To correct this ratio for target thickness, it was assumed that the target consisted of 63 thin lamina. An energy distribution for singly charged particles emitted from a compound sulfur nucleus[8] was attributed to each layer. Using range-energy relations[9] for sulfur, the energy distributions for protons and for deuterons that leave the thick target from each layer were determined. The contributions from all of the layers were combined to give composite energy distributions for the protons and the deuterons that emerge from the thick target.

The cloud chamber physically limited the solid angle available for particles of different ranges and therefore different energies. The range-energy relation for the cloud-chamber vapor was different for protons than for deuterons and gave a different energy range available to be measured for each type of particle. Corrections for these two effects were applied to the theoretical energy distributions of particles from the thick target. The ratio of the number of protons to the number of deuterons in the two respective energy intervals is the correction that was applied to the experimental ratio. The corrected ratio of deuterons to protons from a thin sulfur target when irradiated by 65-Mev bremsstrahlung is 0.15 ± 0.04.

The author wishes to express his gratitude to Dr. L. J. Laslett and Dr. D. J. Zaffarano for many helpful discussions and for advice during the course of this experiment.

[8] V. Weisskopf and D. Ewing, Phys. Rev. **57**, 472 (1940).
[9] J. Lindhard and M. Scharff, Phys. Rev. **85**, 1058 (1952).

PHYSICAL REVIEW VOLUME 99, NUMBER 1 JULY 1, 1955

Elastic Scattering of Protons by F^{19}†

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The differential cross section for the elastic scattering of protons by F^{19} has been measured for proton energies from 550 to 1800 kev at center-of-mass angles of 90, 125.3, and 159.8 degrees and for proton energies from 1300 to 1500 kev at 53.2, 60, 70, 80, 100, 110, and 136 degrees. Marked scattering anomalies were observed for proton energies near 669 (1^+), 843 (0^+), 873 (2^-), 935 (1^+), 1346 (2^-), 1372 (2^-), 1422 (1^+), and 1700 kev. The indicated spin and parity assignments for the corresponding levels in Ne^{20} are required by the results of this experiment or are consistent with them. Observations of the elastic scattering have also been made in the regions of 340 and 480 kev at 159.8 degrees, and no anomaly was observed in either case. The ambiguity in the choice of Γ_p/Γ has been resolved for several of the Ne^{20} levels. An approximate method of correcting the observed cross sections for the effects of finite energy resolution has been developed, and the relative stopping cross section for protons in lithium fluoride has been measured for proton energies from 400 to 1600 kev.

I. INTRODUCTION

THE study of the elastic scattering of protons by F^{19} was undertaken in connection with the recent investigations[1] at this laboratory of the low excited states in F^{19}. The spin and parity assignments for these states as determined from the $F^{19}(p,p'\gamma)$ reaction depend upon the assignments for the Ne^{20} states involved as resonances in the reaction, and the study of $F^{19}(p,p)$ was made to assist in the determination of the Ne^{20} assignments.

In addition to the information regarding spin and parity, the study of the elastic scattering yields useful information in many cases concerning the partial widths of the levels in the compound nucleus. Measurements of the reaction cross sections, $F^{19}(p,p')$ and $F^{19}(p,\alpha)$ and the total width, Γ, permit the determination of the sum and product of the proton width, Γ_p, and the reaction width, $\Gamma_\alpha+\Gamma_{p'}$. The solution of these relations yields two sets of values for Γ_p and $\Gamma_\alpha+\Gamma_{p'}$, and the size of the elastic scattering anomaly at the resonance may usually be used to resolve this ambiguity. A large anomaly generally indicates $\Gamma_p>\Gamma_\alpha+\Gamma_{p'}$ and a small anomaly, $\Gamma_p<\Gamma_\alpha+\Gamma_{p'}$.

The present article describes the experimental procedure and results, and the analysis and interpretation of these data will be discussed in the following paper.[2] Preliminary results of this experiment have been presented to the American Physical Society, and similar measurements and results have recently been reported by Dearnaley.[3]

In the course of this work, it was found desirable to

† Assisted by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.
* Dow Chemical Company Fellow, 1953–1954; International Business Machines Corporation Fellow, 1954–1955.
[1] Peterson, Barnes, Fowler, and Lauritsen, Phys. Rev. **94**, 1075 (1954); Thirion, Barnes, and Lauritsen, Phys. Rev. **94**, 1076 (1954); Sherr, Li, and Christy, Phys. Rev. **94**, 1076 (1954) and **96**, 1258 (1954); R. F. Christy, Phys. Rev. **94**, 1077 (1954); Peterson, Fowler, and Lauritsen, Phys. Rev. **96**, 1250 (1954); and C. A. Barnes, Phys. Rev. **97**, 1226 (1955).
[2] E. Baranger, following paper [Phys. Rev. **99**, 145 (1955)].
[3] Webb, Hagedorn, Fowler, and Lauritsen, Phys. Rev. **96**, 851(A) (1954); G. Dearnaley, Phil. Mag. **45**, 1213 (1954).

investigate the effects of finite energy resolution on the observed cross sections. A discussion of this problem and a description of an approximate correction procedure are given in Appendix B.

