V. FERROMAGNETIC MATERIALS

Table I also lists the resistivities of the ferromagnetic stoichiometric compounds. It is to be noted that these substances have resistivities 5 to 14 orders of magnitude lower. In addition, the conduction is metallic in sign. In order to see that the order of magnitude of the resistivity is correct, let us make a calculation on a simple model. Considering (see Zener's[6] double exchange) conductivity to be a diffusion process, Zener showed that one could write the following relation:

$$\sigma = Ne^2a^2J/kTh,$$

where N is the number of electrons per unit volume participating in the diffusion process (e.g., equal to the number of Ni^+ ions in NiO), a is the lattice constant, and J is the integral representing the transition of an electron from the anion to the cation. Taking Van Vleck's[7] estimate of 10^3 cm^{-1} for J and assuming that $N \simeq 0.1 \times$ number of anions, one finds $10^3 \Omega^{-1}$ cm^{-1}. This is the correct order of magnitude for the ferromagnetic materials.

SUMMARY

The present model treats NiO on an atomic basis, the lack of conduction arising from the action of the Pauli exclusion principle. This treatment should be valid as long as cation states of principle quantum number greater than three can be neglected.

The data in Table I show that the predicted qualitative correlation between the magnetic and electric properties is borne out.

[6] C. Zener, Phys. Rev. **82**, 403 (1951).

[7] J. H. Van Vleck, Grenoble Conference 114 (1951).

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Avalanche Breakdown in Germanium

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It is shown that all germanium junctions studied break down as the result of the same avalanche process found in silicon. An empirical expression for the multiplication inherent in this breakdown process is given for step junctions. Ionization rates for holes and electrons in Ge are derived with the use of this expression. The ionization rate for holes is larger than that for electrons by about a factor of two. The agreement between these ionization rates as a function of field and the theory of Wolff is excellent. It is determined that the threshold for electron-hole pair production is about 1.50 ev and the mean free path for electron (or hole)-phonon collisions is about 130 A.

INTRODUCTION

When *pn* junctions are reverse-biased to sufficiently high voltage, a breakdown occurs and large currents begin to flow. The principal theories for this breakdown depend on internal field emission or a solid state analogue of the Townsend β avalanche breakdown in gases.[1,2] The former mechanism was first proposed by Zener[3] for dielectric breakdown. It involves the direct excitation of electrons from the valence to the conduction bands in high electric fields. In the latter mechanism, electrons or holes in high fields interact with valence electrons to produce electron-hole pairs. An important difference between the two mechanisms is that in the latter case multiplication of charge injected into the junction takes place at voltages below the breakdown voltage. In fact, breakdown is defined as the point at which this multiplication becomes very large. The Zener theory has no such multiplicative effects associated with it.

Recently, McKay and McAfee[1] have demonstrated that charge multiplication takes place in some germanium and silicon junctions at prebreakdown voltages. McKay[2] has found that all silicon junctions in the range studied have multiplicative breakdowns. Breakdowns possibly attributable to internal field emission have been reported in some narrow germanium junctions.[4]

The measurements described in this paper were undertaken to understand the breakdown mechanism in germanium junctions. However, the analysis developed should be applicable to other semiconductors. As a result of the measurements, there is strong evidence that the breakdown process in germanium is the avalanche process and that internal field emission has not been observed. It has also been possible to differentiate between the roles of holes and electrons in the avalanche breakdown process by an extension of the Townsend β theory and to compare the ionization rates obtained with the recent theory of Wolff.[5]

[1] K. G. McKay and K. B. McAfee, Phys. Rev. **91**, 1079 (1953).

[2] K. G. McKay, Phys. Rev. **94**, 877 (1954).

[3] C. Zener, Proc. Roy. Soc. (London) **145**, 523 (1934).

[4] McAfee, Ryder, Shockley, and Sparks, Phys. Rev. **83**, 650 (1951).

