The Maser—New Type of Microwave Amplifier, Frequency Standard, and Spectrometer*†

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(Received May 4, 1955)

A type of device is described which can be used as a microwave amplifier, spectrometer, or oscillator. Experimental results are given. When operated as a spectrometer, the device has good sensitivity, and, by eliminating the usual Doppler broadening, a resolution of 7 kc/sec has been achieved. Operated as an oscillator, the device produced a frequency stable to at least 4 parts in 10^{12} in times of the order of a second, and stable over periods of an hour or more to at least a part in 10^{10}. The device is examined theoretically, and results are given for the expected sensitivity of the spectrometer, the stability and purity of the oscillation, and the noise figure of the amplifier. Under certain conditions a noise figure approaching the theoretical limit of unity, along with reasonably high gain, should be attainable.

INTRODUCTION

A TYPE of device is described below can be used as a microwave spectrometer, a microwave amplifier, or as an oscillator. As a spectrometer, it has good sensitivity and very high resolution since it can virtually eliminate the Doppler effect. As an amplifier of microwaves, it should have a narrow band width, a very low noise figure and the general properties of a feedback amplifier which can produce sustained oscillations. Power output of the amplifier or oscillator is small, but sufficiently large for many purposes.

The device utilizes a molecular beam in which molecules in the excited state of a microwave transition are selected. Interaction between these excited molecules and a microwave field produces additional radiation and hence amplification by stimulated emission. We call an apparatus utilizing this technique a "maser," which is an acronym for "microwave amplification by stimulated emission of radiation."

Some results obtained with this device have already been briefly reported.[1] An independent proposal for a system of this general type has also been published.[2] We shall here examine in some detail the general behavior and characteristics of the maser and compare experimental results with theoretical expectations. Particular attention is given to its operation with ammonia molecules. The preceding paper,[3] which will hereafter be referred to as (I), discusses an investigation of the hyperfine structure of the microwave spectrum

of $N^{14}H_3$ with this apparatus. Certain of its properties which are necessary for an understanding of the relative intensities of the hyperfine structure components are also discussed there.

BRIEF DESCRIPTION OF OPERATION

A molecular beam of ammonia is produced by allowing ammonia molecules to diffuse out a directional source consisting of many fine tubes. The beam then transverses a region in which a highly nonuniform electrostatic field forms a selective lens, focusing those molecules which are in upper inversion states while defocusing those in lower inversion states. The upper inversion state molecules emerge from the focusing field and enter a resonant cavity in which downward transitions to the lower inversion states are induced. A simplified block diagram of this apparatus is given in Fig. 1. The source, focuser, and resonant cavity are all enclosed in a vacuum chamber.

For operation of the maser as a spectrometer, power of varying frequency is introduced into the cavity from an external source. The molecular resonances are then observed as sharp increases in the power level in the cavity when the external oscillator frequency passes the molecular resonance frequencies.

At the frequencies of the molecular transitions, the beam amplifies the power input to the cavity. Thus the maser may be used as a narrow-band amplifier. Since the molecules are uncharged, the usual shot noise existing in an electronic amplifier is missing, and essentially no noise in addition to fundamental thermal noise is present in the amplifier.

If the number of molecules in the beam is increased beyond a certain critical value the maser oscillates. At the critical beam strength a high microwave energy density can be maintained in the cavity by the beam alone since the power emitted from the beam compensates for the power lost to the cavity walls and coupled wave guides. This oscillation is shown both experimentally and theoretically to be extremely monochromatic.

[1] Gordon, Zeiger, and Townes, Phys. Rev. **95**, 282 (1954).
[2] N. G. Bassov and A. M. Prokhorov, J. Exptl. Theoret. Phys. (U.S.S.R.) **27**, 431 (1954). Also N. G. Bassov and A. M. Prokhorov, Proc. Acad. of Sciences (U.S.S.R.) **101**, 47 (1945).
[3] J. P. Gordon, preceding paper [Phys. Rev. **99**, 1253 (1955)].

APPARATUS

The geometrical details of the apparatus are not at all critical, and so only a brief description of them will be made. Two ammonia masers have been constructed with somewhat different focusers. Both have operated satisfactorily.

A source designed to create a directional beam of the ammonia molecules was used. An array of fine tubes is produced in accordance with a technique described by Zacharias,[4] which is as follows. A $\frac{1}{4}$ in. wide strip of 0.001-in. metal foil (stainless steel or nickel, for example) is corregated by rolling it between two fine-toothed gears. This strip is laid beside a similar uncorregated strip. The corregations then form channels leading from one edge of the pair of strips to the other. Many such pairs can then be stacked together to create a two-dimensional array of channels, or, as was done in this work, one pair of strips can be rolled up on a thin spindle. The channels so produced were about 0.002 in. by 0.006 in. in cross section. The area covered by the array of channels was a circle of radius about 0.2 in., which was about equal to the opening into the focuser. Gas from a tank of anhydrous ammonia was maintained behind this source at a pressure of a few millimeters of mercury.

This type of source should produce a strong but directed beam of molecules flowing in the direction of the channels. It proved experimentally to be several times more effective than a source consisting of one annular ring a few mils wide at a radius of 0.12 in., which was also tried.

The electrodes of the focuser were arranged as shown in Fig. 1. High voltage is applied to the two electrodes marked V, while the other two are kept at ground. Paul et al.[5,6] have used similar magnetic pole arrangements for the focusing of atomic beams.

In the first maser which was constructed the inner faces of the electrodes were shaped to form hyperbolas with 0.4-in. separating opposing electrodes. The distance of closest approach between adjacent electrodes
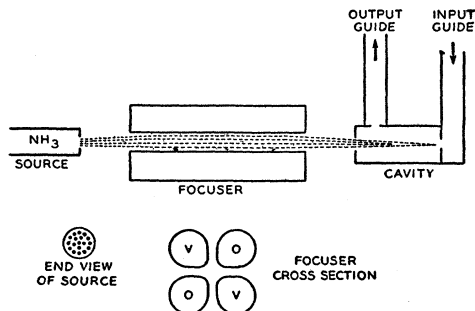

FIG. 1. Simplified diagram of the essential parts of the maser.

[4] J. R. Zacharias and R. D. Haun Jr., Quarterly Progress Report, Massachusetts Institute of Technology Research Laboratory of Electronics, 34, October, 1954 (unpublished).
[5] H. Friedberg and W. Paul, Naturwiss. 38, 159 (1951).
[6] H. G. Bennewitz and W. Paul, Z. Physik 139, 489 (1954).

was 0.08 in., and the focuser was about 22 in. long. Voltages up to 15 kv could be applied to these electrodes before sparking occurred. In the second maser the electrodes were shaped in the same way, but were separated from each other by 0.16 in. This allowed voltages up to almost 30 kv to be applied, and somewhat more satisfactory operation was obtained since higher field gradients could be achieved in the region between the electrodes. This second focuser was only 8 in. long. Teflon spacers were used to keep the electrodes in place. To provide more adequate pumping of the large amount of ammonia released into the vacuum system from the source the focuser electrodes were hollow and were filled with liquid nitrogen.

