Intermediate Coupling Shell Model for Be^9†

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Energy levels, nuclear moments, and relative deuteron reaction cross sections are calculated for Be^9 using a Rosenfeld central and a one-particle spin-orbit interaction. The level structure and the magnetic dipole moment are sensitive functions of the magnitude of the spin-orbit interaction. The best fit to the experimentally found spectrum is given at a value $a/K=1.7$ for the intermediate coupling parameter of Inglis and at $K=1.33$ Mev for the Slater integral parameter. The 2.43-Mev state, in particular, is predicted to be $5/2-$. The observed magnetic moment is fitted either in extreme jj coupling or once again at $a/K=1.7$. Certain relative cross sections in the reactions $B^{10}(n,d)Be^9$ and $B^{10}(p,d)B^9$ are calculated and found to be in agreement with experiment.

1. INTRODUCTION

BECAUSE of the low dissociation energy of Be^9, a simple single-particle model has often been favored for its ground state.[1] This model finds the unpaired neutron moving in a predominantly central field supplied by the other nucleons, which themselves form an inert ${}^1S_0$ core. The same model has also been applied to the 2.43 Mev level, assigned a $d_{\frac{5}{2}}$ character by Longmire.[2] Later measurements[3] have indicated a width for this level which cannot be reconciled with a $d_{\frac{5}{2}}$ assignment, and recent determinations[4,5] of negative parity both for this level and the corresponding 2.37-Mev level in B^9 exclude such an assignment. Recently, moreover, several other low-lying levels in Be^9 have been found,[6] and by now the spectrum is so complex that a one-particle model is quite inadequate for its description. A more complex model in which a neutron moves in the field of two alpha particles has been considered by Haefner,[7] who predicts $5/2-$ for the 2.43-Mev level. The full consequences of this "molecular" model have not been worked out.

In this paper we give the results of an intermediate-coupling shell-model calculation. From the standpoint of energy levels this work is a refinement of the work of Inglis,[8] who drew curves of the intermediate-coupling behavior of energy levels by extrapolating between LS and jj limits. We have instead explicitly calculated energy levels and wave functions in the intermediate-coupling region. These wave functions have been used for the calculation of magnetic dipole and electric quadrupole moments, and also for the calculation of cross-section ratios for different excitations of the final-state nucleus in the reaction $B^{10}(n,d)Be^9$ and its analog $B^{10}(p,d)B^9$. For the latter calculation ground-state wave functions for B^{10} are needed, and these wave functions we use also in investigating the moments of B^{10} in intermediate coupling.

Before discussing the explicit calculations we should remark that while, following Inglis, we have been willing to consider a shell model even for quite highly excited states, there is evidence that for some purposes this model may be inadequate. For example, many cases have been reported[6] where, in inelastic scattering even of high-energy particles, some of the low-lying levels are not observed, although the ground state and the 2.43-Mev level are always seen. Of course such processes involve highly excited states of an intermediate nucleus. Calculations for such reactions are beyond the scope of the present work, but it seems questionable whether these results can be reconciled with the use of shell-model functions for all the states occurring.

TABLE I. Examples of normalized wave functions for the lowest $T=1/2$ states of Be^9 with the J values shown. The numbers listed are the amplitudes for the multiplets in the first column. (For $J=3/2$, $\zeta=\infty$, the exact values are of the form $\pm(x/540)^{\frac{1}{2}}$, where $x=$integer.)

	$J=3/2$			$J=5/2$		$J=1/2$
$LST\alpha$	$\zeta=1.4$	$\zeta=5.7$	$\zeta=\infty$	$\zeta=1.4$	$\zeta=5.7$	$\zeta=1.4$
${}^{22}P(41)$	0.929	0.756	0.471	···	···	0.991
${}^{22}D(41)$	−0.335	−0.466	−0.322	0.950	0.732	···
${}^{22}F(41)$	···	···	···	0.262	0.500	···
${}^{22}G(41)$	···	···	···	···	···	···
${}^{22}P(32)$	0.037	−0.074	−0.236	···	···	−0.033
${}^{22}D(32)$	0.078	0.252	0.385	0.066	0.168	···
${}^{22}F(32)$	···	···	···	−0.062	−0.158	···
${}^{24}P(32)$	0.090	0.237	0.224	−0.098	−0.221	0.061
${}^{24}D(32)$	0.045	0.088	0.136	0.075	0.192	0.102
${}^{24}F(32)$	0.031	0.140	0.228	−0.027	0.047	···
${}^{22}S(311)$	···	···	···	···	···	−0.033
${}^{24}S(311)$	−0.066	−0.203	−0.430	···	···	···
${}^{22}D(311)$	0.003	0.008	−0.086	0.001	−0.097	···
${}^{24}D(311)$	0.033	0.096	0.136	−0.068	−0.220	0.026
${}^{22}P(221)$	0.018	−0.042	−0.304	···	···	0.023
${}^{24}P(221)$	0.008	0.033	0.043	0.009	0.107	0.003
${}^{26}P(221)$	0.010	0.077	0.211	−0.006	−0.056	···