[5] P. A. Wolff, Phys. Rev. **95**, 1415 (1954).

THEORY

In the field emission hypothesis, breakdown occurs sharply when a certain critical field is reached anywhere in the junction. This critical field is a constant in a given semiconductor at a given temperature. In a step junction, where the conductivity type varies abruptly from p to n and the impurity concentration is very much greater on one side than the other, the maximum electric field is given by[6]

$$E_M=2V/W=2V/W_1V^{\frac{1}{2}}=2V^{\frac{1}{2}}/W_1 =KV^{\frac{1}{2}}(|N_D-N_A|)^{\frac{1}{2}}, \quad (1)$$

where W = width of the space charge region, W_1 = width constant of the junction $=(1.77\times10^7/|N_D-N_A|)^{\frac{1}{2}}$ for Ge, $|N_D-N_A|=N_I$ = the net impurity concentration on the high resistivity side of the junction, and V = the voltage applied across the junction plus the built-in voltage of the junction.

Thus the critical electric field would be reached when $V=V_B$, the breakdown voltage. Then

$$(E_M)_{\text{critical}}=\text{a constant}=KV_B^{\frac{1}{2}}(|N_D-N_A|)^{\frac{1}{2}},$$

or

$$V_B=\text{constant}/|N_D-N_A|. \quad (2)$$

Hence, the Zener theory results in a breakdown voltage which is inversely proportional to the net impurity concentration on the high-resistivity side for this type of junction. In the range of resistivities where mobility is essentially constant, this means that V_B would be proportional to resistivity. This relation does not hold even for the narrowest germanium junctions investigated.

McKay has applied a modified form of the Townsend β discharge theory for gases to the multiplicative breakdown process in silicon junctions. Under the assumption that electrons and holes have equal ionization rates he obtained an expression for the ionization rate at the maximum field in the junction, $\alpha_i(E_M)$. In step junctions, like those discussed above, numerical values for $\alpha_i(E_M)$ could then be determined from measurements of the multiplication, M, of a junction *vs* E_M or alternatively from measurements of E_M at $M=\infty$ or breakdown *vs* W_1 for various junctions. The multiplication experiments were performed on diodes with the carriers injected by α-particle bombardment.

It has been observed experimentally in this research that the multiplication of minority carriers coming from the high-resistivity side of germanium step junctions closely follows the empirical expression:

$$M(V)=1/[1-(V/V_B)^n], \quad (3)$$

where V_B is the body breakdown of the junction and the parameter n is a number which depends on the resistivity and resistivity type of the high-resistivity side of the junction. It will be shown below that n is quite different for an np^+ and a pn^+ step junction with the same impurity concentration on the high-resistivity side for germanium. This experimental fact can only be interpreted as proof that the ionization rate for electrons is different from that of holes. Furthermore, the existence of a good analytic approximation to the variation of multiplication with voltage makes possible the derivation of analytical expressions for the ionization rates themselves.

$\alpha_i(E)$ is now defined as the number of electron-hole pairs produced by an electron per centimeter travelled in the direction of the field, E. $\beta_i(E)$ is the analogous quantity for holes. Following the notation of McKay, n_0 is the number of electrons entering the junction at $x=0$, n_1 is the number of electrons produced by electrons or holes between 0 and x, and n_2 is the number of electrons produced between x and W. Then the number of electrons produced between x and $x+dx$ is

$$dn_1=(n_0+n_1)(\alpha_i-\beta_i)dx+(n_0+n_1+n_2)\beta_i dx.$$

This is integrated with the boundary conditions $n_1=0$ at $x=0$ and $n_2=0$ at $x=W$. We then obtain

$$1-\frac{1}{M}=\int_0^W \alpha_i \exp\left[-\int_0^x(\alpha_i-\beta_i)dx'\right]dx, \quad (4)$$

where $M=(n_0+n_1+n_2)/n_0$ = the multiplication factor.