II. EXPERIMENTAL PROCEDURE

The protons were accelerated by the 2-Mv electrostatic generator at this laboratory and were analyzed by an 80 degree electrostatic analyzer which maintained the beam homogeneous in energy to 0.05 percent. The proton beam was scattered from a thick LiF target placed at the object position of a 180-degree double-focussing magnetic spectrometer which is mounted to allow a continuously variable scattering angle from 0 to 160 degrees with respect to the incident beam direction. The scattered protons were detected by a zinc sulfide scintillation counter placed at the exit slit of the magnetic spectrometer. The energy scale of the electrostatic analyzer was calibrated by observing the gamma radiation associated with the 873 and 1372-kev resonances[4] in the $F^{19}(p,\alpha\gamma)$ reaction. The magnetic spectrometer energy calibration and the effective solid angle were then determined by observing the protons elastically scattered from copper, assuming pure Rutherford scattering. A detailed description of the calibration procedure is given in Appendix A.

LiF targets were prepared by pressing powdered LiF into a circular recess in a copper backing. The opposite face of the copper was polished so that the target could be rotated through 180 degrees to observe the scattering by copper for energy and solid angle calibration. Some evaporated LiF targets were used but it was found that for targets of sufficient thickness to mask the protons scattered from the copper backing, a few microcoulombs of bombardment would produce cracks in the LiF layer. The behavior of the pressed targets was, in general, satisfactory although new targets frequently gave high and erratic yields for the first one or two bombardments. This effect is probably due to difficulty in charge collection caused by the insulating nature of the target material before bombardment, although it could also be caused by the loss of fluorine from the target due to an initial fluorine excess or decomposition of LiF in the surface layers. As this effect was usually small (5–10 percent) and not reproducible, the equilibrium values, reached after one or two bombardments, were used. In addition to this rapid change, a slow, reproducible decrease in yield with bombardment was observed. This was probably due to decomposition of the LiF and subsequent escape of fluorine or to dilution of the surface layers by carbon and oxygen deposited during bombardment. Corrections were made for this effect by linear extrapolation to zero bombardment. This

[4] Herb, Snowden, and Sala, Phys. Rev. **75**, 246 (1949). The value 1372 kev is the result of recent measurements at this laboratory by Barnes, Mills, and Hilton [C. A. Barnes, Phys. Rev. **97**, 1226 (1955)]. F. S. Mozer of this laboratory and H. B. Willard of Oak Ridge National Laboratory give 1373 kev (private communication). The previously accepted value was 1381 kev.

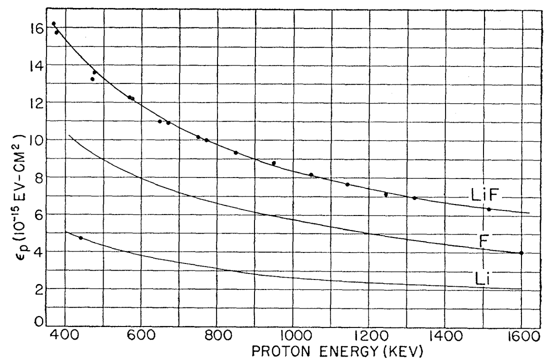

FIG. 1. The stopping cross section for protons (ϵ_p) in fluorine, lithium, and lithium fluoride in units of 10^{-15} ev-cm^2. The relative values for LiF were measured in this experiment and normalized to 8.4×10^{-15} ev-cm^2 at 1 Mev. The values for lithium are from Bader *et al.*[8] and Warters.[7] The values for fluorine are the difference between these.

correction was usually kept to less than 5 percent by frequently changing to new target areas. Attempts were made to improve the target behavior by heating the LiF before pressing, but no noticeable improvement resulted from this treatment. The targets exhibited rapid discoloration on bombardment and would occasionally chip after long use.

Thin target yields were obtained from the thick LiF targets by observing a given interval of the momentum spectrum of the scattered protons using the magnetic spectrometer. This technique has been previously discussed by Brown *et al.*[5] and Snyder *et al.*[6] who give the appropriate expressions for obtaining the cross section and reaction energy.

The calculation of the elastic scattering cross section involves the stopping cross section for protons in the target material, and therefore a determination of this quantity was necessary. The relative stopping cross section was obtained by evaporating a thin layer of LiF on a copper backing and observing the protons elastically scattered from the copper after traversing the LiF layer.[7] The relative values were normalized to 8.42×10^{-15} ev-cm^2 at 1 Mev by assuming the stopping cross section of LiF to be the sum of the stopping cross sections for lithium and fluorine at this energy. The lithium value was obtained from the absolute measurement by Bader *et al.*[8] at 440 kev which was used to normalize the relative values measured by Warters *et al.*[7] The fluorine value was obtained from the measured proton stopping cross section for neon,[9,10] using 4-Mev alpha-particle data for neon[11,12] and

[5] Brown, Snyder, Fowler, and Lauritsen, Phys. Rev. **82**, 159 (1951).
[6] Snyder, Rubin, Fowler, and Lauritsen, Rev. Sci. Instr. **21**, 852 (1950).
[7] Warters, Fowler, and Lauritsen, Phys. Rev. **91**, 917 (1953).
[8] Bader, Wenzel, and Whaling (private communication).
[9] Chilton, Cooper, and Harris, Phys. Rev. **93**, 413 (1953).
[10] Reynolds, Dunbar, Wenzel, and Whaling, Phys. Rev. **92**, 742 (1953).
[11] R. W. Gurney, Proc. Roy. Soc. (London) **A107**, 340 (1925).
[12] G. Mano, Ann. Phys. **1**, 408 (1934).