The resonant cavities used in most of this work were circular in cross section, about 0.6 in. in diameter by 4.5 in. long, and were resonant in the TE_{011} mode at the frequency of interest (about 24 kMc/sec). Each cavity could be turned over a range of about 50 Mc/sec by means of a short section of enlarged diameter and variable length at one end. A hole 0.4 in. in diameter in the other end allowed the beam to enter. The beam traversed the length of the cavity. The cavities were made long to provide a considerable time for the molecules to interact with the microwave field. Only one-half wavelength of the microwave field in the cavity in the axial direction was allowed for reasons which will appear later in the paper. Since the free space wavelength of 24-kMc/sec microwaves is only about 0.5 in., and an axial wavelength of about 9 in. was required in the cavity, the diameter of the cavity had to be very close to the cut-off diameter for the TE_{01} mode in circular wave guide. The diameter of the beam entrance hole was well beyond cutoff for this mode and so very little loss of microwave power from it was encountered. The cavities were machined and mechanically polished. They were made of copper or silver-plated Invar, and had values of Q near 12 000. Some work was also done with cavities in the TM_{01} mode which has some advantages over the TE_{01} mode. However, the measurements described here all apply to the TE_{011} cavities.

Microwave power was coupled into and out from the cavities in several ways. Some cavities had separate input and output wave guides, power being coupled into the cavity through a two-hole input in the end of the cavity furthest from the source and coupled out through a hole in the sidewall of the cavity. In other cavities the sidewall hole served as both input and output, and the end-wall coupling was eliminated. About the same spectroscopic sensitivity was obtained with both types of cavities.

Three MCF 300 diffusion pumps (Consolidated Vacuum Company, Inc.) were used to maintain the necessary vacuum of less than 10^{-5} mm Hg. Nevertheless, due to the large volume of gas released into the system through the source, satisfactory operation has

not yet been attained without cooling the focuser electrodes with liquid nitrogen. At 78°K the vapor pressure of ammonia is considerably less than 10^{-6} mm Hg and so the cold electrode surfaces provide a large trapping area which helps maintain a sufficiently low pressure in the vacuum chamber. The pumping could undoubtedly be accomplished by liquid air traps alone; however the diffusion pumps alone have so far proven insufficient. The solidified ammonia which builds up on the focuser electrodes is somewhat of a nuisance as electrostatic charges which distort the focusing field tend to build up on it, and crystals form which can eventually impede the flow of gas. For the relatively short runs, however, which are required for spectroscopic work, this arrangement has been fairly satisfactory.

EXPERIMENTAL RESULTS

Experimental results have been obtained with the maser as a spectrometer and as an oscillator. Although it has been operated as an amplifier, there has as yet been no measurement of its characteristics in this role. Its properties as an amplifier are examined theoretically below.

The reader is referred to (I) for the results obtained from an examination of the hyperfine structure of the $N^{14}H_3$ inversion spectrum with the maser. Resolution of about seven kc/sec was obtained, which is a considerable improvement over the limit of about[7] 65 kc/sec imposed by Doppler broadening in the usual absorption-cell type of microwave spectrometer. This resolution can be improved still further by appropriate cavity design. The sensitivity of the maser was considerably better than that of other spectrometers which have had comparably high resolution.[8-10]

The factors which determine the sensitivity and resolution of the maser spectrometer are discussed in detail below, but we may make a general comment here. The sensitivity of the maser depends in part on the physical separation of quantum states by the focuser and thus on the forces exerted by the focuser on molecules in the various quantum states. For this reason its sensitivity is not simply related to the gas absorption coefficient for a given molecular transition. Each individual case must be examined in detail. Due to the focuser, for example, the sensitivity of the maser varies more rapidly with the dipole moment of the molecule to be studied than does that of the ordinary absorption spectrometer.

The experimental results obtained with the maser in its role as an oscillator agree with the theory given below and show that its oscillation is indeed extremely monochromatic, in fact more monochromatic than any

other known source of waves. Oscillations have been produced at the frequencies of the 3–3 and 2–2 inversion lines of the ammonia spectrum, those for the 3–3 line being the stronger. Tests of the oscillator stability were made using the 3–3 line, so we shall limit the discussion to oscillation at this frequency. Other ammonia transitions, or transitions of other molecules could, of course, be used to operate a maser oscillator.

The frequency of the $N^{14}H_3$ 3–3 inversion transition is 23 870 mc/sec. The maser oscillation at this frequency was sufficiently stable in an experimental test so that a clean audio-frequency beat note between the two masers could be obtained. This beat note, which was typically at about 30 cycles per second, appeared on an oscilloscope as a perfect sine wave, with no random phase variations observable above the noise in the detecting system. The power emitted from the beams during this test was not measured directly, but is estimated to be about 5×10^{-10} watt.

The test of the oscillators was made by combining signals from the two maser oscillators together in a 1N26 crystal detector. A heterodyne detection scheme was used, with a 2K50 klystron as a local oscillator and a 30-Mc/sec intermediate-frequency (IF) amplifier. The amplified intermediate frequency signals from the two maser oscillators were then beat together in a diode detector, and their difference, which was then a direct beat between the two maser oscillator frequencies, displayed on an oscilloscope. The over-all band width of this detecting system was about 2×10^4 cps, and the beat note appeared on the oscilloscope with a signal to noise ratio of about 20 to 1.

It was found that the frequency of oscillation of each maser could be varied one or two kc/sec on either side of the molecular transition frequency by varying the cavity resonance frequency about the transition frequency. If the cavity was detuned too far, the oscillation ceased. The ratio of the frequency shift of the oscillation to the frequency shift of the cavity was almost exactly equal to the ratio of the frequency width of the molecular response (that is, the line width of the molecular transition as seen by the maser spectrometer) to the frequency width of the cavity mode. This behavior is to be expected theoretically as will be shown below. The two maser oscillators were well enough isolated from one another so that the beat note could be lowered to about 20 cps before they began to lock together. The appearance of this beat note has been noted above. As perhaps $\frac{1}{10}$-cycle phase variation could have been easily detected in a time of a second (which is about the time the eye normally averages what it observes), the appearance of the beat indicates a spectral purity of each oscillator of at least 0.1 part in 2.4×10^{10}, or 4 parts in 10^{12} in a time of the order of a second.

By using Invar cavities maintained in contact with ice water to control thermal shifts in their resonant frequencies, the oscillators were kept in operation for

[7] Gunther-Mohr, White, Schawlow, Good, and Coles, Phys. Rev. 94, 1184 (1954).
[8] G. Newell and R. H. Dicke, Phys. Rev. 83, 1064 (1951).
[9] R. H. Romer and R. H. Dicke, Phys. Rev. 98, 1160(A) (1955).
[10] M. W. P. Strandberg and H. Dreicer, Phys. Rev. 94, 1393 (1954).

periods of an hour or so with maximum variations in the beat frequency of about 5 cps or 2 parts in 10^{10} and an average variation of about one part in 10^{10}. Even these small variations seemed to be connected with temperature changes such as those associated with replenishing the liquid nitrogen supply in the focusers. Theory indicates that variations of about 0.1°C in temperature, which was about the accuracy of the temperature control, would cause frequency deviations of just this amount.

