ^1P phase shifts are set equal to 1.2 times the ^3S and ^3P shifts respectively (Born approximation). The circles represent an acceptable set of solutions the triangles an unacceptable set.

with what is known about the corresponding integrals which occur in the theory of p-d and n-d scattering.

The consequences of these assumptions and Eqs. (12)–(15) are the following. For p-He^3, the S phase shift should be negative and the P phase shifts positive. For p-T, the ^3P phase shift should be positive and the ^1P phase shift should be negative, and these should have the same magnitude:

$$\delta(^3P)=-\delta(^1P). \qquad (18)$$

$\delta(^3S)$ should be negative, and $\delta(^1S)$ should be positive. We might expect a very strong attraction in the ^1S state corresponding to the strong binding of He^4 [$\delta(^1S)$ should be large and positive].

Except for Coulomb effects, the triplet δ's should be equal for p-T and p-He^3.

III. PHASE SHIFT ANALYSIS INCLUDING SOME OF THE QUALITATIVE FEATURES OF BORN'S APPROXIMATION

We have made a phase shift analysis of experimental p-T data[3,4] and p-He^3 data,[5,6] using an IBM 701 calculator.

The code was designed to allow for spin-orbit splitting.

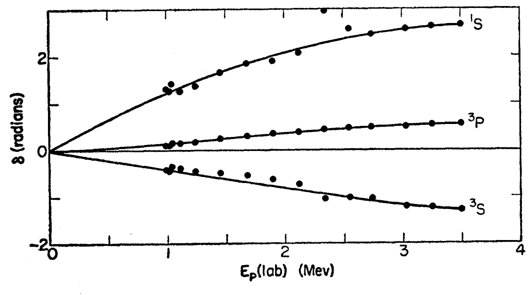

FIG. 2. The ^1S, ^3S, and ^3P phase shifts found for p-T scattering when the ^1P phase shift is the negative of the ^3P shift. Several additional possible phase shifts were found at each energy; however, only the set shown has an acceptable energy dependence.

When spin-orbit splitting is allowed, six parameters ($^1S_0$,$^1P_1$,$^3S_1$,$^3P_0$,$^3P_1$,$^3P_2$) are available to fit the data even when l is restricted to $l\leqslant 1$. A confusion of solutions results, none of which suggests anything (to us anyway) in terms of resonances. We therefore gave up trying to find out about possible spin-orbit splitting in the problem.

This reduces the number of parameters to four (^1S,^1P,^3S,^3P), and several solutions are found at each energy. The phase shifts must be reasonable functions of energy, and this eliminates most of the solutions. The results of these analyses with no further restrictions imposed are given in the last section.

We find it possible to impose the condition Eq. (16) on both the S and P phase shifts as an auxiliary condition on the analysis of the p-He^3 data and still get good fits. This can be anticipated in view of Lowen's results.

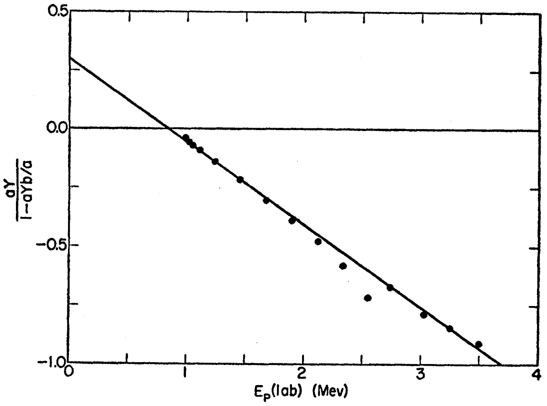

FIG. 3. Plot of $aY/(1-bY)$ vs E for the $^1S_0$ p-T phase shift. b/a is taken to be 1.5, with a equal to 3.0×10^{-13} cm. The slope of the line gives a value of $\gamma_\lambda^2/a=2.85$ Mev and the intercept is $E_\lambda=0.84$ Mev.

The δ's which result when this is done are shown in Fig. 1 as a function of energy.

The condition Eq. (18) can be imposed on the P phase shifts in the p-T case. For the S phase shifts, it is reasonable to impose Eq. (16) in the p-He^3 case but not in the p-T case. The reasons are as follows. Both S phase shifts are repulsive in the p-He^3 case, but the $\delta(^1S)$ is attractive in the p-T case. Born's approximation may be valid for small negative δ's (repulsive), but almost certainly not for large positive δ's (attractive). Even more important, Eq. (18) depends on the assumption that J_2 dominates J_1 and J_3, which is probably not correct for $l=0$, but Eq. (16) depends on no such assumption. The δ's which result when Eq. (18) is imposed on the P-δ's are shown in Fig. 2 as a function of energy.

All of the predictions of Born's approximation are satisfied by these solutions. All of the phase shifts have the expected sign. [It should be noted that in imposing Eqs. (16) and (18) we do not fix the sign of any phase shifts, but only ratios of phase shifts.] $\delta(^3P)$ for p-T

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[3] M. E. Ennis and A. Hemmendinger, Phys. Rev. **95**, 772 (1954).