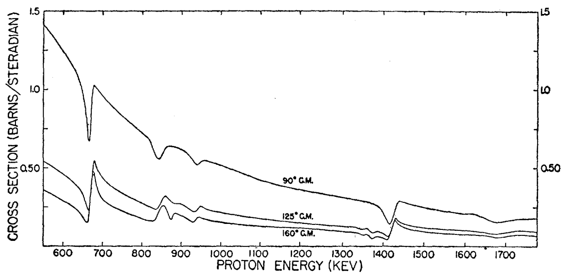

FIG. 2. The differential cross section for $F^{19}(p,p)$ as a function of proton bombarding energy at scattering angles of 90°, 125.3° and 160° n the center-of-mass system.

oxygen[11,13] to determine the value for fluorine relative to neon. This procedure was believed to be more reliable at high energies and for this reason the normalization was made at 1 Mev, the highest energy for which proton data for neon was available. Oxygen[10] and neon[9,10] proton data are both available at 600 kev, and normalization at this point gives values for LiF about 3 percent lower than those used. The final values for LiF are presented in Fig. 1, together with the values for lithium from Warters *et al.*[7] and Bader *et al.*,[8] and the values for fluorine are taken as the difference of these.[14]

The major uncertainty in the cross section values is due to the uncertainty in the value of the LiF stopping cross section which we estimate to be about 5 percent. The probable error in the solid angle (involving the ratio of the elastic scattering cross section of copper to the stopping cross section for protons in copper) is about 3 percent. The uncertainty due to target composition and behavior, estimated from the agreement of results from different targets, is also about 3 percent. This gives a probable error in the absolute values of 6 percent. In addition, the irregularity of the target surface implied by the energy resolution of the experiment (Appendix B) could introduce a systematic error in the cross section values. Rough estimates of the size of this effect indicate that it is negligible at scattering angles larger than 90 degrees but that it could increase the observed yield by 5 to 10 percent at 60 degrees.

The uncertainty in the relative cross sections is about 5 percent arising from the uncertainty in the relative stopping cross sections (4 percent) and the possible variation in the effective solid angle (3 percent). The statistical uncertainty of each point is 1–2 percent, and the current integrator reproducibility is about one percent.

III. EXPERIMENTAL RESULTS

The results of these investigations are presented as the differential cross section in Fig. 2 and as the ratio of the observed cross section to Rutherford cross section in Figs. 3 and 4. The Rutherford differential cross section can be written as

$$\frac{d\sigma_R}{d\Omega_C}=1.296\left(\frac{Z_1Z_0}{E_1}\right)^2\left(\frac{M_1+M_0}{M_0}\right)^2\csc^4\left(\frac{\theta_C}{2}\right)\times10^{-3}\frac{\text{barn}}{\text{sterad}},\quad(1)$$

$$\frac{d\sigma_R}{d\Omega_L}=1.296\left(\frac{Z_1Z_0}{E_1}\right)^2\left[\csc^4\left(\frac{\theta_L}{2}\right)-2\left(\frac{M_1}{M_0}\right)^2+O\left(\frac{M_1}{M_0}\right)^4\right]\times10^{-3}\frac{\text{barn}}{\text{sterad}}.\quad(2)$$

For $F^{19}+p$:

$$\frac{d\sigma_R}{d\Omega_C}=0.1164\left(\frac{1}{E_1}\right)^2\csc^4\left(\frac{\theta_C}{2}\right)\frac{\text{barn}}{\text{sterad}}.\quad(3)$$

In these expressions, E_1, Z_1, and M_1 are the laboratory energy, charge, and mass respectively of the incident particle; Z_0 and M_0 apply to the nucleus at rest, and the subscripts C and L designate measurements in the center-of-mass and laboratory systems, respectively.

The cross section was measured at center-of-mass angles of 90, 125.3, and 159.8 degrees for proton energies from 550 to 1800 kev and at angles of 53.2, 60, 70, 80, 100, 110, 136 degrees for energies from 1300 to 1500 kev. The Legendre polynomials $P_1(\cos\theta)$ and $P_2(\cos\theta)$ vanish at 90 degrees and 125.3 degrees respectively. Pronounced anomalies were observed which we identify with known levels[15] of Ne^{20} at proton energies of 669,

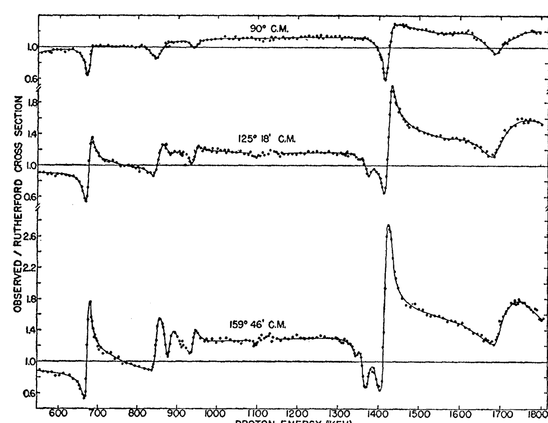

FIG. 3. Ratio of observed cross section to Rutherford cross section for the elastic scattering of protons by F^{19} at scattering angles of 90, 125.3, and 159.8 degrees in the center-of-mass system. In addition to the marked anomalies small structures are observed near 900, 1092, and 1137 kev.