It was found that the oscillation frequency was slightly dependent on the source pressure and the focuser voltage, both of which affect the strength of the beam. These often produced frequency changes of the order of 20 cycles per second when either voltage or pressure was changed by about 25%. As the cavity was tuned, however, both these effects changed direction, and the null points for the two masers coincide to within about 30 cps. The frequency at which these effects disappear is probably very near the center frequency of the molecular response, so this may provide a very convenient way of resetting the frequency of a maser oscillator without reference to any other external standard of frequency.

THE FOCUSER

In (I) it was shown that forces are exerted by the nonuniform electric field of the focuser on the ammonia molecules, the force being radially inward toward the focuser axis for molecules in upper inversion states and radially outward for molecules in lower inversion states. Molecules in upper inversion states are therefore focused by the field, and only these molecules reach the cavity. Moreover, the quadrupole hyperfine splitting of the upper inversion state was shown to affect the focusing since the flight of the molecules through the focuser is adiabatic with respect to transitions between the different quadrupole levels. As a result the higher energy quadrupole levels are focused considerably more strongly than the lower energy ones. The further slight splitting of the various quadrupole states by the magnetic hyperfine interactions of the hydrogen nuclei has little effect since the molecules make many transitions between these closely spaced levels as they enter and leave the focuser. In regions of high field strength where hyperfine effects are unimportant and can be neglected the energy of the molecules in an electric field may be written as

$$W = W_{\text{rotation}}(J,K) \pm \left\{ \left(\frac{h\nu_0}{2}\right)^2 + \left(\frac{M_J K}{J(J+1)}\mu\mathcal{E}\right)^2 \right\}^{\frac{1}{2}}, \quad (1)$$

where ν_0 is the zero-field inversion frequency, J, K, and M_J specify the rotational state of the molecule relative to the direction of the field, μ is the molecular dipole moment, and $\mathcal{E}$ is the magnitude of the electric field.

With these considerations in mind, an approximate calculation of the total number of molecules in the upper inversion state which are trapped by the potential well of the focuser and delivered to the cavity is fairly straightforward. It involves some computation, since the line used for the oscillation (the main line of the $J=K=3$ inversion transition) is composed of three different but unresolved component transitions between quadrupole sublevels of the inversion states, and therefore the number of molecules trapped by the focuser must be calculated for each of these three sublevels and the results added. This calculation is outlined below. We shall consider in detail the properties of the first maser oscillator, with which the work reported in (I) was done.

The focuser electrodes form approximate equipotentials of the potential $V = V_0 r^2 \cos 2\theta$, where r and θ are cylindrical coordinates of a system whose z axis coincides with the axis of the focuser. 15 kv applied to the high-voltage focuser electrodes establishes an electric field whose magnitude is given by

$$\mathcal{E} = 200r, \quad (2)$$

where $\mathcal{E}$ is measured in esu and r is in cm. For simplicity we shall assume that the source is small in area and is located on the axis of the focuser. We shall also assume that all molecules which can travel farther than 0.5 cm from the focuser axis collide with the focuser electrodes and are removed from the beam. From (1) and (2) it is seen that the force $(\mathbf{f} = -\text{grad}W)$ on the molecules is radial, and for small field strength is proportional to r. Furthermore it can be seen from energy considerations that all molecules which emerge from the source with radial velocity v_r less than v_{max}, where $\frac{1}{2}mv_{\text{max}}^2 = W(r=0.5 \text{ cm}) - W(r=0)$ are held within the focuser by the electric field, while all molecules whose radial velocity is greater than v_{max} collide with the electrodes. Since v_{max} is a function of M_J (M_J is the projection of $\mathbf{J}$ on the direction of the electric field of the focuser) the number of molecules focused from a given zero-field quadrupole level depends on the high field distribution of these molecules among the various possible M states.

From kinetic theory, the number of molecules per second emerging from a thin-walled source of area S with radial velocity less than v_{max} is given by

$$N = PSv_0\Omega/(2\pi)^{\frac{3}{2}}kT, \quad (3)$$

where P is the source pressure, $v_0 = (kT/m)^{\frac{1}{2}}$ is the most probable velocity of molecules in the beam, T is the absolute temperature, and Ω is a solid angle defined by $\Omega = \pi(v_{\text{max}}/v_0)^2$. The number of molecules per second in a given quadrupole level which are focused is therefore

$$N(F_1) = \frac{PSv_0}{(2\pi)^{\frac{3}{2}}kT} f(JKF_1)\sum_{M_J}\varphi(F_1 M_J)\Omega(M_J), \quad (4)$$

where $f(JKF_1)$ is the fraction of molecules emerging from the source in the quadrupole state characterized by J, K, and F_1 ($\mathbf{F}_1 = \mathbf{J} + \mathbf{I}_N$, where I_N is the spin of the nitrogen nucleus), and $\varphi(F_1, M_J)$ is that fraction of these molecules which, according to the discussion in (I), go adiabatically into the state characterized by the quantum number M_J as they enter the high electric field of the focuser. The total number of molecules per second in the upper inversion state which are delivered to the cavity by the focuser is then just the sum of the $N(F_1)$ for the three quadrupole levels, and so is

$$N(J,K) = \frac{PSv_0}{(2\pi)^{\frac{3}{2}}kT}f(JK)\Omega(JK),\qquad(5)$$

where

$$\Omega(JK) = \sum_{F_1}\frac{f(JKF_1)}{f(JK)}\sum_{M_J}\varphi(F_1 M_J)\Omega(M_J)$$

is an average solid angle for the upper inversion state, and $f(JK)$ is the fraction of molecules emerging from the source in the upper inversion state of the JK rotational level.

If each of these N molecules could be induced to make a transition to the lower inversion state while in the resonant cavity the total power delivered by the beam would be just $N(JK)h\nu_0$. Actually only about 50 to 75% average transition probability for the molecules in the beam can be obtained due to the variation of their velocities and spatial orientations. Assuming 50% transition probability, a source temperature of 300°K, and the geometry and voltage of the focuser given above, a calculation of the solid angle for the 3–3 line gives $\Omega(3\text{–}3) = 4 \times 10^{-3}$ steradian, and available power of 1.5×10^{-9} watt per square millimeter of source area at 1 mm Hg source pressure.

It is estimated that the total number of molecules emerging from the source in the solid angle from which the upper inversion state molecules are selected is about 10^{15} per second. This estimate comes from knowledge of the number of molecules necessary to induce oscillation. This indicates that the present source is operating fairly inefficiently.

