and

$$\frac{2n_\beta}{E_{MB}}\left(\frac{E_M}{E_{MB}}\right)^{2n_\beta-1}=\frac{W_1^2}{2}\frac{\beta_i(E_M)}{I(E_M)}, \quad (15)$$

where

$$I(E)=\exp\left(-\frac{W_1^2}{2}\int_0^E(\alpha_i-\beta_i)dE\right).$$

But

$$\frac{W_1^2}{2}\alpha_i(E_M)I(E_M)=-\frac{dI(E_M)}{dE_M}+\frac{W_1^2}{2}\beta_i(E_M)I(E_M).$$

When this relation is substituted into Eq. (14) it becomes

$$\frac{dI(E_M)}{dE_M}-\frac{2n_\beta}{E_{MB}}\left(\frac{E_M}{E_{MB}}\right)^{2n_\beta-1}[I(E_M)]^2 =-\frac{2n_\alpha}{E_{MB}}\left(\frac{E_M}{E_{MB}}\right)^{2n_\alpha-1}. \quad (16)$$

Now a new variable $y=(E_M/E_{MB})^{2n_\beta}$ is introduced. Equation (16) then simplifies to

$$\frac{dI(y)}{dy}-[I(y)]^2=-\frac{n_\alpha}{n_\beta}y^{(n_\alpha/n_\beta)-1}. \quad (17)$$

This can be made linear by the substitution $I(y)=-H'(y)/H(y)$. The result is

$$H''-\frac{n_\alpha}{n_\beta}y^{(n_\alpha/n_\beta)-1}H=0. \quad (18)$$

Equation (18) is a variation of Bessel's equation whose solutions are of the form

$$H(y)=K_1(\sqrt{y})J_{n_\beta/(n_\alpha+n_\beta)}\left[\frac{2i(n_\alpha/n_\beta)^{\frac{1}{2}}}{(n_\alpha/n_\beta)+1}y^{\frac{1}{2}(n_\alpha/n_\beta)+\frac{1}{2}}\right] +K_2(\sqrt{y})J_{-n_\beta/(n_\alpha+n_\beta)}\left[\frac{2i(n_\alpha/n_\beta)^{\frac{1}{2}}}{(n_\alpha/n_\beta)+1}y^{\frac{1}{2}(n_\alpha/n_\beta)+\frac{1}{2}}\right]. \quad (19)$$

The boundary condition which this solution must obey is that

$$-H'(y)/H(y)=1 \quad \text{when} \quad y=0.$$

Since the highest value of y which is of physical interest is unity, the series expansion for the Bessel function is highly convergent and can be used effectively to evaluate the Bessel functions of fractional order and imaginary argument which appear.

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Gyromagnetic Ratio of Iron at Low Magnetic Intensities

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It is shown that the measured value for the gyromagnetic ratio of pure iron as determined by a direct magneto-mechanical method, undergoes a change for low values of the induced magnetic intensity. The effective value of this ratio extrapolated to zero intensity, checks the value which is theoretically expected from a consideration of recent ferromagnetic resonance experiments. For higher values of the induced magnetic intensity, the gyromagnetic ratio approaches a constant value which checks the value obtained in previous investigations based upon the Einstein-de Haas effect.

INTRODUCTION

For a number of years an investigation of the gyromagnetic ratios of the iron-nickel alloy series has been under way at the General Motors Research Laboratories. It was noted during this investigation that the measured value of the gyromagnetic ratio (ρ) for a given alloy always increased for low values of magnetic intensity. Although the shape of the curve of magnetic intensity *vs* gyromagnetic ratio appeared to be different for different concentrations of nickel in iron, there was nevertheless, always an increase in ρ for small induced magnetic intensities. Also when the largest obtained ρ values were plotted against concentration of nickel in iron it was found that all points from 0 percent Fe 100 percent Ni to 90 percent Fe 10 percent Ni fell on a smooth curve. The value for pure iron which had been previously determined was, however, an exception. It was, therefore, decided to undertake an extensive investigation of pure iron varying the induced magnetic intensity, and going down to as low a value of magnetic intensity as was practicable. It was found that a similar effect exists for pure iron also. Furthermore, when the curve of ρ *vs* intensity, was extrapolated to zero intensity, the gyromagnetic ratio was found to have a value which checks the value which is theoretically expected from a consideration of recent work in ferromagnetic resonance. For high values of magnetic intensity the gyromagnetic ratio approaches a constant value, which checks previous work done at the General Motors Research Laboratories, and also the work of other investigators.