[13] G. E. Gibson and H. Eyring, Phys. Rev. 30, 553 (1927).

[14] We are indebted to W. Whaling for making available to us the collection of stopping cross sections compiled by Fuchs and Whaling, and for many valuable suggestions regarding the determination of the stopping cross section of LiF.

[15] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. 24, 321 (1952) and Revs. Modern Phys. 27, 77 (1955).

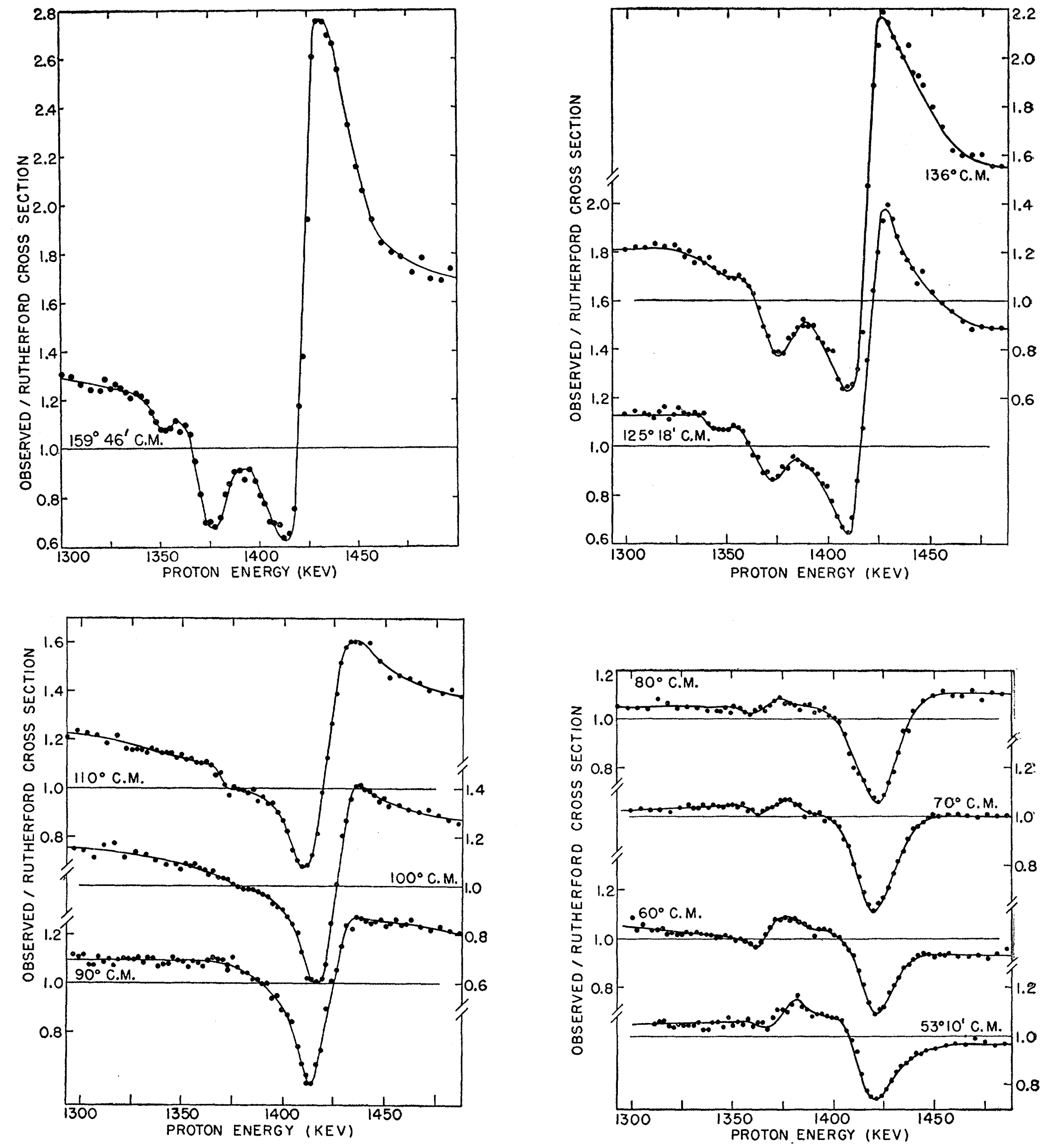

FIG. 4 Ratio of the observed cross section to Rutherford cross section in the energy range 1300 to 1500 kev at ten angles. The anomalies occur near 1346, 1372, and 1422 kev. $F^{19}(p,\alpha\gamma)$ resonance occur at 1346 and 1372 kev and an $F^{19}(p,p'\gamma)$ resonance occurs at 1422 kev. Note the absence of an anomaly near 1346 kev at forward angles and the angular variation of the 1372-kev resonance which disappears at 90 degrees.

843, 873, 935, 1346, 1372, and 1422 kev[16]; in addition a broad structure was observed in the region near 1700 kev which could be associated with one or more reported levels in this region.[15,17] There also appear to be small unresolved structures in the cross-section curve corresponding to resonances at proton energies of 900, 1092, and 1137 kev. In addition to the data shown, some investigations have been made of the regions around 340 and 480 kev. Target deterioration and carbon contamination at these energies cause a much more rapid decrease in yield with bombardment than at higher energies which makes the interpretation of these data difficult. A scattering anomaly of 10 to 20 percent of the Rutherford scattering could have been detected, however, whereas no reproducible structure was observed near either energy. The 340-kev resonance is known to correspond to a Ne^{20} state of $J=1$ and even parity,[15] and the observed reaction cross sections[15] give values of Γ_p/Γ of 0.016 or 1.0. Considering the effects of energy resolution, the larger value would be expected to give rise to an anomaly of 30 to 50 percent

[16] The resonance energies 1346 and 1422 are determined relative to the 1372-kev resonance (see reference 4). The 843-kev anomaly was incorrectly reported by Webb *et al.* and Dearnaley (reference 3) as 831 kev. The assignment 843 kev has been verified by Dearnaley (private communication).