[4] Classen, Brown, Frier, and Stratton, Phys. Rev. **82**, 589 (1951).

[5] Famularo, Brown, Holmgren, and Stratton, Phys. Rev. **93**, 928(A) (1954).

[6] D. R. Sweetman (private communication).

scattering is slightly larger than the $\delta(^3P)$ for p-He^3, as it should be in view of the stronger Coulomb repulsion for p-He^3. The ^3S phase shifts for p-T and p-He^3 are about equal in magnitude.

IV. RESONANCES

The repulsive S phase shifts found in the phase shift analyses including the predictions of Born's approximation are about those for scattering from a hard sphere of radius 3×10^{-13} cm, as shown in Figs. 1 and 2.

According to Wigner,[7] the reciprocal of the logarithmic derivative evaluated at some radius should have the following dependence on energy:

$$R=\sum_\lambda \frac{\gamma_\lambda^2}{E_\lambda-E}. \tag{19}$$

TABLE I. Resonance parameters for the ^1S state in p-T.

a		$b/a=1.50$			$b=4.5\times10^{-13}$ cm	
cm	E_λ Mev	b/a	γ_λ^2/a Mev	E_λ Mev	b/a	γ_λ^2/a Mev
2.6×10^{-13}	0.84	1.50	3.13	0.84	1.73	3.51
3.0×10^{-13}	0.84	1.50	2.85	0.84	1.50	2.85
3.4×10^{-13}	0.84	1.50	2.50	0.84	1.32	2.02

Only one E_λ is likely to lie in the neighborhood of a narrow range of energies (such as we have here), so that

$$R=\frac{\gamma_\lambda^2}{E_\lambda-E}+b, \tag{20}$$

where E_λ is the nearby resonance and b represents the effects of all distant resonances. Thus

$$\frac{aY}{1-(b/a)(aY)}=\frac{E_\lambda-E}{\gamma_\lambda^2/a}, \tag{21}$$

where $Y=1/R$ and a is the radius at which the logarithmic derivative is evaluated. To evaluate the logarithmic derivative, we use

$$aY=\rho\frac{F'+G'\tan\delta}{F+G\tan\delta}, \tag{22}$$

where F and G are the regular and irregular Coulomb functions, F' and G' their derivatives, $\rho=ka$, and δ is the phase shift.

For the ^1S state in p-T a plot of the left hand side of Eq. (21) *vs* E is shown in Fig. 3 for a radius of 3×10^{-13} cm. The quantity b/a is adjusted to make the plot a straight line. From this it is possible to determine E_λ and γ_λ^2/a. Two other values of the radius were tried; these are 2.6 and 3.4×10^{-13} cm. The plots are not extremely sensitive to the value of b/a and so two prescriptions were tried for this parameter. The first was to keep b/a the same as that used for $a=3.0\times10^{-13}$, the second to keep b the same. The results are shown in Table I. It will be noted that the value of E_λ is found to be independent of both a and b.

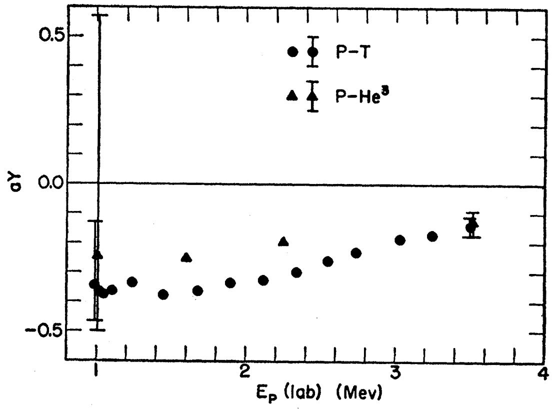

FIG. 4. aY is plotted against E for the ^3P phase shift for p-He^3 and p-T. The vertical lines at the highest and lowest energies show the effect on aY of changing the phase shift by ±2.0 degrees.

Plots of aY *vs* E are shown in Fig. 4 for the attractive P-states in p-T and p-He^3. It is not possible to say much in terms of resonances. Probably the best thing is to say that the levels are all distant and aY is just the negative constant read off the graph. This means that there is at least one negative E_λ, and the corresponding level is broad.

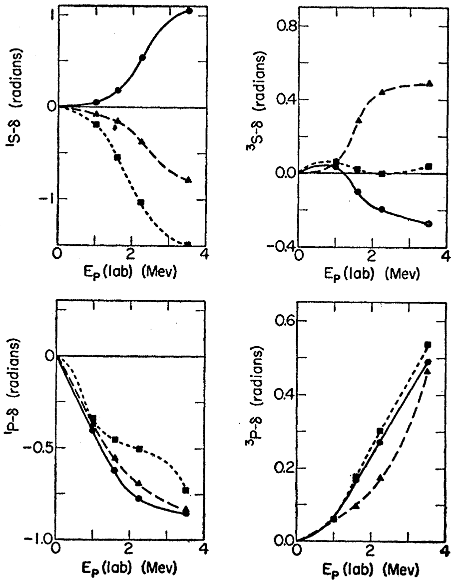

FIG. 5. The ^1S, ^3S, ^1P, and ^3P phase shifts found for p-He^3 scattering when the only restriction placed on the phase shifts is that there is no spin-orbit splitting. In addition to the three sets shown several more possible sets of phase shifts are found at each energy; however, these do not have a reasonable energy dependence.

