Spin Resonance in Metals as a Function of the Overhauser Effect

JEROME I. KAPLAN
University of California, Berkeley, California, and Naval Research Laboratory, Washington, D. C.
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The polarization of the nuclear spin states by saturating the electron spin levels as predicted by Overhauser is shown to alter the electron spin resonance absorption line shape. It is shown that under certain stringent conditions the line shape may depend upon the sweep direction of the magnetic field H_0. The theory is developed under the assumption of no skin-depth broadening.

IT will be shown that under certain conditions the Overhauser[1] effect causes the electron spin resonance absorption line shape in metal particles, small compared to the skin depth, to be dependent on the sweep direction of H_0, the applied static field. The rate of energy absorption P for the spin system, assuming no skin-depth broadening,[2] is given by[3]

$$P=\frac{\omega\hbar}{2H_1^2}\frac{\alpha^2 g(\omega-\omega_0')n_0}{1+\alpha^2 g(\omega-\omega_0')}=\frac{\omega\hbar}{2H_1^2}n_0 s, \tag{1}$$

where $\alpha^2=(\pi/4)\gamma_e^2H_1^2T_1$ and the saturation factor s is

$$s=\frac{\alpha^2 g(\omega-\omega_0')}{1+\alpha^2 g(\omega-\omega_0')}.$$

ω is the applied frequency, H_1 the rf magnetic field, T_1 the spin lattice relaxation time, ω_0' the electronic Larmor frequency, γ_e the electronic magnetomechanical ratio, n_0 the population difference of the electronic spin states as determined by the total static field H_0' and $g(\omega-\omega_0')$ the normalized shape factor for the transition. For a Fermi gas of electrons, the population difference is

$$n_0=3N_e\hbar\gamma_e H_0'/4E_f, \tag{2}$$

where E_f is the Fermi energy and N_e the electron density.

It is assumed that the line-shape function $g(\omega-\omega_0')$ has a Lorentzian form which expressed in terms of

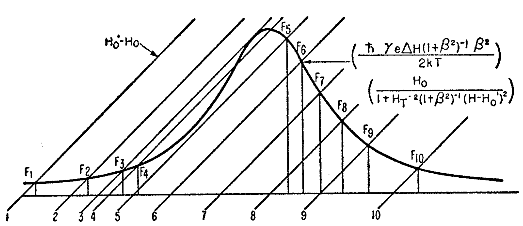


FIG. 1. Graphical solution of the equation
$$H_0'-H_0=\hbar\gamma_e\Delta H(1+\beta^2)^{-1}\beta^2\times\frac{H_0}{1+H_T^{-2}(1+\beta^2)^{-1}(H-H_0')^2},$$
where it is assumed that $\hbar\gamma_e\Delta H(1+\beta^2)^{-1}\beta^2H_0=4/3$ and $H_T^{-2}\times(1+\beta^2)^{-1}=1$.

[1] A. Overhauser, Phys. Rev. **92**, 411 (1953).
[2] F. Dyson, Phys. Rev. **98**, 439 (1955).
[3] A. Portis, Phys. Rev. **91**, 1071 (1953).

magnetic fields becomes

$$g(\omega-\omega_0')=\frac{(\pi\gamma_e H_T)^{-1}}{1+H_T^{-2}(H-H_0')^2}, \tag{3}$$

where $\omega=\gamma_e H$ and $1/T_1=\gamma_e H_T$. Substitution of $g(\omega-\omega_0')$ in the expression for the saturation factor s gives the result that

$$s=\beta^2/[(1+\beta^2)+H_T^{-2}(H-H_0')^2], \tag{4}$$

where

$$\beta^2=\alpha^2/\pi\gamma_e H_T.$$

The static field acting on the electron is the sum of the applied field and the contribution arising from the polarization of the nuclei. According to Overhauser, H_0' is given as

$$H_0'=H_0+\mathcal{P}\Delta H, \tag{5}$$

where

$$\Delta H=(8\pi/3)N|\psi_0^2|I$$

is the maximum local field and

$$\mathcal{P}=\sum_{m=-I}^{m=I} m\exp(m\hbar\gamma_{\text{eff}}/2)\Big/\left(I\sum_{m=-I}^{m=I}\exp(m\hbar\gamma_{\text{eff}}/2)\right) \tag{6}$$

is the measure of nuclear polarization.

$$\gamma_{\text{eff}}=(\gamma_e+s\gamma_n)H_0/kT,$$

N is the density of atoms, ψ_0 the value of the electronic wave function at the nucleus, I the nuclear spin, γ_n the nuclear magnetomechanical ratio, k the Boltzmann constant, and T the temperature. For $I=\frac{1}{2}$, $s\gamma_e\gg\gamma_n$, and $\hbar s\gamma_e H_0\ll kT$,

$$H_0'-H_0\simeq\tfrac{1}{2}(s\hbar\gamma_e H_0)\Delta H/kT. \tag{7}$$

The saturation factor s in Eq. (7) is replaced by its value given in Eq. (1). The resulting equation,

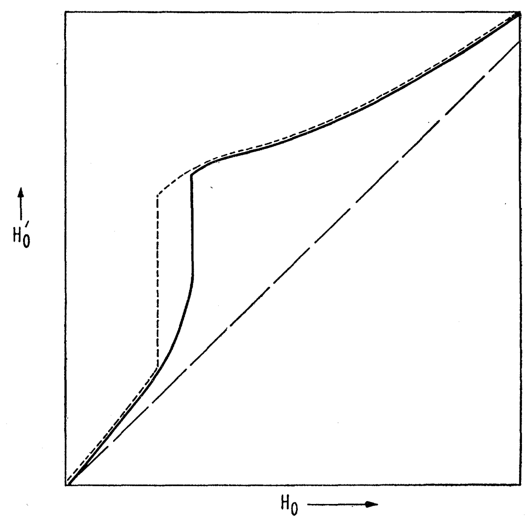
$$H_0'-H_0\simeq\frac{\hbar\gamma_e\Delta H(1+\beta^2)^{-1}\beta^2}{2kT}\times\frac{H_0}{1+H_T^{-2}(1+\beta^2)^{-1}(H-H_0')^2}, \tag{8}$$

relates H_0' to H_0, the other factors being taken as constants.