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[1] E. Guth, Phys. Rev. 55, 411 (1939); P. Caldirola, Nuovo cimento 4, 39 (1947).
[2] C. Longmire, Phys. Rev. 74, 1773 (1948).
[3] Van Patter, Sperduto, Huang, Strait, and Buechner, Phys. Rev. 81, 233 (1951); Browne, Williamson, Craig, and Donahue, Phys. Rev. 83, 179 (1951).
[4] F. L. Ribe and J. D. Seagrave, Phys. Rev. 94, 934 (1954).
[5] J. B. Reynolds and K. G. Standing, Phys. Rev. 95, 639(A) (1954).
[6] For the experimental data see F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. 27, 77 (1955).
[7] R. R. Haefner, Revs. Modern Phys. 23, 228 (1951).
[8] D. R. Inglis, Revs. Modern Phys. 25, 390 (1953). See also Revs. Modern Phys. 27, 76 (1955).

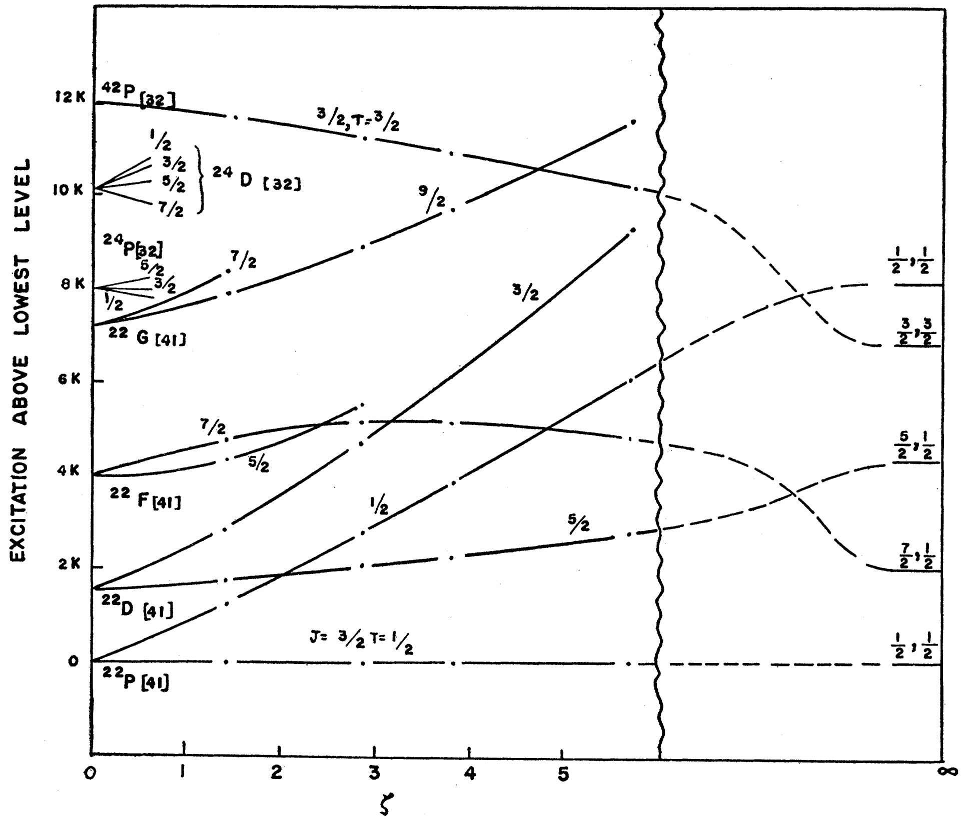

FIG. 1. Intermediate coupling transition of energy levels for the nuclear configuration p^5. Levels have $T=\frac{1}{2}$ unless otherwise specified. Dots indicate values of the intermediate coupling parameter where eigenvalues and eigenfunctions of the interaction energy matrix were calculated. Multiplet labels give the predominant character of the wave functions near the LS limit. The behavior in the jj limit is schematically indicated by the section to the right of the jagged line, where excitations of levels belonging in this limit to the configuration $p_{\frac{3}{2}}{}^5$ are shown.