[17] C. A. Barnes, Phys. Rev. **97**, 1226 (1955).

TABLE I. Resonance parameters for $F^{19}(p,p)$. A dash (—) indicates that no anomaly was observed. σ_R=Rutherford cross section.

Resonance energy[a]	Ne^{20*} J, π, l_p	Resonance width[a] (Γ) (kev)	Partial width[a] (Γ_p/Γ)	Angle (c.m.)	σ_{max}/σ_R Theory[c]	σ_{max}/σ_R Observed	σ_{min}/σ_R Theory[c]	σ_{min}/σ_R Observed	$(\sigma_{max}-\sigma_{min})/\sigma_R$ Theory[c]	$(\sigma_{max}-\sigma_{min})/\sigma_R$ Observed	$(\sigma_{max}-\sigma_{min})/\sigma_R$ Corrected[b]	Full window width[b] ($\alpha\Gamma$) (kev) Observed	Calculated Cohen	Calculated Upper limit for surface irregularity
669	1^+			90°	1.04	1.02	0.41	0.65	0.63	0.37	0.61	14.1	4.8	16
	$l_p=0$	7.5	0.98	125°	1.64	1.35	0.42	0.55	1.22	0.80	1.01	11.8	4.1	12
				160°	2.31	1.77	0.49	0.53	1.82	1.21	1.51	10.6	3.6	10
	0^+			90°	1.03	1.07	0.80	0.86	0.23	0.21				
843	$l_p=0$	23	0.996	125°	1.20	1.27	0.72	0.88	0.48	0.39				
				160°	1.54	1.55	0.82	0.89	0.72	0.66				
	2^-			90°		—		—	0.02					
873	$l_p=1$	5.2	0.21	125°		d	d	d	0.26	~0.08[e]				
				160°					0.50	~0.37[e]				
	1^+			90°					0.17	0.10	0.20	17.9	6.5	18
935	$l_p=0$	8.0	0.18	125°	d	d	d	d	0.27	0.22	0.31	15	6.0	16
				160°					0.34	0.28	0.33	9.4	5.2	12
	2^-			≤110°[f]		—		—	≤0.034	—				
1346	$l_p=1$	4.5	0.067	125°	1.10[g]	1.09[g]	1.01	1.07	0.09	0.02				
				160°	1.06[g]	1.12[g]	0.87	1.07	0.19	0.05				
	2^-			70°	1.06	1.07	0.98	1.01	0.08	0.06				
1372	$l_p=1$	15	0.17	90°		—		—	≤0.02	—				
				125°	0.92[g]	0.96[g]	0.79	0.87	0.13	0.09				
				160°	0.91[g]	0.92[g]	0.55	0.67	0.36	0.25				
				53°	1.01	1.08	0.64	0.74	0.37	0.34	0.39			
				60°	1.00	1.03	0.56	0.69	0.44	0.34	0.37			
				70°	1.01	1.00	0.48	0.61	0.53	0.39	0.52	16.1	8.5	30
	1^+			80°	1.08	1.11	0.41	0.66	0.67	0.45	0.60			
1422	$l_p=0$	14.6	0.85	90°	1.23	1.27	0.37	0.58	0.86	0.69	0.79			
				100°	1.42	1.41	0.36	0.60	1.06	0.81	0.88	11.4	8.4	28
				110°	1.69	1.61	0.36	0.68	1.33	0.93	1.15			
				125°	2.15	1.98	0.39	0.64	1.76	1.34	1.47	12.3	8.0	24
				136°	2.49	2.17	0.42	0.63	2.07	1.54	1.64			
				160°	3.06	2.74	0.44	0.63	2.62	2.11	2.18	6	7.4	18

[a] Resonance energies, widths, and partial widths are from Barnes, reference 17. The resonance energies 1346, 1372, and 1422 are new determinations (see reference 4) corresponding to the older values 1355, 1381, and 1431.
[b] See appendix B.
[c] See reference 2.
[d] Values for the maximum and minimum cross sections for the 873- and 935-kev resonances are not given since the theoretical values are calculated neglecting the broad resonance at 843 kev.
[e] Obtained by subtracting the estimated contribution of the 843-kev resonance from the observed cross section.
[f] Measured at 110°, 100°, 90°, 80°, 70°, 60°, and 53.2°.
[g] The quoted values are for the maximum in the cross section curve occurring at an energy higher than the minimum.

of Rutherford scattering, and we thus conclude that Γ_p/Γ is small for this resonance.