TABLE I. Condensed data for determination of gyromagnetic ratios (ρ).[a] $\rho\frac{e}{m}=\left(\frac{\pi Id}{4Pxk(m/e)}-2i_e\Sigma A_e\right)\Big/(M_e-i_e\Sigma A_e)$.

i_e	M_e	P	d	$\rho e/m$
4.0004	20832	27.517	0.034141	1.0465
2.0005	9695	27.520	0.016021	1.0542
2.0001	9693	27.515	0.016068	1.0579
10.0031	54571	27.532	0.089126	1.0429
7.0008	37676	27.536	0.061691	1.0453
7.0006	37674	27.536	0.061646	1.0446
1.0000	4276	27.520	0.007099	1.0572
1.0000	4276	27.520	0.007049	1.0495
4.0005	20832	27.523	0.034198	1.0481
2.0002	9694	27.515	0.016126	1.0618
2.0005	9695	27.515	0.016085	1.0588
1.0000	4276	27.514	0.007150	1.0653
1.0000	4276	27.515	0.007125	1.0614
1.0000	4276	27.515	0.007145	1.0644
10.0033	54572	27.532	0.088769	1.0386
3.0002	15213	27.487	0.025025	1.0509
3.0000	15212	27.490	0.025057	1.0522
3.0000	15212	27.490	0.025003	1.0499
5.0007	26415	27.486	0.043237	1.0462
5.0005	26414	27.486	0.043359	1.0492
6.0004	32017	27.493	0.052351	1.0449
2.5001	12412	27.482	0.020482	1.0542
2.5000	12411	27.481	0.020476	1.0535
2.5002	12412	27.481	0.020563	1.0585
3.5003	17980	27.481	0.029491	1.0482
3.5003	17980	27.481	0.029597	1.0521
2.5000	12411	27.483	0.020587	1.0598
3.5001	17979	27.485	0.029548	1.0502
5.0001	26412	27.484	0.043331	1.0485
4.0003	20802	27.485	0.034135	1.0487
4.0001	20801	27.481	0.034061	1.0466
5.0003	26413	27.482	0.043230	1.0462
5.0003	26413	27.482	0.043287	1.0476
7.0016	37628	27.493	0.061746	1.0489
7.0010	37624	27.493	0.061557	1.0457
1.0001	4249	27.477	0.007074	1.0614
1.0003	4250	27.477	0.007148	1.0726
1.0000	4249	27.484	0.007161	1.0749
1.0001	4249	27.484	0.007093	1.0641

Moment of inertia, first group	$I=203.16$ g cm^2
Moment of inertia, second group	$I=203.08$ g cm^2
Optical length	$x=1576.9$ cm
Phase angle constant	$k=0.99941$
Winding constant	$\Sigma A_e=78115$ cm^2
Mass-charge ratio of electron	$m/e=5.6844\times10^{-9}$ g coulomb^{-1}

[a] i_e = magnetizing current, milliamperes. M_e = magnetic moment, amp cm^2. P = period, sec. d = amplitude change per reversal of i_e, cm.

EXPERIMENTAL PROCEDURE

The general experimental procedure used in these experiments has been previously described.[1] However, since there have been numerous improvements in the technique it appears desirable to give a brief outline of the method used to obtain the major experimental factors required to determine the gyromagnetic ratios in this experimental work.

Angular Momentum Changes

In Table I each value of gyromagnetic ratio given is the result of taking eight different sets of data on change of angular momentum. In four of these sets the amplitude of the torsional pendulum is built up by resonance using square wave impulses, and in four more the amplitude is similarly cut down.

The period of the torsional pendulum is about 27 seconds and the square wave is obtained by using a hand operated reversing switch. Before starting a series of runs a current considerably above any that is ever used in the actual experiment is passed through the magnetizing winding and gradually reduced in strength to zero while the reversing switch is rapidly reversed. This demagnetization process is necessary in order to obtain a high degree of correlation between the change in magnetizing current and the resulting change in magnetic moment.

The amplitude of the swinging torsional pendulum is built up for 30 complete periods; it is then cut down for 30 complete periods. All of the amplitude values are recorded for both sets. The azimuth is then changed 180° and two more sets obtained in a similar manner. The second half of the data required for one value is obtained by reversing the switch located at the lower end of the element[1] again demagnetizing, and repeating the aforementioned steps.

The data given in the tables were taken in two groups. The first group was obtained between November 11, 1954 and November 26, 1954. The second group was obtained between December 13, 1954 and January 7, 1955.

Data correlating changes in magnetizing current with changes in magnetic moment were taken both before and after each one of these two groups.