2. ENERGY LEVELS

We use for Be^9 the configuration p^5 (thus restricting ourselves to the odd-parity levels) and for B^{10} the configuration p^6. In company with many other authors[9] we adopt a combination of the two-particle central interaction of Rosenfeld,[10]

$$H_c(1,2)=\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(0.1+0.23\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)J(r_{12}), \qquad (1)$$

and a one-particle spin-orbit interaction,

$$H_{\mathrm{s.o}}(1)=a(r_1)\mathbf{s}_1\cdot\mathbf{l}_1. \qquad (2)$$

Then $J(r_{12})$ enters only by the two well-known parameters L and K,[11] and we assume $L=6.1K$.[12] Two free parameters remain, namely K and the intermediate coupling parameter $\zeta=a/K$, where a is the average value of $a(r)$.

The general procedure is well known and is described by Auerbach and French.[13] We construct the Hamiltonian matrix for a definite J, T pair and then find its lowest few eigenvalues (energy values) and the corresponding eigenvectors. As in 13, we use LS wave functions for the matrix representation, and consequently the eigenvectors appear as a sum of functions with definite L, S, and space symmetry α. The coefficients in the expansion for a given eigenvector we call $K(JT, \gamma\alpha LS)$, where γ is a parameter distinguishing between two or more multiplets of the same α, L, and S. [This label is needed for the B^{10} functions, where there occur two ^{13}D multiplets with space symmetry characterized by the partition (42).] In order to explore the region between LS and jj coupling, eigenvalues and eigenfunctions are found for matrices corresponding to several values of the parameter. From the purely arithmetical point of view the problems involved are not trival because of the large size of the matrices encountered—up to 13 by 13 in the present case. The matrices themselves along with the eigenvalues, eigenvectors, and other numerical details are contained in unpublished reports.[14] Some wave functions for the lowest $T=\frac{1}{2}$ states of Be^9 are listed in Table I.

Figure 1 shows the energy levels which result from this procedure. The excitation in units of K is plotted *versus* ζ for the levels arising from the first four doublets and also the lowest $T=\frac{3}{2}$ level. Besides these we

[9] For a list of references on intermediate coupling see reference 13 below.

[10] L. Rosenfeld, *Nuclear Forces* (North-Holland Publishing Company, Amsterdam, 1948).

[11] E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937).

[12] H. H. Hummel and D. R. Inglis, Phys. Rev. **77**, 736 (1950).

[13] T. Auerbach and J. B. French, Phys. Rev. **98**, 1276 (1955).

[14] French, Halbert, and Pandya, U. S. Atomic Energy Commission Report NYO-7133, 1955 (unpublished). The matrices, along with those for other p-shell nuclei, are given by M. E. Mandl, U. S. Atomic Energy Commission Report NYO-7135, 1955 (unpublished).

indicate the LS limits and slopes of the other two multiplets which in LS coupling lie below the $T=\frac{3}{2}$ level. In extreme jj coupling ($\zeta=\infty$), the lowest $T=\frac{3}{2}(J=\frac{3}{2})$ level and one $T=\frac{1}{2}$ level each for $J=\frac{1}{2}$, $\frac{3}{2}$, 5/2, and 7/2 belong to the $(p_{\frac{3}{2}})^{-3}$ configuration. The other $T=\frac{1}{2}$ levels belong to a higher configuration and consequently their excitation energy in units of K increases without limit as $\zeta\rightarrow\infty$. The jj energy levels for $(p_{\frac{3}{2}})^{-3}$ have been taken from a paper by Kurath.[15]

Figure 2 shows the known levels up to 15 Mev. The position of the first $T=\frac{3}{2}$ state is not known but a mass comparison with Li^9 indicates its position to be 15 Mev,[16] and we adopt this value. The 2.43-Mev level has negative parity, $\frac{3}{2}\leqslant J\leqslant 9/2$,[4] and a width $\Gamma\leqslant 3$ kev.[3] For the other levels we have Γ(4.8 Mev)$\sim$1 Mev, Γ(1.8 Mev)$<$400 kev, Γ(3.1 Mev)$<$300 kev.[17]

Since only one J value and two parity values are known, one needs a certain amount of optimism in specifying a best value of ζ as determined by comparison of the curves in Fig. 1 with the known spectrum. But if we assume that the 1.8-Mev level has negative parity (and that there are no undiscovered negative parity levels below 2.43 Mev), then a value for ζ of about 1.7 is the only choice which can give reasonably good correspondence between our theory and the observed spectrum. The other alternative would be to place the lowest $\frac{1}{2}-$ level above the 2.43-Mev level, but in this case we find rather poor agreement with the observed spectrum. Besides, as will be seen later, the magnetic moment calculation also favors a small value of ζ. We thus adopt the value $\zeta=1.7$ and then, determining K to fit the 2.43-Mev level, make the comparison of theory with experiment as given in Fig. 2.