Breakdown occurs or $M\rightarrow\infty$ when

$$\int_0^W \alpha_i \exp\left[-\int_0^x(\alpha_i-\beta_i)dx'\right]dx=1. \quad (5)$$

It should be noted that in the case of those junctions in which the injected carriers are holes instead of electrons, equations (4) and (5) hold with the α_i's and β_i's reversed. When $\alpha_i=\beta_i$ these equations reduce to McKay's result. The assumptions implicit in them are the same as those of McKay:

(1) The ionization rates of both holes and electrons are only functions of the electric field.

(2) The loss of carriers in the junction by recombination is negligible.

(3) Mutual interactions between carriers are negligible.

(4) The density of carriers in the junction is small enough so that there is no change in the field configuration in the junction. That is, there are no space-charge effects.

In the case of a step junction like that already discussed, essentially all of the space-charge region is on the high-resistivity side and the electric field distribution is given by

$$E=E_Mx/W, \quad (6)$$

where $W-x$ is the distance from the junction. This distribution is shown in Fig. 1. The $x=0$ point has been chosen as the point where $E=0$ since the particles

[6] The reader is referred to W. Shockley, Bell System Tech. J. **28**, 435 (1949) for a detailed treatment of p-n junction theory.

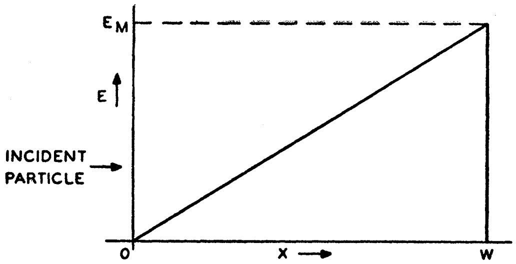

FIG. 1. Field distribution in *np* step junctions.

which cause the avalanche come from that direction in both an isolated reverse biased junction or in the collector of a transistor made by the alloy diffusion method which contains such a junction. Equation (4) then becomes

$$1-\frac{1}{M}=\tfrac{1}{2}W_1^2\int_0^{E_M}\alpha_i\exp\left[-\tfrac{1}{2}W_1^2\int_0^{E}(\alpha_i-\beta_i)dE'\right]dE. \tag{7}$$

The quantity

$$\alpha_i\exp\left[-\tfrac{1}{2}W_1^2\int_0^{E}(\alpha_i-\beta_i)dE\right],$$

which appears in the above equation can be thought of as an effective ionization rate. This ionization rate is no longer only a function of E. It is a function of the width constant of the junction and the electric field. For the same W_1 and V/V_B values, M would be different depending on whether the avalanche started with holes or electrons since the effective ionization rate expression is not symmetrical in α_i and β_i.

Equation (7) is differentiated with respect to E_M, yielding:

$$\alpha_i(E_M)=\frac{2}{W_1^2}\frac{d(1-1/M)}{dE_M}\exp\left[\tfrac{1}{2}W_1^2\int_0^{E_M}(\alpha_i-\beta_i)dE\right]. \tag{8}$$

The expression $(2/W_1^2)[d(1-1/M)/dE_M]$ reduces to $(2n/W_B)(E_M/E_{MB})^{2n-1}$ when the empirical analytic form for M is used, and Eq. (8) becomes

$$\alpha_i(E_M)=\frac{2n}{W_B}\left(\frac{E_M}{E_{MB}}\right)^{2n-1}\exp\left[\tfrac{1}{2}W_1^2\int_0^{E_M}(\alpha_i-\beta_i)dE\right], \tag{9}$$

where W_B is the width of the junction and E_{MB} the maximum field in the junction at breakdown.