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[7] E. P. Wigner, Proc. Cambridge Phil. Soc. 47, 790 (1951).

V. OTHER SOLUTIONS FOUND IN THE PHASE SHIFT ANALYSIS

Even in the cases where we impose the predictions of Born's approximation on the phase shifts, we find several solutions at each energy all of which fit the cross sections within the experimental error. Only one set of these has a reasonable energy dependence as shown in Figs. 1 and 2. However, if we use as a restriction only that all ^3P phase shifts shall be equal (that is, no spin-orbit splitting), we find a large number of possible solutions at each energy. For the case of p-He^3 scattering, those solutions which have a reasonable energy dependence are shown in Fig. 5. The solutions which do not have a reasonable energy dependence are not shown to keep the figure readable.

Since good agreement with experiment was found by using the Born approximation, no attempt was made in the p-T case to find solutions with other types of restrictions on the δ's.

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Particle Binding Energies and the Diffuseness of the Nuclear Boundary*

Alex E. S. Green
The Florida State University, Tallahassee, Florida

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The approximate eigenvalues for a spherical well with an exponentially diffuse boundary and with spin-orbit splittings are applied to the study of particle binding energies. If the A-value locations of the low velocity $3s$ and $4s$ maxima in the neutron cross-section surface are taken at $A=55$ and $A=150$, the general trends of experimental binding energies and experimental radii sharply restrict the degrees of diffuseness that can be allowed. It would appear that the general trends of both proton and neutron binding energies as well as their discontinuities can be accounted for if a diffuseness parameter (i.e., tail length to e^{-1} point divided by inner radius) is chosen which drifts gradually from $\delta=0.3$ for light nuclei to $\delta=0.2$ for heavy nuclei. The diffuseness parameter needed goes to somewhat smaller values ($\delta\sim 0.13$) if the critical $4s$ A-value is taken at 170. For heavy elements the diffuseness of the potential well obtained here is comparable to the diffuseness of the nuclear charge distribution obtained in recent studies.

1. INTRODUCTION

THE approximate eigenvalues for a spherical well with an exponentially diffuse boundary have been obtained in a previous study.[1] To apply these results to the investigation of particle binding energies in complex nuclei one must first determine the magnitudes of the well strength parameter ϵ_0 and the radius parameter a for various values of A. It should be obvious that for a given A, as the diffuseness parameter δ is increased, the well strength parameter ϵ_0 and the radius parameter a needed to account for specific experimental observations will grow smaller. The precise relationship $\epsilon_0(\delta,A)$ and $a(\delta,A)$ will depend upon the particular experimental observations which are taken as standard. Let us now consider a basis for arriving at these functions.

2. IDENTIFICATION OF THE WELL PARAMETERS

The particle mass used in this work is taken to be the average of the neutron and proton, i.e.,

$$m=\tfrac{1}{2}(m_n+m_p)=1008.288 \text{ mMU}. \qquad (1)$$

Since the independent-particle model (I.P.M.) places each particle in a field of force set up by the $A-1$ other particles, the natural energy unit takes the form

$$E_0=\hbar^2/2\mu a^2=U_0[1+1/(A-1)]/a^2(\delta,A), \qquad (2)$$

where

$$U_0=22.267 \text{ mMU}=20.734 \text{ Mev}, \qquad (3)$$

and $a(\delta,A)$ is a dimensionless distance parameter obtained by dividing a itself by 1×10^{-13} cm. The dimensionless well strength parameter is now given by

$$\epsilon_0=(V_0/E_0)^{\frac{1}{2}}=(V_0/U_0)^{\frac{1}{2}}a(\delta,A)/[1+1/(A-1)]^{\frac{1}{2}}, \qquad (4)$$

where the constant V_0 represents the depth of the uniform portion of the potential function. It is not unreasonable to expect V_0 to be a universal constant which measures the strength of the interaction of a single nucleon in condensed nuclear matter. Accordingly $\epsilon_0(\delta,A)$, apart from the small reduced mass effect, is expected to vary simply as $a(\delta,A)$. To proceed further, it shall be assumed that[2]

$$\epsilon_0(\delta,A)=f_1(\delta)A^{\frac{1}{3}}+f_2(\delta), \qquad (5)$$

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* This work is supported by a contract with the U. S. Atomic Energy Commission.

[1] A. E. S. Green and Kiuck Lee, Phys. Rev. 99, 772 (1955). This work will henceforth be called GLI.

[2] It would perhaps be more desirable to let $a(\delta,A)$ be given by Eq. (5) and to let $\epsilon_0(\delta,A)$ embody the reduced mass effect. Equation (5) however, was chosen because it brings the $2s$ resonance closer to $A=11$.