Our choice of parameters satisfactorily predicts the first $T=\frac{3}{2}$ level at 15.2 Mev,[18] a $\frac{3}{2}-$ level at 4.2 Mev which we would associate with the observed 4.8-Mev level, and above this two levels arising from the inverted F doublet which could correspond to the observed levels at 6.8 and 7.9 Mev. The latter correspondence is not particularly good and could perhaps be improved by variation of some of the parameters; on the other hand many effects, in particular interaction with other configurations, can seriously perturb the positions of higher levels for a given T value. On this account too (and also because it is probable that all the high levels below 15 Mev are not known), we prefer not to take seriously the predicted levels which arise from the three highest $T=\frac{1}{2}$ multiplets of Fig. 1. We point out too that we have omitted consideration of the Ehrman-Thomas[19] correction to the level positions, and this could be of consequence expecially for the broad levels. The one major discrepancy between prediction and observation is that we do not predict a level at 3.1 Mev; this level would then presumably be a positive parity state not belonging to the p^5 configuration. As a curiosity we point out that if later measurements do indeed indicate a coupling scheme close to the LS limit as above, it could be particularly interesting to see whether the F doublet levels are inverted, for probably not all forms of spin orbit coupling will predict this inversion.

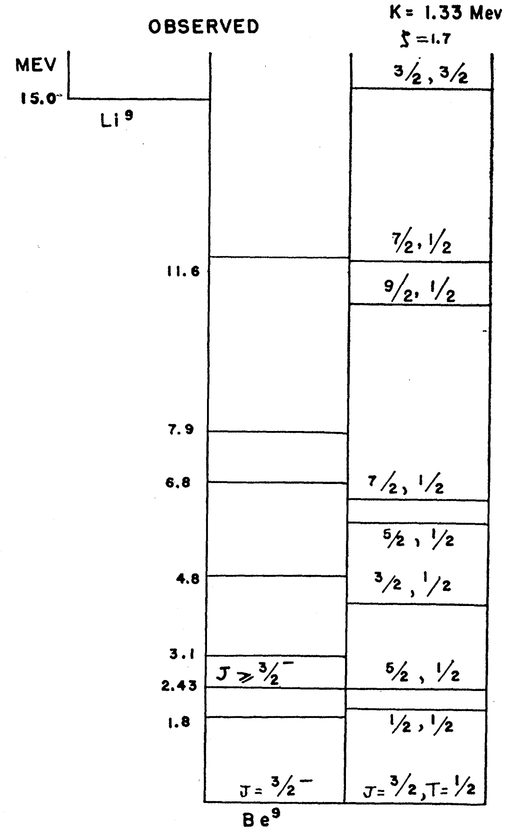

FIG. 2. Comparison of the observed level scheme with the theoretical spectrum for $\zeta=1.7$ and $K=1.33$ Mev.

None of the assignments made above is incompatible with known data about the level widths. The 2.43-Mev level would decay by $l=3$ neutrons to the ground state of Be^8 with a maximum Wigner limit width[20] of about 1 kev. If the shell model is at all valid, we would in fact expect an actual width much less than this because of the small probability of finding an f neutron in Be^9. It is difficult to give limiting values for the widths of the 1.8-Mev and 4.8-Mev levels, for in the first case the available energy of decay to the ground state of Be^8 is comparable with the level width, and in the second case the same is true for decay to the 2.90-Mev level in Be^8. Rough calculation, however, indicates that

[15] D. Kurath, Phys. Rev. **88**, 804 (1952).

[16] F. Ajzenberg and T. Lauritsen, Quarterly Progress Report No. 4, Appendix B, Boston University, Department of Physics, September 1954 (unpublished).

[17] Moak, Good, and Kunz, Phys. Rev. **96**, 1363 (1954); Almqvist, Allen, and Bigham, Bull. Am. Phys. Soc. **30**, No. 3, 31 (1955).