Equation (9) was derived for the case in which the initial particles entering the junction are electrons. The analogous expression for holes is obtained by interchanging the α's and β's throughout.

$$\beta_i(E_M)=\frac{2n}{W_B}\left(\frac{E_M}{E_{MB}}\right)^{2n-1}\exp\left[\tfrac{1}{2}W_1^2\int_0^{E_M}(\beta_i-\alpha_i)dE\right]. \tag{10}$$

In these expressions, the n value and the breakdown voltages (and hence W_B and E_{MB}) appropriate to different junctions can be determined by experiment. The exponential term, however, cannot be evaluated directly. It can be determined from the solution of a differential equation derived from the experimental data.[7]

Equations (9) and (10) are a set of simultaneous integral equations which in principle determine the values of α_i and β_i. The problem of evaluating the exponential term can be simplified considerably by choosing two junctions with the same net density of impurity centers on the high-resistivity side, i.e., with the same W_1. Two such junctions, a pn^+ and an np^+, will be called complementary junctions. The n values appropriate to these junctions will be designated respectively by n_α and n_β. There is an additional benefit from this choice since E_{MB} has been found to be the same within experimental accuracy for complementary junctions.

It is shown in the Appendix in a purely mathematical exercise that a differential equation,

$$H''-\left(\frac{n_\alpha}{n_\beta}\right)y^{(n_\alpha/n_\beta)-1}H=0, \tag{11}$$

can be derived from a set of such integral equations. Here

$$y=(E_M/E_{MB})^{2n_\beta},$$

and

$$\frac{H'(y)}{H(y)}=\exp\left(-\tfrac{1}{2}W_1^2\int_0^{E_M}(\alpha_i-\beta_i)dE\right).$$

The solution of this equation, subject to the appropriate boundary condition, gives the exponential term in Eqs. (9) and (10) in the form of Bessel functions which can be evaluated by series expansion. Therefore a complete solution for $\alpha_i(E_M)$ and $\beta_i(E_M)$ for any value of $E_M \leq E_{MB}$ is obtainable from the knowledge of n_α, n_β and the breakdown voltage for a set of complementary junctions.

EXPERIMENT

A series of experiments has been performed with transistors to determine n in the expression for multiplication (3). The transistors were made by the alloy diffusion method[8] which gives step junctions similar to those discussed above. The carriers were injected into the reverse biased collector junction from the nearby emitter junction. The current arriving at the collector is given by the emitter junction current times the current transport efficiency or current gain of the transistor, α. The multiplication measurements could then be made by determining the charge flowing across

[7] The author is indebted to P. A. Wolff for this method of attack.

[8] J. S. Saby and W. C. Dunlap, Jr., Phys. Rev. 90, 630 (1953).

the collector barrier as a function of the bias with the emitter current held constant. Figures 2 and 3 give the results of such measurements on collector characteristics and the values of n determined in this way for representative *pnp* and *npn* transistors. For reasons cited below, this is at best an approximate way to determine the parameter n appropriate to a given junction. However the difference between the multiplication of the two collector junctions shown is unmistakable. These values are inaccurate because a slight uncertainty in the body breakdown voltage, V_B, would affect n greatly. Furthermore, the alpha of a transistor varies with the bias on the collector junction for several reasons. The principal effect is that caused by changes in the majority carrier flow pattern in the base with increasing multiplication of the collector. There are voltage drops in the base region which tend to concentrate the emission of minority carriers at the center of

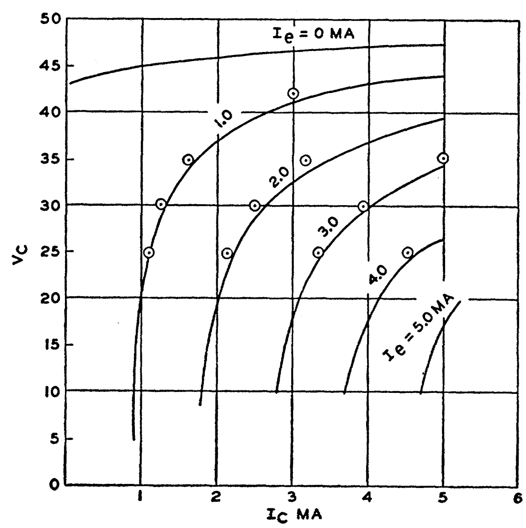

FIG. 2. Collector characteristics for a *pnp* transistor with about 0.7 Ω-cm base layer resistivity. ⊙ designates points calculated from $M=[1-(V/V_B)^{2.6}]^{-1}$.