RESONANT CAVITY AND LINE WIDTH

The beam of molecules which enters the resonant cavity is almost completely composed of molecules in the upper inversion state. During their flight through the cavity the molecules are induced to make downward transitions by the rf electric field existing in the cavity. The transition probability for any particular molecule at low field strengths is given from first-order perturbation theory by

$$P_{ab} = \hbar^{-2}|\mu_{ab}|^2\left|\int_0^{L/v}\mathcal{E}(t)e^{-i2\pi\nu_0 t}dt\right|^2,\qquad(6)$$

where μ_{ab} is the dipole matrix element for the transition, L is the length of the cavity, v is the velocity of the

molecule, and $\mathcal{E}(t)$ is the rf electric field at the position of the molecule.

An average transition probability $\bar{P}_{ab}$ can be obtained for all molecules in the beam by averaging over the various velocities, trajectories, and values of $|\mu_{ab}|$ for the molecules in the several states which contribute to each spectral line. The power emitted from the beam is then just

$$P = Nh\nu\bar{P}_{ab},\qquad(7)$$

so $\bar{P}_{ab}$ as a function of the frequency of the applied field determines the line width of the molecular response.

Under the simplifying assumptions that the molecules all travel axially down the length of the cavity, that their velocity is uniform and equal to v_0, and that the cavity is a perfect cylinder with only one-half wavelength of rf field in the axial direction, we find that the emitted power has a maximum at the natural transition frequency ν_B, and a total width at half-maximum of $1.2v_0/L$. If the field is assumed to be uniform along the axis rather than one-half of a sine wave, the corresponding total width at half-maximum is $0.9v_0/L$. This line width of about v_0/L can alternatively be obtained from the uncertainty principle and the finite time of interaction of the molecules with the rf field. Thus $\Delta\nu \approx 1/\Delta t$, where Δt is the time of flight of the molecule in the cavity, or L/v_0. The identity of the "Doppler broadening" of the spectral line has essentially disappeared. The sharpness of the molecular response as opposed to that obtained in the usual spectrometer may alternatively be attributed to the long wavelength of the rf field in the cavity in the direction of travel of the beam. If the cavity is excited in a mode in which there is more than one-half wavelength in the direction of travel of the beam, then the molecular emission line, as given by Eq. (6), has two peaks symmetrically spaced about the transition frequency. The frequency separation of these peaks can be associated with the Doppler shift.

Equations (6) and (7) show that for small rf field strengths the emitted power P is proportional to $\mathcal{E}_{max}^2$ and thus to the energy stored in the cavity. For larger field strengths, of course, the molecular transitions begin to saturate and Eq. (6) is no longer sufficient for the calculation of P_{ab}. The effects of this saturation will be considered in detail in a later paper; however, we can say that P_{ab} must certainly be less than 1, and that if a high field strength is maintained in the cavity then the average transition probability $\bar{P}_{ab}(\nu_B)$ will be about 0.5. The total power available from the beam is therefore about $Nh\nu_B/2$. Power saturation in this case is rather similar to that in the usual molecular beam experiment, for which it has been considered by Torrey.[11]

Associated with the power emitted from the beam is an anomalous dispersion, that is, a sharp variation

[11] H. C. Torrey, Phys. Rev. 59, 293 (1941).

in the dielectric constant of the cavity medium due to the beam. These two effects can be considered at the same time by thinking of the beam as a polarizable medium introduced into the cavity, whose average electric susceptibility is given by $\chi = \chi' + i\chi''$. The power emitted from the beam can then be shown directly from Maxwell's equations to be[12]

$$P = 8\pi^2 \nu_B W \chi'', \tag{8}$$

where W is the energy stored in the cavity. Thus, from Eqs. (7) and (8), χ'' is related to $\bar{P}_{ab}$ by

$$\chi'' = Nh\bar{P}_{ab}/8\pi^2 W. \tag{9}$$

The value of χ' is given from χ'' by Kramer's relation,[13] which for a sharp resonance line can be approximated by[14]

$$\chi'(\nu) = \frac{1}{\pi}\int_0^\infty \frac{\chi''(\nu')d\nu'}{\nu' - \nu}. \tag{10}$$

Figure 2 shows the form of χ' and χ'', calculated with the assumptions that all molecules are traveling parallel to the axis of the cavity with uniform velocity, that the cavity is excited in the TE_{011} mode, and that there is a small field strength in the cavity so that $P_{ab} \ll 1$ and Eq. (6) is valid. χ' and χ'' can also be found directly by calculation of the induced dipole moments of the molecules as they traverse the cavity.

If the Q of the cavity is defined in terms of net power loss (i.e., $dW/dt = -2\pi\nu W/Q_C$) then the presence of the
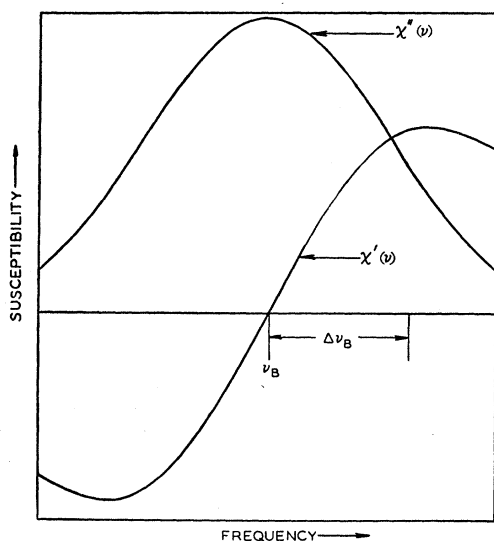

FIG. 2. Real and imaginary parts of the electric susceptibility χ of the molecular beam in the cavity. $\chi = \chi' + i\chi''$.

[12] J. C. Slater, *Microwave Electronics* (D. Van Nostrand Company, Inc., New York, 1950).
[13] J. H. Van Vleck, *Massachusetts Institute of Technology Radiation Laboratory Report*, 735, see also *Radiation Laboratory Series* (McGraw-Hill Book Company, Inc., New York, 1948), Vol. 13, Chap. 8.
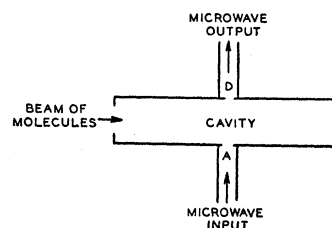[14] G. E. Pake and E. M. Purcell, Phys. Rev. **74**, 1184 (1948).

FIG. 3. Schematic diagram of the resonant cavity and molecular beam.

beam can be considered as causing a change in the effective Q_C given by $1/Q_{CB} = 1/Q_C - 4\pi\chi''$, where Q_{CB} and Q_C are respectively the cavity Q's with and without the beam, along with a shift in the resonant frequency of the cavity given by $\nu_{CB} = \nu_C(1 - 2\pi\chi')$ if $\chi' \ll 1$. These relations can also be easily derived directly from Maxwell's equations, and they will prove important in determining the properties of the maser.

