constant, Δ will be appreciable. This affords a new criterion for deciding whether electrolytic transport takes place interstitially or by vacancies. The fact that for NaBr Δ was found to vanish within the accuracy of the experiments[1] supports the vacancy transport assumption for this material. The fact that for AgCl Δ was found experimentally[2] to range between 1 and 2 is strong support for the interstitial transport assumption for the silver salt.

[1] See for example H. W. Schamp and E. Katz, Phys. Rev. **94**, 828 (1954).
[2] W. D. Compton, doctoral thesis under R. J. Maurer, University of Illinois, Urbana, Illinois, 1955 (unpublished).
[3] See for example N. F. Mott and R. W. Gurney, *Electronic Processes in Ionic Crystals* (Oxford University Press, Oxford, 1940), p. 63.

Outgoing and Ingoing Waves in Final States and Bremsstrahlung*

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IT is well known that, in calculating transition probabilities between stationary states, a scattered particle in the final state should be represented by a wave function whose asymptotic form is plane wave plus ingoing spherical wave, rather than by the usual scattering wave function. The importance of this point has been stressed repeatedly, and in particular in connection with the recent calculation of bremsstrahlung at high energies.[1]

On the other hand, it has been pointed out[2] that this particular choice of final state wave functions is only required if the differential cross section for a given direction of the scattered particle (final electron in the case of bremsstrahlung) is to be calculated. If it is asked for the cross section integrated over the direction of the scattered particle, then one can equally well use the outgoing wave type of wave function for the final state. This may be understood from the fact that in this case the direction of motion of the scattered particle merely represents a degeneracy of the final state. One may therefore use any complete set of states to represent the particle; in particular the outgoing wave modification may be chosen.

In the bremsstrahlung problem, these statements hold if the total cross section for emission of a quantum of a given energy and direction is desired. We need in this case a complete set of wave functions for the electron of a given energy. The statement that outgoing waves may be used for the electron will now be proved explicitly for a Dirac electron in a spherical symmetric potential.

Let $\psi_{\pm}{}^{\mu}(\mathbf{p},\mathbf{r})$ denote the Dirac wave function with asymptotic form: plane wave with momentum $\mathbf{p}$ together with $\binom{\text{out}}{\text{in}}$-going spherical wave, μ being the spin label. In an arbitrary coordinate system this can clearly be written as

$$\psi_{\pm}{}^{\mu}(\mathbf{p},\mathbf{r})=\sum_{jj_zlm} i^l\langle jj_z|lm\tfrac{1}{2}\mu\rangle \times Y_{l,m}{}^*(\mathbf{p}/p)e^{\pm i\delta_{l,j}}\Phi_{ljj_z}(\mathbf{r}), \quad (1)$$

where $Y_{l,m}$ are the spherical harmonics, $\langle jj_z|lm\frac{1}{2}\mu\rangle$ the Clebsch-Gordan coefficients, $\delta_{l,j}$ the phases and Φ_{ljj_z} the standing wave solutions

$$\Phi_{ljj_z}(\mathbf{r})=4\pi\left(\frac{E+m}{2E}\right)^{\frac{1}{2}}\begin{pmatrix} i\dfrac{p}{E+m}f_{l,j}(r)(\boldsymbol{\sigma}\mathbf{r}/r)\chi_{ljj_z} \\ g_{l,j}(r)\chi_{ljj_z}\end{pmatrix},$$

χ_{ljj_z} being the central field spinors,[3] and $f_{l,j}$ and $g_{l,j}$ the usual radial functions with asymptotic behavior:

$$f_{l,j}(r)\rightarrow(1/pr)\cos(pr-\tfrac{1}{2}l\pi+\delta_{l,j}),$$
$$g_{l,j}(r)\rightarrow(1/pr)\sin(pr-\tfrac{1}{2}l\pi+\delta_{l,j}).$$

The probability for a process described by a perturbation $\mathcal{O}$, integrated over angles and summed over spins in the final state, is proportional to

$$I_{\pm}=\sum_{\mu=\pm\frac{1}{2}}\int d\Omega_{pf}\left|\int d^3r(\psi_{\pm}{}^{\mu})^{\dagger}(\mathbf{p}_f,\mathbf{r})\,\mathcal{O}\psi_i\right|^2. \quad (2)$$

On account of the orthogonality of the spherical harmonics and of the Clebsch-Gordan coefficients, the factor $\exp(\pm i\delta_{l,j})$ depending upon the choice of out- or ingoing wave modifications, disappears from the expression

$$I_{\pm}=\sum_{ljj_z}\left|\int d^3r\Phi_{ljj_z}{}^{\dagger}(\mathbf{r})\,\mathcal{O}\psi_i\right|^2, \quad (3)$$

giving the matrix element in terms of standing waves.