[18] We are indebted to Dr. D. R. Inglis for suggesting that we should consider the first $T=3/2$ state.

[19] J. B. Ehrman, Phys. Rev. **81**, 412 (1951); R. G. Thomas, Phys. Rev. **88**, 1109 (1952).

[20] E. P. Wigner, Am. J. Phys. **17**, 99 (1949).

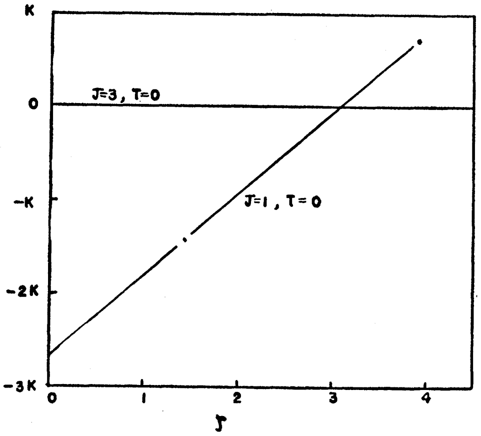

FIG. 3. Excitation of lowest $J=1$ level above lowest $J=3$ level for B^{10} in intermediate coupling.

the assignment given is satisfactory for the 4.8-Mev level and probably so for the 1.8-Mev level.

We remark now on the magnitude of the Slater integral parameter K. At $\zeta=1.7$, a value $K=1.33$-Mev places the lowest $J=5/2$ level at 2.43 Mev. The values of K found by Inglis for nuclei with $A=6$, 7, 8, 10 are in every case between 1.2 and 1.4 Mev, so that we have here satisfactory agreement. We note that, if one assumes that the Li^9-Be^9 mass difference properly predicts the position of the first $T=\frac{3}{2}$ level, an interpretation of Be^9 near the jj coupling extreme is unfavored because to fit a $T=\frac{3}{2}$ state at 15 Mev there, requires a large value of K (2.2 Mev in the jj limit). This last point will be of interest later in connection with the discussion of magnetic moments.

For analysis of the deuteron reactions, the wave function for the ground state of B^{10} will be needed. We use here the same interaction and the same values of L and K as for Be^9, and consider the first two states of B^{10}. As has been emphasized by Zeldes,[21] who has given explicit calculations for these two levels, there is a critical value of ζ below which the theoretical ground state has $J=1$, in contradiction with the experimental determination of $J=3$. From the plot in Fig. 3 of the excitation of the $J=1$, $T=0$ level we find this critical value to be $\zeta=3.0$. The experimental value, 0.72 Mev, of the excitation is fitted at $\zeta=3.6$.

3. MAGNETIC DIPOLE AND ELECTRIC QUADRUPOLE MOMENTS

The magnetic moment may be written as the sum of an orbital part and a spin part. For the configuration l^n these terms are given in terms of Racah coefficients $W(abcd;ef)$, coefficients of fractional parentage

$$\langle\gamma\alpha LST|\gamma_1\alpha_1L_1S_1T_1\rangle$$

and Clebsch-Gordan coefficients $(j_1j_2m_1m_2|jm)$ by

$$\begin{aligned}\mu_L=&\tfrac{1}{2}n\mu_0(J+1)^{-1}(-1)^{l-J}[l(l+1)(2l+1)\\&\times J(J+1)(2J+1)]^{\frac{1}{2}}\sum_{\substack{\gamma\alpha LS\\\gamma'\alpha'L'S'}}K(JT,\gamma\alpha LS)\\&\times K(JT,\gamma'\alpha'L'S')\delta_{SS'}(-1)^S[(2L+1)(2L'+1)]^{\frac{1}{2}}\\&\times W(JJLL';1S)\sum_{\gamma_1\alpha_1L_1S_1T_1}\langle\gamma\alpha LST|\gamma_1\alpha_1L_1S_1T_1\rangle\\&\times\langle\gamma'\alpha'L'S'T|\gamma_1\alpha_1L_1S_1T_1\rangle(-1)^{L_1}\\&\times W(llLL';1L_1)[1+F(T,T_z,T_1)],\end{aligned}\quad(3)$$