The moment of inertia was measured at the end of each group.

Magnetic Moment Determinations

The changes in magnetic moment occurring in these experiments are correlated with changes in the current flowing in the magnetizing winding which is wound on the rod. To do this a separate experiment is performed wherein the wound rod is supported in a horizontal position on a critically damped torsional system.

The apparatus is arranged so that the gyromagnetic rod can be readily replaced by an air core coil whose dimensions and number of turns are accurately known. A current of 16 milliamps is passed through the air core coil; the resulting magnetic moment can then readily be computed. This is the standard of magnetic moment which has been used for all of the work on gyromagnetic ratios at this laboratory.

In determining magnetic moments for the gyromagnetic rod, the standard is first placed in the torsional system. The current in the large horizontal NS Helmholtz coil system is then adjusted so that a torsional deflection of 50.00 cm is obtained on the 52 ft optical lever system, when the current in the air core standard is reversed. The air core standard is then replaced by the gyromagnetic rod. This rod is first demagnetized, and then the desired current is passed through its

[1] G. G. Scott, Phys. Rev. 82, 542 (1951).

magnetizing winding. After several preliminary reversals, the current in the NS system is again adjusted so as to obtain 50.00 cm upon reversal of the current in the gyromagnetic winding. Allowance is, of course, made for the earth's horizontal field. This is done by taking these readings both with and against the direction of the earth's horizontal component. The magnetic moment of the rod is found by multiplying the magnetic moment of the standard by the ratio of the NS field values required to produce the common torque. A separate determination of magnetic moment is made for each current value that is used in the experiments to determine changes in angular momentum.

Moment of Inertia

In determining, moment of inertia, a series of four brass cylinders is used. These cylinders are chosen to differ in mass by a few grams. The mass and radius of each cylinder is measured and its moment of inertia computed. The period of at least 3 of these cylinders and that of the unknown gyromagnetic pendulum system is measured on the same suspension that is used in the angular momentum experiment. The mass of the gyromagnetic pendulum system is also carefully determined. This mass lies within a few grams of that of the cylinders. From the periods of the three known cylinders the variation in torsional constant of the suspension ribbon with suspended mass can be determined. This can then be applied to the unknown pendulum system to determine the desired moment of inertia. Accuracy of 1 part in 2000 is readily obtainable.

Period

The period of the swinging pendulum system is determined once each day. Time signals from WWV are utilized, and an elapsed time between readings of from $\frac{1}{2}$ hour to 1 hour is used. High accuracy is easily obtainable.

Element Current

The current which is reversed in the magnetizing winding on the gyromagnetic rod is measured to five places on an L and N type K potentiometer. This is done both before and after each "build up" and "cut down" run. This gives a total of 16 current readings for each value of gyromagnetic ratio reported in the table.

INSTRUMENTATION

The instrumentation used in this experiment is much the same as has been recently reported (reference 1). That which has been changed will be briefly described.

Vacuum Chamber

In previous work on measuring gyromagnetic ratios it was not possible to support the ferromagnetic rod in a horizontal position in the vacuum chamber which was used for the measurement of angular momentum changes. It was, therefore, necessary to use a separate chamber for the measurement of magnetic moment. This situation has been remedied by the construction of a new vacuum chamber in which the central section is somewhat spherical in shape, so that the rod may be supported in a horizontal as well as in a vertical position. Other changes were made so as to increase the accessibility of the pendulum system for the purpose of making the necessary adjustments.

Gradient Detector

A gradient detector was constructed. A closely matched and accurately aligned pair of coils was made into an astatic system and supported on a critically damped torsional system in the main vacuum chamber. The current through this coil system was reversed and the resulting rotation observed on the regular 52-ft optical system. The sensitivity of this system was such that a vertical gradient in the horizontal component of 8×10^{-8} gauss per cm, produced a 1-mm deflection of the system. The normal uniform horizontal component of the earth's field produced only a few mm deflection.

By the use of this instrument it was found that several pieces of ferromagnetic material used in the building construction which were previously considered to be far enough away from the main instrument, were causing gradients several times larger than the gradients inherent to the large Helmholtz coil system. This ferromagnetic material was, of course, removed.

Helmholtz Coil System

The homogeneity of the magnetic fields produced by the 8 ft and 9 ft diameter Helmholtz coil systems has been considerably improved by using in connection with each one of these systems an additional Helmholtz coil system concentric with the large system but about $\frac{1}{2}$ the size. The field produced by the small coil set is made to oppose that produced by the large system. By an inspection of the equations for the fields produced by a Helmholtz coil pair[2] it can readily be seen that two systems of different sizes connected in series opposition can be made to produce a much more uniform field than that produced by the large one alone, if the number of turns on the small and large systems is proportional to the fifth power of the radii of the two systems.