Turning now to the bremsstrahlung, it will be seen that the present remark introduces considerable simplifications:

Although it is necessary to use the final state wave function $\psi_-{}^f$ in order to obtain the correct differential cross section,[1] it is now clear that as soon as the cross section is integrated over angles of the electron, we can forget about the difference between $\psi_-{}^f$ and $\psi_+{}^f$. We may therefore take advantage of using $\psi_+{}^f$ in the final state, and hence we can deduce the bremsstrahlung cross section integrated over angles of the electron from the corresponding pair cross section by the familiar transformations $d\epsilon_1/k^3\rightarrow dk/kp_1{}^2$ and the change of sign of p_2 and ϵ_2. This conclusion holds whether the screening is absent, partial or complete. In particular

then, from the pair cross section formula, Eq. (43), of Davies, Bethe, and Maximon,[4] we thus find the bremsstrahlung spectrum at high energies:

$$(d\sigma/dk)=(d\sigma/dk)_{BH} -4a^2\frac{e^2}{\hbar c}\left(\frac{\hbar}{mc}\right)^2\frac{1}{kp_1^2}(\epsilon_1^2+\epsilon_2^2-\tfrac{2}{3}\epsilon_1\epsilon_2)f(Z). \quad (4)$$

Here $(d\sigma/dk)_{BH}$ is the Bethe-Heitler spectrum as given by Bethe,[5] including the effect of screening. Otherwise the notation is as in reference 4.

The Coulomb correction, which is given by the last term in (4), is therefore the same irrespective of the strength of the screening, and the relative correction is the same as for the pair cross section. We may furthermore use the same argument as was used by Davies, Bethe, and Maximon[4] in the case of pair production, to show that the form of the bremsstrahlung spectrum is essentially unchanged by the Coulomb correction.

In the case of no screening, the present result has in fact been obtained by Maximon[6] by integration of the differential bremsstrahlung cross section of Bethe and Maximon.[1] This, then, provides a good check on their calculation. On the other hand Bethe and Maximon[1] stated that there should be zero Coulomb correction to the Bethe-Heitler formula in the case of complete screening. This discrepancy between their result and ours arises from the fact that in bremsstrahlung the corrections due to screening and to the Coulomb effect cannot simply be superposed in the differential cross section. This is because the Coulomb correction occurs for small values of the momentum transfer q when the ingoing wave modification is used,[1] i.e., under the same conditions as the screening. But the customary expression for screening, *viz.*, the multiplication of the cross section by a form factor, is only valid in the Born approximation, i.e., only when the matrix element is *not* modified by the Coulomb correction. Therefore only the calculation of the cross section integrated over electron angles, with *outgoing* waves in the final state is reliable, and at present no result is available for the differential cross section of bremsstrahlung in the presence of both screening and Coulomb correction.

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Preliminary Study of the Electrical Properties of a Semiconducting Diamond

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THE Hall effect, electrical conductivity, and point contact rectification of a semiconducting diamond have been examined as the initial studies of electrical conduction in diamonds. The specimen investigated is a blue-white, rectangular parallelepiped $0.124\times0.104\times0.356$ cm^3, and is apparently a Type IIb diamond.[1,2]

The Hall effect was determined using a standard ac Hall apparatus operating at 27 cps and a dc magnetic field. Both indium solder and silver paint electrodes have been used, but neither type seems to be entirely ohmic at all temperatures. The specimen temperature was adjusted by immersion in a cooled or heated helium stream and the temperature determined by means of a thermocouple imbedded in the sample mounting cement. The temperature excursion in the present work was limited by contact instability at low temperatures and softening of the mounting cement at high temperatures.

The Hall constant is found to be independent of magnetic field up to the highest value employed, 10 000 oersteds, both at room temperature and at -40°C, the lowest temperature reached. The sample is p-type over the entire range. The logarithm of the Hall constant as a function of reciprocal temperature is found to be

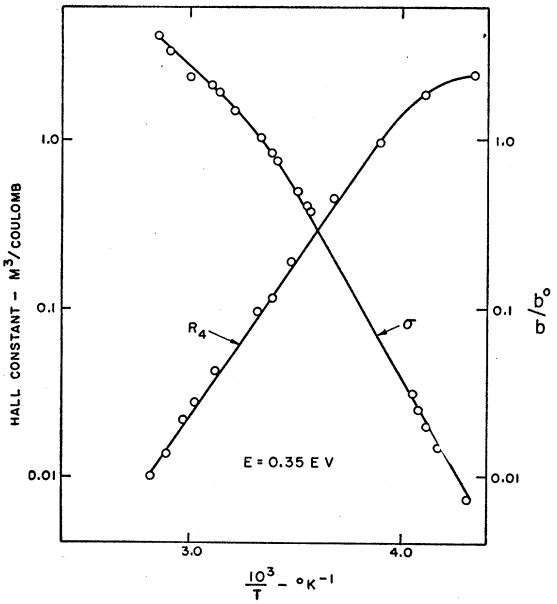

FIG. 1. Hall constant and conductivity as a function of reciprocal temperature for diamond sample.