$$\begin{aligned}\mu_S=&\tfrac{1}{2}n\mu_0(J+1)^{-1}(-1)^{J+\frac{1}{2}}[\tfrac{3}{2}J(J+1)(2J+1)]^{\frac{1}{2}}\\&\times\sum_{\substack{\gamma\alpha LS\\\gamma'\alpha'L'S'}}K(JT,\gamma\alpha LS)K(JT,\gamma'\alpha'L'S')\delta_{LL'}\\&\times(-1)^{S+S'+L}[(2S+1)(2S'+1)]^{\frac{1}{2}}W(JJSS';1L)\\&\times\sum_{\gamma_1\alpha_1L_1S_1T_1}\langle\gamma\alpha LST|\gamma_1\alpha_1L_1S_1T_1\rangle\\&\times\langle\gamma'\alpha'L'S'T|\gamma_1\alpha_1L_1S_1T_1\rangle(-1)^{-S_1}\\&\times W(\tfrac{1}{2}\tfrac{1}{2}SS';1S_1)[(g_n+g_p)\\&-(g_n-g_p)F(T,T_z,T_1)],\end{aligned}\quad(4)$$

where we use a positive T_z to indicate proton excess and

$$\begin{aligned}F(T,T_z,T_1)=&(-1)^{T_1+T_z-\frac{1}{2}}(2T+1)\\&\times\sqrt{2}(TT-T_zT_z|10)W(\tfrac{1}{2}\tfrac{1}{2}TT;1T_1).\end{aligned}\quad(5)$$

Here g_p and g_n are the nucleon gyromagnetic ratios. Expressions which are equivalent to the above have already been given by Lane.[22] The fractional parentage sums may in fact be carried out immediately for those terms which do not have F as a factor. These terms, arising from the isotopic spin independent part of the magnetic dipole operator and therefore depending upon the expectation value of L_z and S_z, are calculable immediately in terms of a Landé g factor. They contribute to μ a part

$$\begin{aligned}&\tfrac{1}{4}\mu_0(1+g_n+g_p)J+\tfrac{1}{4}(J+1)^{-1}(1-g_n-g_p)\\&\times\sum_{\gamma\alpha LS}[K(JT,\gamma\alpha LS)]^2[L(L+1)-S(S+1)].\end{aligned}\quad(6)$$

Since $F(0,0,T_1)=0$, the expression (6) is the total moment for $T=0$ levels.

TABLE II. Quadrupole moments in units of $e\langle r_i^2\rangle$ for Be^9 and B^{10} in intermediate coupling.

ζ =	0	1.4	2.8	3.8	5.7	∞
Be^9	6/25	0.43	0.43	0.41	0.39	4/15
B^{10}	0	0.71	0.71	0.68	0.64	2/5

[21] N. Zeldes, Phys. Rev. **90**, 416 (1953).

[22] A. M. Lane, Proc. Phys. Soc. (London) **A66**, 977 (1953).

Figure 4 shows the variation of μ with ζ for Be^9. For B^{10} the values of μ/μ_0 obtained for the $J=3$, $T=0$ state at $\zeta=0$, 1.4, 2.8, 3.8, 5.7, and ∞ are 1.88, 1.85, 1.81, 1.79, 1.78, and 1.88 respectively. (Since the $J=1$ state lies lowest for $\zeta<3$, the first three values listed are of academic interest only.) Here μ is not a sufficiently sensitive function to determine an optimum value of ζ, but the value of 3.6 which we have adopted for B^{10} does give satisfactory agreement with the experimentally observed $\mu=1.80\mu_0$.[23] For Be^9, Fig. 4 shows that μ is quite sensitive to ζ: we may take ζ either quite large (close to jj coupling) or once again about 1.7, which we already favor for the level structure. The Be^9 magnetic moment is a good example of the fact that LS and jj values for μ and other quantities do not always give a trustworthy indication of the actual coupling scheme.

The quadrupole moment for the configuration l^n is given by

$$
\begin{aligned}
Q=\tfrac{1}{5}ne\langle r_l^2\rangle(2l+1)(2J+1)(ll00|20)(JJ-JJ|20)\\
\times\sum_{\substack{\gamma\alpha LS\\ \gamma'\alpha'L'S'}} K(JT,\gamma\alpha LS)K(JT,\gamma'\alpha'L'S')(-1)^S\\
\times\delta_{SS'}[(2L+1)(2L'+1)]^{\frac{1}{2}}W(JJLL':2S)\\
\times\sum_{\gamma_1\alpha_1L_1S_1T_1}\langle\gamma\alpha LST|\gamma_1\alpha_1L_1S_1T_1\rangle\\
\times\langle\gamma'\alpha'L'ST|\gamma_1\alpha_1L_1S_1T_1\rangle(-1)^{L_1}W(llLL':2L_1)\\
\times[1+F(T,T_z,T_1)], \quad (7)
\end{aligned}
$$

where e is the protonic charge and $\langle r_l^2\rangle$ is the average square of the radial distance for a single particle with angular momentum l. The experimental values given (in units of $e\times10^{-24}$ cm^2) are 0.02 for the Be^9 nucleus[24] and 0.06 for the B^{10} nucleus,[25] but in each case the uncertainties are very large. Table II gives in units of $e\langle r_l^2\rangle$ the calculated results for both nuclei, where as