THE MASER SPECTROMETER

Observed Line Shape as a Function of the Cavity Resonant Frequency

Consider the situation shown in Fig. 3. Power P_0 is incident on the cavity from wave guide A, and the power transmitted on out through wave guide D is detected as a function of the frequency of the input power. The power transmitted through the cavity in the absence of the beam is given by[12]

$$P_D(\nu) = \frac{P_0}{Q_A Q_D} \Bigg/ \left[\left(\frac{1}{2Q_L}\right)^2 + \left(\frac{\nu - \nu_C}{\nu_C}\right)^2\right], \tag{11}$$

where Q_A and Q_D are defined in terms of the power losses from the cavity to wave guides A and D respectively, and Q_L is the loaded Q of the cavity, given by $1/Q_L = 1/Q_C + 1/Q_A + 1/Q_D$. As was shown in the last section, the change in $P_D(\nu)$ caused by the presence of the beam can be described through variations in Q_C and ν_C near the transition frequency ν_B. Thus in the presence of the beam we find P_D modified to

$$P_{DB}(\nu) = \frac{P_0}{Q_A Q_D} \Bigg/ \left[\left(\frac{1}{2Q_L} - 2\pi\chi''(\nu)\right)^2 + \left(\frac{\nu - \nu_C + 2\pi\nu_C\chi'(\nu)}{\nu_C}\right)^2\right]. \tag{12}$$

As long as the power output P_{DB} is not so high that nonlinearities in the molecular response are important, (12) gives the output power as a function of frequency in the presence of the beam and represents the spectrum which may be observed. For most spectroscopic applications we are interested in the case for which $\chi''(\nu) \ll 1/4\pi Q_L$ for all ν. For this case, an appropriate expansion of Eq. (12) shows that if $\nu_C = \nu_B$, where ν_B is the center frequency of the molecular response, then the presence of the beam shows up as a pip of the shape
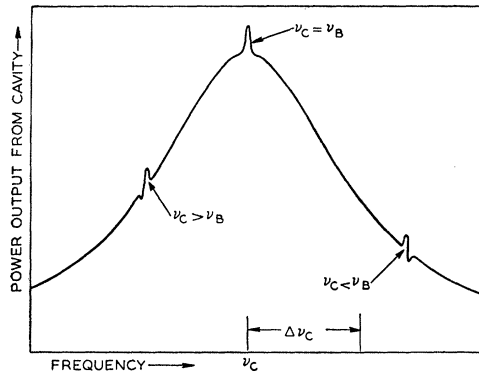
FIG. 4. Spectral line shapes as observed in the maser.

of χ'' superimposed on the cavity mode. The half-width $\Delta\nu_B$ of the molecular response is therefore defined as half the frequency separation of the half-maximum points of $\chi''(\nu)$. If the cavity frequency is altered so that the molecular line appears on the wings of the cavity mode, then the pip due to the beam assumes the shape of $\pm\chi'(\nu)$, the $+$ or $-$ sign depending on whether the line appears on the low or high frequency side of the cavity mode. These three situations are illustrated in Fig. 4.

Sensitivity of the Maser Spectrometer

We have here the usual problem occurring in microwave spectroscopy of detecting a small change in output power P_0 caused by the presence of the molecules. Assume that $\nu_C = \nu_B = \nu$ in Eq. (12). Then

$$P_{DB} = \frac{P_0}{Q_A Q_D} \Big/ \Big(\frac{1}{2Q_L} - 2\pi\chi''(\nu_B)\Big)^2. \quad (13)$$

The change in output power which must be detected is then

$$\delta P_D = P_{DB} - P_D \cong P_0(4Q_L{}^2/Q_A Q_D)$$
$$\times [8\pi Q_L\chi''(\nu_B)]. \quad (14)$$

Consider now the change in output voltage δV_D, which is proportional to $(P_D + \delta P_D)^{\frac{1}{2}} - P_D^{\frac{1}{2}}$ or to $\delta P_D/2P_D^{\frac{1}{2}}$ when $\delta P_D \ll P_D$. The noise voltage at the output of a linear detector is just $(FkT\Delta\nu)^{\frac{1}{2}}$, where $\Delta\nu$ is the band width of the detector and F is its over-all noise figure. Thus the voltage signal-to-noise ratio is given by

$$\delta V_D/V_N = [(\delta P_D)^2/4FkTP_D\Delta\nu]^{\frac{1}{2}}. \quad (15)$$

As long as the change in power δP_D due to the beam increases linearly with the power level P_D, the signal to noise ratio from (15) continues to increase. Hence the power input to the cavity should be increased until the transition begins to saturate. For saturation and a given Q_C, it can be shown that maximum sensitivity is achieved by using a small input coupling $(Q_A \gg Q_C)$

and a matched output coupling $(Q_D = Q_C)$. If we approximate the saturation condition by setting W equal to the level at which the power emitted by the beam is just $\frac{1}{2}Nh\nu_B$, then with a little algebra we find, from Eqs. (8), (14), and the known relationship of W to P_0,[12] that the change in output power δP_D is just $\frac{1}{2}Nh\nu_B$ (the beam emits $\frac{1}{2}Nh\nu_B$ of power, and due to the change in the input match caused by the beam, $\frac{1}{2}Nh\nu_B$ more power enters the cavity through the input coupling hole A. Thus the increase in power input to the cavity is twice the power emitted from the beam. Half of this increase emerges into wave guide D since it was assumed to be matched to the cavity. The power level P_D is now determined by the required energy W from the relation

$$P_D = 2\pi\nu_B W/Q_C. \quad (16)$$

This, with (8), gives

$$P_D = P/4\pi\chi''(\nu_B)Q_C = Nh\nu_B/8\pi\chi''(\nu_B)Q_C. \quad (17)$$

Inserting these values for P_D and δP_D in (15) yields

$$\delta V_D/V_N = [\pi Q_C Nh\nu_B\chi''(\nu_B)/2FkT\Delta\nu]^{\frac{1}{2}}. \quad (18)$$

This relation gives the sensitivity of the maser, once the value of $\chi''(\nu_B)$ for a given molecule is calculated. χ'' is, of course, related to the average transition probability $\bar{P}_{ab}$ by Eq. (9), so that Eq. (18) can easily be rewritten in terms of the transition probability.

For the ammonia 3–3 line, a calculation of the number of molecules in the 3–3 state necessary to make $\delta V_D/V_N = 1$ was done, assuming $F = 100$, $\Delta\nu = 1$ cps, $Q_C = 12\,000$, $T = 300°$ and using an approximate calculation of $\chi''(\nu_B)$ based on the considerations of the previous section. The result was 10^9 molecules per second. It is estimated from the value of $\chi''(\nu_B)$ which is necessary to cause oscillation (see next section) that the number of upper inversion state molecules in the 3–3 rotational state in the beam when oscillations occur is at least 10^{13} molecules per second, and experimentally a number about four times this great was achieved. Thus, for ammonia, the maser should have good sensitivity, and the results described in paper (I) show that this is indeed the case.