That a very considerable increase in homogeneity was achieved, was verified by measurements made with the gradient detector described previously.

RESULTS

In Table I is given a condensation of all of the data taken in making these gyromagnetic determinations. Table II gives an average value for $\rho e/m$ for each

[2] Ruark and Peters, J. Opt. Soc. Am. **13**, 205 (1926).

TABLE II. Gyromagnetic ratios (ρ) and Landé factors (g') for various induced magnetic intensities in iron.[a]

i_e	$\mathcal{I}$	$\rho e/m$	g'
1.0	107.7	1.063	1.881
2.0	245.3	1.058	1.890
2.5	314.2	1.056	1.893
3.0	385.2	1.051	1.903
3.5	455.4	1.050	1.904
4.0	527.4	1.048	1.909
5.0	669.3	1.047	1.909
6.0	811.4	1.045	1.914
7.0	954.3	1.046	1.912
10.0	1383.5	1.041	1.922

[a] i_e = winding current, milliamperes. $\mathcal{I}$ = average induced magnetic intensity, amp cm^{-1}. $\mathcal{I} = (M_e - i_e \Sigma A_e)/v$, $v = 38.88$ cm^3.

value of the magnetizing current used. The corresponding values of the Landé factor (g') are also given in Table II. Figure 1 is a plot of the data shown in Table II. In this curve g' is plotted against the average induced magnetic intensity of the rod. The average magnetic intensity is obtained by dividing the total moment of the rod in amp cm^2 by the volume of the rod.

Since this rod is magnetized by a uniformly distributed winding the resulting induced magnetization will, of course, not be uniform but will rather approximate an elliptical distribution of magnetic intensity. This nonuniformity of magnetization, of course, has its effect on the shape of the g' *vs* intensity curve. It is planned to repeat this experiment using a magnetizing winding of a shape that will more closely induce uniform intensity of magnetization. Although this should change the shape of the curve, it is considered unlikely that either the extrapolated zero intensity value or the limiting value at higher intensities, would be effected by such a change in winding shape.

The value of g' extrapolated to zero intensity is 1.873 ± 0.005. This value is in good agreement with the value expected from recent experiments in ferromagnetic resonance.[3]

The limiting value at higher values of magnetic intensity appears to be about 1.919, corresponding to $\rho e/m$ of 1.042 ± 0.003. This is in reasonable agreement with previous work at this laboratory and with the work of Barnett and Meyer.[4] It should be here noted that the values given in reference 4 differ slightly from those previously reported by the present author. It was pointed out by Dr. Sheldon Brown that a small systematic error involving the optical system existed in the original work. Corrected values are given in reference 4.

CONCLUSION

It has been shown that the effective value of g' as measured by an Einstein-de Haas experiment, changes from one value at zero intensity of magnetization to a higher value as the magnetic intensity increases. The value obtained by extrapolation to zero magnetic intensity checks the value to be expected by a consideration of recent work in the field of ferromagnetic resonance. For higher magnetic intensities the effective value of g' approaches a limiting value which checks previous work done at the General Motors Laboratory and also the work of other investigators who have determined gyromagnetic ratios by magneto-mechanical methods. The zero intensity value for g' from the present series of experiments is 1.873 ± 0.005.

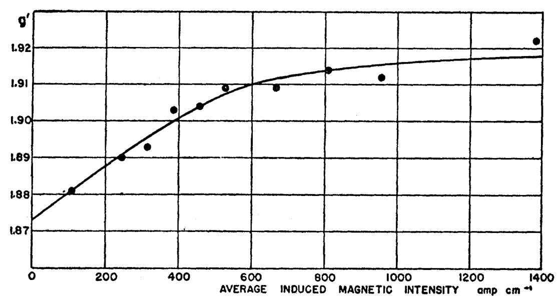

FIG. 1. Plot of Landé Factor (g') *vs* average induced magnetic intensity. Iron rod 1.5-cm diameter, 22 cm long.

This same effect has been noted for seven different alloys in the iron nickel series. In each case a larger g' was noted for larger values of the intensity of magnetization. This work on alloys will be reported when more information has been obtained.

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[3] C. Kittel, *Introduction to Solid State Physics* (John Wiley and Sons, Inc., New York, p. 171.

[4] Brown, Meyer, and Scott, Compt. rend. **238**, 2502 (1954).