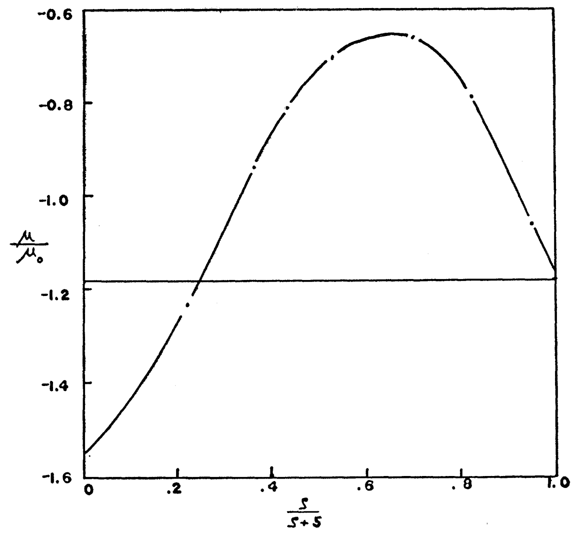

FIG. 4. Magnetic dipole moment of Be^9 in intermediate coupling. The theoretical value of μ/μ_0 is -1.18. A satisfactory fit is found at $\zeta=1.7$ or near $\zeta=\infty$.

before we consider always the $J=3$, $T=0$ state for B^{10}. For any reasonable value of $\langle r_l^2\rangle$, we have results not in disagreement with experiment for the values of ζ listed.

Finally we remark (once again as a matter of academic interest) that the zero value for Q of B^{10} in the LS limit comes about because the p shell is half-filled in this case. However, for $J=3$, $T=0$ the p^6 configuration has two multiplets of the type $^{13}D(42)$, and for our choice of the central force these are almost degenerate in the LS limit, being separated by an amount $0.008K$. This has the consequence that a very small admixture of spin-orbit force changes the character of the wave function radically, and with it the size of the quadrupole moment.

TABLE III. S = the factor by which the single-particle deuteron pickup cross section increases on considering all the p-shell nucleons. S is for the ground state reaction and S^* for the reaction to the lowest $J=5/2-$ state, which in Be^9 we associate with the 2.43-Mev level. The factors are given for various values of the intermediate coupling constants of both nuclei. For $\zeta(Be^9)=1.4$, $\zeta(B^{10})=3.8$, we have also, for the reaction to the second $J=3/2-$ state, $S^{**}=0.035$.

$\zeta(Be^9)$	$\zeta(B^{10})$	S	S^*	S/S^*
0	0	0.0012	1.84	0.0007
1.4	1.4	1.18	1.35	0.87
2.8	2.8	1.12	1.24	0.90
3.8	3.8	1.09	1.24	0.88
5.7	5.7	1.03	1.26	0.82
1.4	3.8	1.03	0.95	1.08

4. DEUTERON REACTIONS

The relative cross sections for deuteron pickup or stripping reactions from a single target nucleus to a series of final levels is in some cases calculable as a function of the intermediate coupling parameters of the two nuclei involved.[13,20] Using the procedures of reference 13 we consider the pick-up reaction from the lowest $J=3$, $T=0$ state of B^{10} to the lowest $J=\frac{3}{2}$ and $J=5/2$ levels of Be^9 or B^9. We define S (or S^* for the excited state) as the factor by which the cross section for the single particle model increases when we take account of the many particle nature of the wave functions involved. In the notation of reference 13, we have $S=n\sum_z\beta_z^2$, and an explicit form is given there.