In the case of the ammonia inversion spectrum, the focuser can effect an almost complete separation of the upper states from the lower states of the transitions. For some other transitions, this ideal state of affairs may not be attainable, but yet the focuser may preferentially focus one of the two states of the transition. In such a case all of the above considerations apply so long as one uses for N just the excess number of molecules in one of the two states. It is, of course, unimportant for spectroscopic purposes whether the more highly focused state is the upper or lower state of the transition. The high sensitivity attained in the observation of the ammonia spectrum with the maser gives promise that it may be generally useful as a microwave spectrometer of very high resolution.

THE MASER OSCILLATOR AND AMPLIFIER

By extending the considerations of the previous section to include amplification of the thermal noise which exists in the cavity, we can discuss the properties of the maser as an oscillator or amplifier. The results of this analysis, which is made below, are as follows:

(1) The center frequency ν_0 of the oscillation is given to a good approximation by the equation

$$\nu_0 = \nu_B + \frac{\Delta\nu_B}{\Delta\nu_C}(\nu_C - \nu_B), \qquad (19)$$

where $\Delta\nu_C$ and $\Delta\nu_B$ are respectively the half-widths of the cavity mode and of the molecular emission line; and $\nu_C - \nu_B$ is the difference between the cavity resonant frequency ν_C and the line frequency ν_B.

(2) The total width at half-power of the spectral distribution of the oscillation is approximately

$$2\delta\nu = 8\pi kT(\Delta\nu_B)^2/P_B \qquad (20)$$

where T is the temperature and P_B is the power emitted from the beam. Inserting in (20) values which approximate the experimental conditions, $T = 300°K$, $\Delta\nu_B = 3 \times 10^3$ cps, $P_B = 10^{-10}$ watt, we find $2\delta\nu \approx 10^{-2}$ cps, or $\nu_B/\delta\nu = 5 \times 10^{12}$.

(3) If the beam is sufficiently strong, the maser may be used as an amplifier with a gain greater than unity and a noise figure very close to unity.

The argument goes along the following lines. Consider the situation of Fig. 3, the cavity with two wave guides. The whole system will be assumed to be in thermal equilibrium in the absence of the beam. Noise power of amount kT per unit band width is incident on the cavity from each wave guide, and the cavity walls emit noise power within the cavity. Of the noise power incident on the cavity from wave guide A, a certain amount within the frequency range of the cavity mode enters the cavity; part of this power is then absorbed by the cavity walls and part is transmitted on out through wave guide D. A similar situation holds for noise power incident on the cavity from wave guide D; some is absorbed in the cavity and some is transmitted through to wave guide A. The cavity walls emit noise power in the region of the cavity mode and some of this power goes out through each wave guide. When the beam is not present we have assumed the system to be in thermal equilibrium, so there must be kT per unit band width of noise power flowing away from the cavity down each wave guide and there must be kT of noise energy in the cavity mode, as required by the equipartition theorem.

In the presence of the beam thermal equilibrium is upset. The beam, since it is composed solely of upper inversion state molecules, and since the probability for spontaneous decay of these molecules to the lower states is negligible during the time they take to traverse the cavity, contributes no random noise of its own to the rf field of the cavity. What it does is merely to amplify, in a way described by its effect on the loaded Q and resonant frequency of the cavity, all of the noise signals which exist in the cavity. Thus to the noise sources in the wave guides the intrinsic Q of the cavity seems to have been altered; whereas to the noise source within the cavity, the loading on the cavity seems to have changed. In fact, the presence of the beam can be duplicated in the imagination by attaching to the cavity a third wave guide, with a negative Q equal to $\frac{1}{4}\pi\chi''(\nu)$ describing its coupling to the cavity, and by simultaneously shifting the resonant frequency of the cavity by an amount $-2\pi\nu_C\chi'(\nu)$.

From these considerations we will show that in the presence of the beam more than kT of power per unit bandwidth travels down each wave guide away from the cavity. The extra power, of course, comes from the beam. At a certain critical beam intensity this power suddenly becomes large, corresponding to sustained oscillations.

Let $\Delta\nu$ be some arbitrarily small element of the frequency spectrum at frequency ν. Within this range noise power of magnitude $kT\Delta\nu$ is incident on the cavity from each wave guide, independent of ν. Let $P_A\Delta\nu$ be the amount of noise power which enters the cavity from the incident power in wave guide A, and let $P_A'\Delta\nu$ be the total noise power re-emitted into wave guide A from inside the cavity. The presence of the beam will be indicated by an added subscript; i.e., P_{AB} will represent the value of P_A when the beam is present, etc. Similar definitions apply to the output guide D. Since the noise powers generated in the wave guides and in the cavity are completely incoherent with one another, we can simply add power coming from various sources to obtain the total power in any element of the system. Thus the total energy $W\Delta\nu$ stored in the cavity is merely the sum $\sum_i W_i\Delta\nu$ of all the energies due to power coming from the various different power sources. (W is energy per unit band width.)

Consider now the flow of power when no beam is present. The noise power entering the cavity from wave guide A is given by[12]

$$P_A = \frac{kT}{Q_A Q_L{}^A}\bigg/\left[\left(\frac{1}{2Q_L}\right)^2 + \left(\frac{\nu - \nu_C}{\nu_C}\right)^2\right], \qquad (21)$$

where $1/Q_L{}^x = \sum_{m \neq x} 1/Q_m$ is just proportional to all the losses from the cavity except that due to Q_x. Of this power P_A, some is absorbed in the cavity walls, and the rest is transmitted on out through wave guide D. The energy per unit band width stored in the cavity due to this power input is

$$W_A = P_A Q_L{}^A/2\pi\nu$$

$$= \frac{kT}{2\pi\nu Q_A}\bigg/\left[\left(\frac{1}{2Q_L}\right)^2 + \left(\frac{\nu - \nu_C}{\nu_C}\right)^2\right], \qquad (22)$$

and the power transmitted on to the output wave guide D is $2\pi\nu W_A/Q_D$, or $P_A Q_L{}^A/Q_D$. Similar expressions hold for the noise power incident on the cavity from wave guide D. Furthermore, the cavity loss associated with Q_C may be assumed to be due to a third wave guide with coupling characterized by Q_C to a perfectly conducting cavity, so that the energy W_C emitted into the actual cavity from its wall has the same form as (22). The total energy stored in the cavity per unit frequency interval is hence

$$W = W_A + W_D + W_C$$

$$= \frac{kT}{2\pi\nu Q_L} \Big/ \left[\left(\frac{1}{2Q_L}\right)^2 + \left(\frac{\nu - \nu_C}{\nu_C}\right)^2\right], \quad (23)$$

where

$$1/Q_L = 1/Q_A + 1/Q_D + 1/Q_C.$$

The total energy stored in the cavity, given by $\int_0^\infty W d\nu$, is easily shown to be equal to kT (we make the assumption that $Q_L \gg 1$, so that in the integration the approximation $\nu \approx \nu_C$ may be made) as required by the equipartition theorem. The net noise power flowing in the wave guides A or D is also easily shown to be zero, so that the system is indeed seen to be in thermal equilibrium.