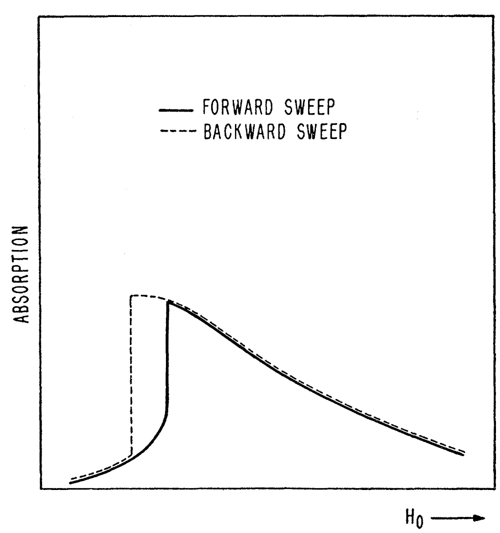
The energy absorption in terms of H_0 and H_0',

$$P=\frac{\omega\hbar}{2H_1^2}\left(\frac{(2kT)}{\gamma_e\Delta H}\right)\left(\frac{3N_e\gamma_e\hbar}{4E_f}\right)\left(\frac{H_0'}{H_0}\right)(H_0'-H_0), \quad (9)$$

is found by using Eqs. (1), (2), and (7).



(a)



(b)

FIG. 2. (a) Plot of H_0' *vs* H_0 as obtained from Fig. 1. (b) Plot of the energy absorption *vs* H_0 as obtained from Fig. 2(a) and Eq. 9.

For each value of H_0, H_0' is calculated from Eq. (8). The qualitative form of the solution can be found graphically as shown in Fig. 1. As H_0 is increased from zero through resonance the solutions for H_0' are marked by F. It is to be noted that the height of the Lorentz curve in Fig. 1 will vary as H_0, but this will only change the magnitude and not the form of the solution. The nteresting point is that there can be a discontinuity in H_0' as H_0 varies from position 4 to 5. The plot of H_0 *vs* H_0' for forward and backward sweeps as found from Fig. 1 is shown in Fig. 2(a). The substitution of these values in Eq. (9) gives the form of the absorption curve for both sweep directions as shown in Fig. 2(b). In Fig. 3 is shown a plot of the absorption for the case where H_T has been increased by a factor of two over the value assumed in Fig. 1. The effect of this change is to broaden out the Lorentzian curve in Fig. 1 also by a factor of two. The broadened curve intersects the straight line $H_0'-H_0$ only at one point, corresponding to a single solution of H_0' as a function of H_0. For this case then the absorption will be independent of the direction of sweep. It is seen that a necessary condition for the asymmetry as regards direction of sweep is that the slope at the inflection point of the Lorentz curve in Fig. 1 be greater than unity. In terms of the physical constants of the problem and the value of H_0 in the resonance region, this condition requires

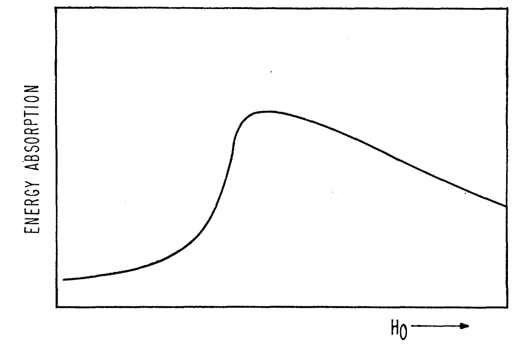


FIG. 3. Plot of the energy absorption *vs* H_0 where H_T is assumed double that used in Fig. 1.

$$\frac{\hbar\gamma_e^2T_1H_1^2\Delta H\sqrt{3}}{8kTH_T^2}\times\frac{H_0}{[(1+\gamma_eT_1H_1^2)/4H_T]^{\frac{3}{2}}}>1. \quad (10)$$

Thus, if $H_0=10^3$, $\Delta H=10^2$, $H_1=1$, $8kT/\sqrt{3}=10^{-15}$, $H_T=1$, $\gamma_e=10^7$, $T_1=10^{-7}$, and $\hbar=10^{-27}$, the expression on the left side of the inequality in Eq. (10) will approximately equal one.

The value of dP/dH_0, to the approximation where terms in $[(H_0'-H_0)/H_0]^2$ are neglected, can be written as

$$\frac{dP}{dH_0}\simeq\frac{\omega\hbar}{2H_1^2}\left(\frac{3N_e\gamma_e\hbar}{4E_f}\right)\left(\frac{2kT}{\gamma_e\Delta H}\right)\times\left(1+\frac{2(H_0'-H_0)}{H_0}\right)\frac{d[H_0'-H_0]}{dH_0}. \quad (11)$$

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