Values of S and S^* are given in Table III for several choices of the intermediate coupling parameters. Near the LS limit, S varies very rapidly with $\zeta(B^{10})$ because, as discussed earlier, the composition of the B^{10} wave function near the LS limit varies very rapidly. Apart

[23] For the references to the experiments on the magnetic moments see the article by N. Ramsay in E. Segrè, *Experimental Nuclear Physics* (John Wiley & Sons, Inc., New York, 1953), Vol. 1.
[24] Gordy, Ring, and Burg, Phys. Rev. **78**, 512 (1950).
[25] W. D. Knight, Phys. Rev. **92**, 539A (1953).

from this, it is clear that S/S^* is not a sensitive function of the two ζ's. The most reasonable way, then, to make use of the information on deuteron reactions is simply to compare the experimental results with the predictions at the ζ values which are already favored by other considerations. The last entry in Table III is for the ζ values 1.4 and 3.8, which are the closest to the favored values for which we have explicit wave functions.

There are two experiments available. The $B^{10}(n,d)Be^9$ experiment with 14-Mev neutrons has been done by Ribe and Seagrave,[4] who measure the differential cross sections to the ground state and to the 2.43-Mev state. Choosing $r_0=4.5\times10^{-13}$ cm and $\theta=23°$, and taking account of the kinematic factors, we find $S/S^*=0.62\times(d\sigma/d\sigma^*)$. (This value incidentally is not very sensitive to r_0; it varies about 5% for $4\times10^{-13}\leqslant r_0\leqslant 4.9\times10^{-13}$ cm). From Figs. 6 and 7 of Ribe and Seagrave we then derive as the experimental value $S/S^*=1.0\pm0.15$.

The conjugate experiment $B^{10}(p,d)B^9$ has been done by Reynolds and Standing[26] with 17.5-Mev protons. With $r_0=4.5\times10^{-13}$ cm, as favored by them, we have at 24° the ratio $S/S^*=0.65(d\sigma/d\sigma^*)$; and then, using the experimental ratio, we have $S/S^*=1.08\pm0.13$.

In both cases the errors attached to these values are based on the errors in the values of the experimental cross sections and do not include any estimation of the error arising from the theory (as, for instance, the error involved in ignoring the fact that part of the observed cross section is due to compound nucleus and interference terms).

It is clear that there is satisfactory agreement between the two experiments and with the calculated value $S/S^*=1.08$. There is also available, however, some negative information: in the $B^{10}(n,d)Be^9$ experiment no other level up to 5.5 Mev was observed, and in particular an upper limit of 5% of the ground state reaction cross section is given for the cross section of a reaction terminating in an undetected level near 1.5 Mev. The $\frac{1}{2}-$ assignment given above to the 1.8-Mev level is consistent with this, for the reaction would involve an $l=3$ neutron. The relative magnitude of this reaction is not calculable within the framework of the present theory, which restricts the configuration of the nuclei to p^n and therefore gives nonzero cross sections only for an $l=1$ momentum of the free particle. But since the contribution of functions with mixed configurations like $p^{n-1}f$ is presumably small for low-lying states, a generalization of the present theory would presumably give a very small cross section for a reaction involving an $l=3$ neutron. Similarly, the assignment of positive parity to the 3.1-Mev state is consistent with a very small relative cross section for the reaction leading to it, since an even value of l would be required for the free particle.

There remains then the 4.8-Mev level, which we regard as the second $\frac{3}{2}-$ level and which is not observed in the (n,d) reaction. In this case we find on calculation the very small value 0.035 for S^{**}; we then predict for the relative cross-section for this level compared with the ground state the ratio $d\sigma^{**}/d\sigma=0.29S^{**}/S=0.01$. Thus the present theory satisfactorily predicts that this level should not appear in the pickup reaction.[27]

5. CONCLUSION

The intermediate coupling theory gives a self-consistent picture of Be^9 and enables us to make rather definite predictions concerning the spins and parties of many levels. Rather surprisingly the results seem to favor a coupling scheme which is quite close to the LS limit. One could extend the investigation further, for example by allowing a different ratio between the Slater integrals L and K, by varying the form of the interparticle interaction, or by taking account of other configurations. There seems however, little advantage in such a procedure until it becomes clear whether or not the level scheme which is predicted has any resemblance to the actual scheme. To this purpose measurements of widths, parities, and spins in either Be^9 or B^9 are very badly needed.

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[26] Private communication from J. B. Reynolds. Only a brief abstract concerning this work has been published (see reference 5 above).

[27] This level as well as the levels below it are observed by Almqvist *et al.* (see reference 17 above) in the reaction $B^{10}(t,\alpha)Be^9$ which in the case of a pick-up reaction is analogous to $B^{10}(n,d)Be^9$. However, the observations are made at 90° and it is unknown whether a pickup process is being observed.