Consider now the case when the beam is present. The noise power incident from each wave guide sees a cavity whose rates of internal loss has been reduced by an amount $4\pi\chi''(\nu)$ by the energy emitted from the beam and whose resonant frequency has been shifted by an amount $-2\pi\nu_C\chi'(\nu)$. Corresponding to Eq. (21), the power entering the cavity from wave guide A in the presence of the beam is

$$P_{AB} = \frac{kT}{Q_{LB}{}^A Q_A} \Big/ \left[\left(\frac{1}{2Q_{LB}}\right)^2 + \left(\frac{\nu - \nu_{CB}}{\nu_{CB}}\right)^2\right], \quad (24)$$

where $1/Q_{XB} = 1/Q_X - 4\pi\chi''(\nu)$ for any x and $\nu_{CB} = \nu_C[1 - 2\pi\chi'(\nu)]$, while that entering from wave guide D is similarly

$$P_{DB} = \frac{kT}{Q_{LB}{}^D Q_D} \Big/ \left[\left(\frac{1}{2Q_{LB}}\right)^2 + \left(\frac{\nu - \nu_{CB}}{\nu_{CB}}\right)^2\right]. \quad (25)$$

The noise energy stored in the cavity due to these two sources is

$$W_{AB} = \frac{P_{AB} Q_{LB}{}^A}{2\pi\nu}, \quad \text{and} \quad W_{DB} = P_{DB} Q_{LB}{}^D/2\pi\nu. \quad (26)$$

At the same time, the energy stored in the cavity due to its own internal noise source is changed as though the loading on the cavity has been altered while its internal loss was unaffected. This energy is therefore given by

$$W_{CB} = \frac{kT}{2\pi\nu Q_C} \Big/ \left[\left(\frac{1}{2Q_{LB}}\right)^2 + \left(\frac{\nu - \nu_{CB}}{\nu_{CB}}\right)^2\right]. \quad (27)$$

Due to the presence of the beam the net noise power emitted from the cavity into the output wave guide D is now no longer zero. The power emerging from the cavity is now

$$P_{DB}' = \frac{2\pi\nu}{Q_D}[W_{CB} + W_{AB}]$$

$$= \frac{kT}{Q_D Q_L{}^D} \Big/ \left[\left(\frac{1}{2Q_{LB}}\right)^2 + \left(\frac{\nu - \nu_{CB}}{\nu_{CB}}\right)^2\right]. \quad (28)$$

Thus the additional noise output in wave guide D due to the beam is, from (25) and (28),

$$P_{DN} \equiv P_{DB}' - P_{DB}$$

$$= \frac{kT}{Q_D} 4\pi\chi''(\nu) \Big/ \left[\left(\frac{1}{2Q_{LB}}\right)^2 + \left(\frac{\nu - \nu_{CB}}{\nu_{CB}}\right)^2\right]. \quad (29)$$

The power which must be emitted from the beam to give this amount of power in wave guide D is just

$$P_N \equiv P_{DN} Q_D/Q_L = \frac{kT}{Q_L} 4\pi\chi''(\nu) \Big/$$

$$\left[\left(\frac{1}{2Q_L} - 2\pi\chi''(\nu)\right)^2 + \left(\frac{\nu - \nu_C + 2\pi\nu_C\chi'(\nu)}{\nu_C}\right)^2\right], \quad (30)$$

where ν_C has replaced ν_{CB} in the denominator of the last term in the denominator of this expression since $\nu_{CB} \approx \nu_C$. Note that (30) is just equivalent to (24) if the beam is thought of as a wave guide coupled to the cavity with a Q of $-1/4\pi\chi''$.

Expression (30) gives the complete spectrum of the power emitted from the beam due to amplification of the noise signals which are always present in the cavity. The necessary condition for the existence of oscillations as some cavity frequency is evidently that $\chi''(\nu_B) \approx \frac{1}{4}\pi Q_L$.

Assume that the cavity is tuned so that $|\nu_C - \nu_B| \ll \Delta\nu_C$, and then let the beam strength slowly increase so that χ'' increases. Then at the critical beam strength where $\chi''(\nu_B) \rightarrow 1/4\pi Q_L$, the total power $\int P_N d\nu$ emitted from the beam approaches infinity accordingly to (30). Obviously, the total power emitted from the beam cannot go to infinity, but is limited to about $\frac{1}{2}Nh\nu_B$. When the power level in the cavity reaches the point at which the molecular transition begins to saturate, χ'' and χ' become functions of the power level, and, of course, vary in such a way that $\int P_N d\nu$ is always less than $\frac{1}{2}Nh\nu_B$. We can, for simplicity, avoid the problem of dealing with this saturation merely by increasing χ'' until $\int P_N d\nu = \frac{1}{2}Nh\nu_B$ and examining the frequency spectrum of the power emitted from the beam at this level of output. Although χ'' is not independent of the electric field strength when saturation occurs, it varies much more slowly with time than does the oscillation, so that it may be

considered constant in treating the short-term behavior of the microwave field.

As the critical number of molecules is reached, P_N becomes very large at frequencies very close to ν_B. Hence it is appropriate to expand χ' and χ'' about the center frequency ν_B. This gives approximately

$$\chi' = \chi_0 \left(\frac{\nu - \nu_B}{\Delta\nu_B}\right) + \cdots,$$

$$\chi'' = \chi_0 \left[1 - \frac{1}{2}\left(\frac{\nu - \nu_B}{\Delta\nu_B}\right)^2 + \cdots\right]. \tag{31}$$

Writing Eq. (30) in terms of (31), and setting $\int P_N d\nu$ equal to P_B, where $P_B = \frac{1}{2}Nh\nu_B$, one obtains

$$P_N \approx 4kT(\Delta\nu_B)^2 \Big/ \left[(\nu - \nu_0)^2 + \left(\frac{4\pi kT}{P_B}(\Delta\nu_B)^2\right)^2\right], \tag{32}$$

where ν_0, the oscillation frequency, is given by the equation

$$\frac{\nu_0 - \nu_B}{\nu_0 - \nu_C} = -\frac{\Delta\nu_B}{\Delta\nu_C}, \tag{33}$$

or, as in (19),

$$\nu_0 = \nu_B + (\nu_B - \nu_C)\Delta\nu_B/\Delta\nu_C \quad \text{if} \quad \Delta\nu_B/\Delta\nu_C \ll 1.$$

The total width $2\delta\nu$ at half-maximum power of this "noise" output is, from (32),

$$2\delta\nu = (8\pi kT/P_B)(\Delta\nu_B)^2 \tag{34}$$

as already stated in (20).