An analysis of the scattering data has been made[2] for several of the observed resonances. This analysis, together with data from the study of the $F^{19}(p,\alpha)$ reactions,[15,18,19] allows the following spin and parity assignments to be made: 669 kev(1^+), 843 kev(0^+), 935 kev(1^+), 1372 kev(2^-), and 1422 kev(1^+). The scattering results are also consistent with the assignments[19,20]: 873 kev(2^-) and 1346 kev(2^-). Table I gives the values of $(\sigma/\sigma_R)_{max}$ and $(\sigma/\sigma_R)_{min}$ as observed in this experiment for the resonances listed above, together with the theoretical values of these quantities calculated for the assignments and proton widths given. Observed values of $(\sigma_{max}-\sigma_{min})/\sigma_R$ after correction for energy resolution effects (Appendix B) are also given in several cases. The values of Γ_p/Γ were determined from the $F^{19}(p,p')$, $F^{19}(p,\alpha)$ and $F^{19}(p,p)$ cross sections by Baranger[2] and Barnes.[17]

We wish to thank C. A. Barnes for many valuable comments and suggestions during the course of this work and the preparation of this paper and to thank E. Baranger and R. F. Christy for many illuminating discussions of the theoretical aspects of this problem.

APPENDIX A. CALIBRATION PROCEDURE

The energy scale of the electrostatic analyzer may be conveniently calibrated by observing the thick target gamma-ray yield over a proton energy range containing a sharp, symmetric resonance of known energy. In a

[18] Chao, Tollestrup, Fowler, and Lauritsen, Phys. Rev. **79**, 108 (1950).
[19] Peterson, Fowler, and Lauritsen, Phys. Rev. **96**, 1250 (1954).
[20] J. Seed and A. P. French, Phys. Rev. **88**, 1007 (1952).

thick target excitation curve the mid-point of the observed step in the yield occurs at the resonance energy, E_R. If the analyzer setting, proportional to the potential across the plates, is S_R at the observed mid-point, then the analyzer constant, C_e, is given to first order by[5]

$$C_e=\frac{E_R}{Z_1S_R}\left(1+\frac{Z_1eV_T+\Delta E_1}{E_R}-\frac{E_R}{2M_1c^2}\right), \quad \text{(A1)}$$

where Z_1e = charge of the bombarding ion, V_T = potential of the target with respect to the analyzer, M_1 = rest mass of the bombarding ion, and ΔE_1 = energy loss of the incident ion in any contamination layer on the target surface.

After calibrating the analyzer it is then possible to calibrate the magnetic spectrometer at any convenient bombarding energy, E_B, by observing the elastic scattering of particles at a known angle, θ, from a clean, pure target, usually copper in these experiments. If the fluxmeter setting (inversely proportional to the magnetic field in the type used in this laboratory) corresponding to the midpoint in the rise of the momentum spectrum of the scattered particles is I, then the spectrometer constant, C_m, is given to first order by[5]

$$C_m=\frac{M_1K}{M_pZ_1^2}E_BI^2\left(1-\frac{\epsilon_2+K\epsilon_1}{K\epsilon_1}\frac{\Delta E_1}{E_B}+\frac{1-K}{K}\frac{Z_1eV_T}{E_B}+\frac{KE_B}{2M_1c^2}\right), \quad \text{(A2)}$$

where M_p = rest mass of proton, taken as the standard particle in specifying C_m, $K=K(\theta)$ = ratio of the energy before scattering to the energy after scattering as determined from the conservation of energy and momentum, ΔE_1 = energy loss of incident particle in contamination layers ($\Delta E_1=0$ for a "clean" target), and ϵ_1, ϵ_2 = stopping cross section for the particles in contamination layers before and after scattering.

Having determined C_e and C_m, the effective solid angle is then found by observing the copper yield and assuming Rutherford cross section for copper. The appropriate formulas are given in Brown *et al.*[5]

The primary uncertainty in the energy calibrations is due to the presence of contamination layers (ΔE_1). The most effective method of avoiding this difficulty is, of course, the use of clean targets which do not become contaminated rapidly. By repeating the observation of the midpoint of the γ-ray step or momentum spectrum several times on the same target it is often possible to determine the rate of contamination and to extrapolate to zero bombardment, or to affirm that contamination is negligible. In cases where the target material is lighter than the contaminants (usually carbon) it is possible to observe the scattering peak of the contamination layer and to determine the thickness from this. Where the layer is appreciable, the magnetic spectrometer may be used to determine the thickness, although this will generally introduce an uncertainty of 1–2 kev in the final determination.

To evaluate ΔE_1 using the magnetic spectrometer, we observe the momentum profile of particles scattered from the target (T) in question and those scattered from a clean target (C) for which ΔE_1 is negligible. Then to first order:

$$\Delta E_1=\frac{K_T\epsilon_1E_B}{K_T\epsilon_1+\epsilon_2}\left[1-\frac{K_CI_C^2}{K_TI_T^2}-(K_C-K_T)\left(\frac{E_B}{2M_1c^2}-\frac{Z_1eV_T}{K_CK_TE_B}\right)\right], \quad \text{(A3)}$$

where $I_{C,T}$ = fluxmeter reading at the half-intensity point in the spectrum of particles scattered by the reference or target material respectively; both observations being made at the same energy, E_B, and angle, θ.

The expression (A3) may also be used to determine the energy difference, $\Delta E_F=E_B-E_1$, in a scattering experiment. Here E_B is the bombarding energy and E_1 is the energy at scattering in the target. In this case, $K_C=K_T$, I_C is the half-intensity point of the spectrum, and ϵ_1 and ϵ_2 refer to the stopping cross section in the target material before and after scattering. (A3) then gives ΔE_F for any fluxmeter setting I_T.

APPENDIX B. EFFECT OF RESOLUTION ON OBSERVED CROSS SECTION CURVES

A discussion of several sources of energy variation in investigations of this type has previously been given by Cohen,[21] who has calculated the size of the energy variation caused by the following effects: (1) finite beam size, (2) energy variation in incoming beam, (3) finite size of the magnetic spectrometer entrance and exit windows, and (4) straggling in the energy loss in the target before and after scattering. The net result of these effects is a root-mean-square variation of about 2 kev (at 1 Mev) under typical conditions. This increases roughly linearly with energy.

Experimental estimates of the over-all energy resolution (as made by the techniques described below) indicate an energy variation which is, in many cases, somewhat larger than that calculated on the basis of (1)–(4) above, and we believe that additional factors, such as the target surface condition, are responsible for this.

No satisfactory calculations of the effect of target surface irregularity have been made, although an upper limit can be estimated by assuming that the surface is sufficiently irregular that the total energy loss in the target (as required by the experimental technique)[5] may take place either before or after scattering. Under

[21] E. R. Cohen, Ph.D. thesis, California Institute of Technology, 1949 (unpublished).

the usual target arrangement (the normal to the target surface bisecting the angle between the incident beam direction and the scattering direction), approximately half the energy loss should occur before scattering. Extreme surface roughness would lead, therefore, to a symmetric spread in the reaction energy with a half-width at half maximum of the order of the "following" depth in the target (usually less than one percent of the bombarding energy). At 1 Mev, this would give a root-mean-square variation in energy of 4–5 kev.

Microscopic examination of the LiF powder used in preparing targets for these investigations indicate that the grain size is about 10^{-4} cm which is of the order of five times the depth at which the observed scattering occurs. This suggests that the surface irregularities (assumed to be caused in part by individual crystals) are of this same order and therefore may affect substantially the energy resolution of the experiment.

Some experimental investigations of this effect have been made by observing the yield at the resonance minimum as a function of the energy loss before scattering ("following depth" in the target). By extrapolating these measurements which were made at 1422 kev at 90° to zero energy loss before scattering and using the relations derived below, these results can be used to calculate δE_F, the root-mean-square energy variation due to energy loss in the target. In a typical experiment, we find for a "following depth" of 15 kev, $\delta E_F=3.3$ kev, while the calculated value[21] (which considers only straggling) is about 2.0 kev. The upper limit for the variation due to surface irregularities would be 8–9 kev in this case, so that a contribution of only about one-third of this would be needed to explain the observed value.

In those cases where the resonance width, Γ, is known independently from measurements of reaction cross sections, the elastic scattering data itself may be used to estimate the over-all resolution. We consider the theoretical expression for the cross section, σ, in the vicinity of a resonance:

$$\sigma=\sigma_0[1+f(x)], \tag{B1}$$

where

$$f(x)=(a+bx)/(1+x^2), \quad x=(E-E_R)/\tfrac{1}{2}\Gamma,$$

E_R is the resonance energy, and a, b, and σ_0 are approximately independent of energy and are to be determined from the experimental results where their values are needed. The observed cross section, σ', will then be

$$\sigma'=\sigma_0[1+F(x)], \tag{B2}$$

where $F(x)$ is obtained by folding $f(x)$ with the energy resolution function which is approximately independent of x over any one resonance.

In comparing the experimental results with the theoretical calculations for various spin assignments, a quantity of considerable interest is the maximum variation of the cross section over the resonance. We define

$$k=(\sigma'_{\max}-\sigma'_{\min})/(\sigma_{\max}-\sigma_{\min})$$

and wish to obtain k as a function of observable parameters of the experimental resonance curve.

For this purpose we assume a particular form of the energy distribution function, a square window of full width $\alpha\Gamma$. Using this, we then obtain a relation between $E_{\max}-E_{\min}$ and k in terms of the parameter A (or $1/A$ if $A>1$), where $A=(\sigma'_{\max}-\sigma_0)/(\sigma_0-\sigma'_{\min})$. The additional parameter, A, is included since we treat both α and a/b as parameters to be determined from the experimental data. In principle, a, b, and σ_0 may be calculated from theoretical considerations if the assignment for the level and the potential scattering phase shift are known. These results are, however, sensitive to the choice of the latter quantity, whereas the value of k is relatively insensitive to A and hence to σ_0. We have found that a visual estimate of σ_0 (to about 5 or 10 percent) is usually satisfactory. In addition, this procedure allows the correction of the data without knowing the assignment for the level.

Carrying out the folding process, we obtain from (B1) and (B2):

$$F(x)=\frac{b}{2\alpha}\left[c\tan^{-1}\frac{2\alpha}{1+x^2-\alpha^2}+\frac{1}{2}\log\frac{1+(x+\alpha)^2}{1+(x-\alpha)^2}\right], \tag{B3}$$

and

$$dF/dx=0 \quad \text{at} \quad x=-c\pm[c^2+1+\alpha^2]^{\frac{1}{2}}, \tag{B4}$$

$$y=(E_{\max}-E_{\min})/\Gamma=(1+\alpha^2+c^2)^{\frac{1}{2}}, \tag{B5}$$

$$A=\frac{\tanh^{-1}(\alpha/y)-c\tan^{-1}[\alpha/(1+c^2+cy)]}{\tanh^{-1}(\alpha/y)+c\tan^{-1}[\alpha/(1+c^2-cy)]}, \tag{B6}$$

$$k=\alpha^{-1}(1+c^2)^{-\frac{1}{2}}[c\tan^{-1}(\alpha c/y)+\tanh^{-1}(\alpha/y)], \tag{B7}$$

where $E_{\max}$ and $E_{\min}$ are the proton energies for which the maximum and minimum cross sections are observed and $c=a/b$.