It should be remembered that (32) involves the assumption that the maser is a linear noise amplifier of very high gain. Actually, the noise properties of an oscillator depend to a considerable·extent on the non-linearities in its response, or the overload, and (32) does not accurately represent the precise noise spectrum of the maser as an oscillator. However, the approximate width of its noise spectrum is properly given by (34). As in the more usual types of oscillators, this oscillator actually maintains a nearly fixed amplitude of oscillation, but its phase slowly varies with time in a random way, corresponding to a noise spectrum of a width given approximately by (34). A more detailed discussion of noise will be given in a later publication.

The half-width of this oscillation signal is not to be confused with the half-width of the molecular response $\Delta\nu_B$. The latter represents the band width of the maser amplifier at low gain, whereas the former gives the band width of the oscillation signal. The oscillation frequency ν_0 can be varied throughout the range over which the molecules will amplify in accordance with (33) or (19). Hence care must be taken to keep the cavity frequency ν_C constant if it is desired to keep the oscillation frequency constant for any extended period of time.

Noise Figure and Band Width of the Amplifier

The noise figure of the maser amplifier may be easily found from the results of the foregoing sections. Assume that $\nu_C = \nu_B = \nu$, where ν is the frequency of the signal to be amplified. Also assume that the detector has a band width $\Delta\nu_{\text{det}}$ such that $\Delta\nu_{\text{det}} \ll \Delta\nu_B$. Equation (13) gives the signal power at the cavity output, while Eq. (29) gives the noise at the output in excess of kT. Thus we see that the signal-to-noise ratio at the output is just

$$\frac{P_0 \Big/ \left[Q_A Q_D \left(\frac{1}{2Q_L} - 2\pi\chi_0\right)^2\right]}{kT\Delta\nu_{\text{det}}\left[1 + \frac{4\pi\chi_0}{Q_D}\Big/\left(\frac{1}{2Q_L} - 2\pi\chi_0\right)^2\right]}, \tag{35}$$

where $\chi''(\nu_B) = \chi_0$. At the input to the cavity, the signal to noise ratio is $P_0/kT\Delta\nu_{\text{det}}$. Therefore the noise figure F, which is just the ratio of these two quantities, is

$$F = Q_A Q_D \left[\frac{4\pi\chi_0}{Q_D} + \left(\frac{1}{2Q_L} - 2\pi\chi_0\right)^2\right]. \tag{36}$$

At the same time, the power amplification available is, from (13), given simply by

$$\mu = P_{DB}/P_0 = \left[Q_A Q_D \left(\frac{1}{2Q_L} - 2\pi\chi_0\right)^2\right]^{-1}. \tag{37}$$

It can be shown from (37) that $\mu < 1$ if $4\pi\chi_0 < 1/Q_C$, i.e., if there is a net loss of power within the cavity itself. Thus unless it is possible to produce oscillation by putting lossless reflections in all the wave guides so that $Q_L \rightarrow Q_C$, it is also impossible to create an amplifier with a gain greater than unity. In order to obtain a large gain μ, one must have $1/Q_L \approx 4\pi\chi_0$. If the gain is large, then a noise figure approaching unity is attainable by making $1/Q_A \approx +4\pi\chi_0 \approx 1/Q_L$. This shows that for high amplification and at the same time a low noise figure, a fairly large input coupling to the cavity and a small output coupling is needed. Furthermore a sufficiently strong beam is required so that the maser is not too far from oscillation.

The maser acts as a regenerative amplifier, as can be seen from (12). Thus under conditions such that $4\pi\chi_0 \approx 1/Q_L$ so that the midband gain is high, the band width becomes substantially smaller than $2\Delta\nu_B$.

It might also be noted that a certain amount of modulation of the amplified output is to be expected due to random variations of the number of molecules in the cavity at any time. These effects, however, are proportional to the input signal strength, and so are quite different from thermal noise signals which have no dependence on input power. Furthermore, they represent a modulation of only about one part in 10^6 since there are 10^{12} or more molecules in the cavity at

any time. This type of modulation can be neglected when small input signals are considered and is not important under most circumstances. This shot effect and also the effect of power flow through the cavity on the frequency dependence of the amplification will be discussed in more detail in a subsequent paper.

Amplification may also be accomplished using one wave guide as both input and output, and the noise figure of such an amplifier can also approach unity. The amplified output signal might be coupled out and detected through a directional coupler, which would have to have a fairly small coupling so that little of the input power was lost to it. Then so long as the amplified input noise appearing at the detector was large compared to kT, the noise figure of this amplifier would be small.

The maser amplifier may be useful in a restricted range of applications in spite of its narrow band width because of its potentially low noise figure. For example,

suppose that the signal to be amplified came from outer space, where the temperature is only a few degrees absolute. Then by making the coupling through the cavity fairly large so that little noise is contributed by the cavity itself, amplification should be attainable while keeping the noise figure, *based on the temperature of the signal source*, fairly low. This might prove to have a considerable advantage over electronic amplifiers. It might also be possible to tune the frequency of a maser amplifier through the use of the Stark or Zeeman effects on the molecular transition frequencies.

ACKNOWLEDGMENTS

Resonance Transitions in Molecular Beam Experiments. I. General Theory of Transitions in a Rotating Magnetic Field*

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(Received March 29, 1955)

The time-dependent Schrödinger equation is solved formally for an atomic or molecular system which is subjected simultaneously to a rotating magnetic field of constant amplitude and angular velocity and to a constant magnetic field along the axis of rotation. The method yields the transition probabilities in terms of the solutions to an eigenvalue problem. This eigenvalue problem is solved both for (a) a normal Zeeman effect and for (b) the case where a transition from a given level is isolated in frequency from other transitions from the same level. Case (a) is exactly soluble and yields a solution which is shown to be the same as that of Bloch and Rabi, but is in a form which is more convenient for integration over the velocity distribution. Case (b) must be solved by an approximate method which results in a prediction of multiple quantum transitions as observed by Kusch.

I N the C-field of a molecular beams apparatus, a system (e.g., an atom or molecule) is subjected, in a region of constant magnetic field, to an oscillating magnetic field. The experimenter observes, as a function of frequency, a quantity proportional to the number of atoms which have left their initial states.

We will be dealing here with the case in which the constant and oscillating fields are perpendicular to each other. In this case, the solution is greatly simplified[1] by replacing the oscillating field by one which is rotating about the direction of the constant field. One must then correct for the effect of the other rotating component,[2] but the correction is usually small. In the present paper, we shall also explicitly assume that the constant magnetic field is uniform and that the rotating field is uniform in both amplitude and phase.

With these assumptions, one may, for a normal Zeeman effect, solve exactly for the probability of a transition being induced by the rotating field.[3] In other cases, however, some approximate method must be used. Allowed transitions may usually be treated by assuming that only the two levels involved interact,[4] but in the case of multiple quantum transitions which take place only by virtue of the existence of intermediate states, it is necessary to have a more general approach.

The approach we shall develop here has the virtue of not only yielding expressions for the probabilities of multiple quantum transitions but also giving corrections to the single quantum transition probabilities and a convenient expression for the exact solution of the normal Zeeman effect problem.

* Research supported by the National Science Foundation.
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