The effects of finite energy resolution are thus seen to be an increase in $E_{\max}-E_{\min}$ and a decrease in $\sigma'_{\max}-\sigma'_{\min}$. Our procedure involves the evaluation of the latter effect by observing the former. The parameter A is fairly insensitive to α and depends mainly on the shape of the theoretical curve (a/b). To obtain the explicit relation between k, A, and y, we first used the expression (B6) to obtain (graphically) a/b as a function of A, treating α as a parameter. We then used the relations (B5) and (B7) to obtain k as a function of y, treating A as a parameter.

An alternative procedure for obtaining k may be found by taking the product x_1x_2; from (B4) this gives

$$-x_1x_2=\frac{(E_{\max}-E_R)(E_R-E_{\min})}{(\Gamma/2)^2}=1+\alpha^2. \tag{B8}$$

Using (B5), (B7), and (B8) we may then obtain α, a/b, and k explicitly in terms of $E_{\max}-E_R$ and $E_{\min}-E_R$. The use of this method requires, however, that the resonance energy (on the energy scale of the experiment) be known with an uncertainty which is small compared to the resonance-width, Γ. In the present experiment, however, the uncertainty in the energy scale is about 3 kev, so that this condition is not well satisfied.

In cases where $E_{\max}$ or $E_{\min}$ is not clearly defined, it is possible to use A and the width at half maximum for determining k. This width is more difficult to determine than $E_{\max}-E_{\min}$ (both experimentally and analytically) and, except for nearly symmetric resonances, this procedure is less satisfactory.

In Table I of the text, we have given corrected values of the cross sections for comparison with the observed and theoretical values for several of the $F^{19}(p,p)$ resonances. Values for the window width, α, have been determined in many of these cases and it is found that these values are always between the values calculated using Cohen's formulation[21] and those estimated as an upper limit for target surface irregularities. The values are close to the lower limit at the higher energies (1400 kev) and large scattering angles and become larger relative to the lower limit as the energy and scattering angle are decreased. Several of these values are given in Table I.

The application of these expressions to obtain relations between experimentally determined quantities may, in part, eliminate the dependence of the results on the particular form we have taken for the resolution function. In any event, the treatment gives a useful description of the qualitative features and at least a first approximation to the quantitative aspects. In addition to the application of these results to the correction of cross sections, they should be of some value in determining resonance energies and widths from elastic scattering measurements.

PHYSICAL REVIEW VOLUME 99, NUMBER 1 JULY 1, 1955

Analysis of the Elastic Scattering of Protons from F^{19}†

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(Received March 21, 1955)

An analysis of the anomalies in the elastic scattering cross section of protons on F^{19} has been carried out. The assignments 1^+ for the resonances at 669, 935, and 1422 kev, and 0^+ for the resonance at 843 kev, are required by the results of the experiment. The assignment of the resonances at 873, 1348, and 1374 kev is 2^-, with 1^- not excluded by these data alone. Unique values of the partial widths are determined for these resonances and several others in this energy range. Reduced widths are given for the various particle reactions which are observed.

IN the preceding paper, Webb, Hagedorn, Fowler, and Lauritsen[1] have discussed an experiment measuring the elastic scattering of protons by F^{19}. The present paper deals with an analysis of their experimental results. The purpose of the analysis is to determine the spins, parities, and partial widths of as many as possible of the excited states of the compound nucleus Ne^{20}. Assignments of the levels examined here, except the resonances at 843 and 1422 kev, have been determined previously by several workers using methods other than elastic scattering. They were, however, unable to determine the partial widths uniquely. Their results are summarized in a review article by Ajzenberg and Lauritsen.[2] Some preliminary work on elastic scattering was done at this laboratory by Peterson *et al.*,[3] which resulted in the assignment of the 1422-kev resonance. Recently, Dearnaley[4] has examined the elastic scattering and his results as to assignment and choice of partial widths are in agreement with ours.

In analyzing proton scattering from a nucleus such as F^{19} where reactions are also possible, certain complications arise in the formulas which have led us to use simplified and not entirely accurate forms for the scattering cross section. Thus, neglecting Coulomb effects, an arbitrary scattering amplitude associated with a given J and parity could be written as $f(e^{2i\delta}-1)$, where $f\leq 1$ and δ is arbitrary. Now in the case of a single resonance we find that $f=\Gamma_p/\Gamma$, the ratio of the elastic proton width to the total width, is independent of energy and $\cot\delta$ is linear in the energy. In the case of two overlapping resonances, however, no simple energy dependence of f and δ which is consistent with resonance theory has been found. What is needed is

† Assisted by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] Webb, Hagedorn, Fowler, and Lauritsen, preceding paper [Phys. Rev. 99, 138 (1955)]. This paper is referred to as Paper A.

[2] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. 27, 77 (1955).

[3] Peterson, Barnes, Fowler, and Lauritsen, Phys. Rev. 94, 1075 (1954).

[4] G. Dearnaley, Phil. Mag. 45, 1213 (1954).