the emitter and also as a secondary effect, give rise to a sweeping field toward the collector. This causes alpha to rise with increasing multiplication. In addition to this, the increase in the width of the collector depletion region with voltage brings the collecting boundary closer to the emitter with a consequent increase in alpha. This latter effect can be minimized in transistors with wide base layers.

A far more efficient and exact method for determining n is given below. The breakdown voltage is measured between the base and collector and between the emitter and collector (with the base floating) on a group of transistors made with the same resistivity base material. These two quantities are different. In the former case the breakdown voltage of the collector junction, that voltage at which $M=\infty$, is seen. In the latter case an apparent breakdown is observed at the voltage V_M,

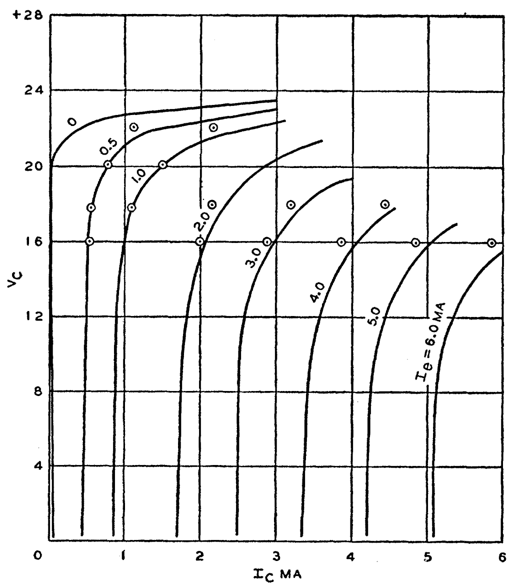

FIG. 3. Collector characteristics for an *npn* transistor with about 0.25 Ω-cm base layer resistivity. ⊙ designates points calculated from $M=[1-(V/V_B)^5]^{-1}$.

at which the low voltage α of the transistor, α_0, times the multiplication in the collector junction equals one, that is

$$\alpha_0 M(V_M)=1. \qquad (12)$$

From Eqs. (12) and (3), the relation

$$1-\alpha_0=(V_M/V_B)^n \qquad (13)$$

is obtained. Thus if $\log(V_M/V_B)$ is plotted *vs* $\log(1-\alpha_0)$ for each group of transistors made on the same material, the points should fall on a straight line of slope $1/n$. The experimental data when plotted in this way do define straight lines. Furthermore, the straight lines go through the $V_M/V_B=1$, $1-\alpha_0=1$ point as they should if the form of equation (3) is correct. Should the breakdown voltage of the junctions actually have been consistently due to a surface breakdown which is lower than the body breakdown of the junction this method of treating the data would have automatically detected it. The line through the experimental points would then have been parallel to the true line but above it by the

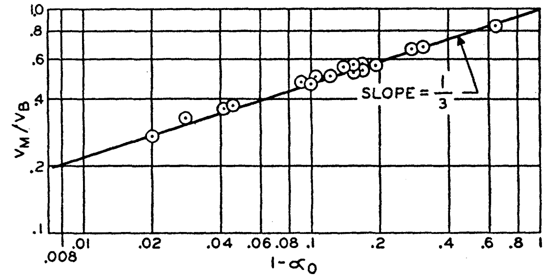

FIG. 4. V_M/V_B *vs* $1-\alpha_0$ for *pnp* transistors with 0.1 Ω-cm base layer resistivity.

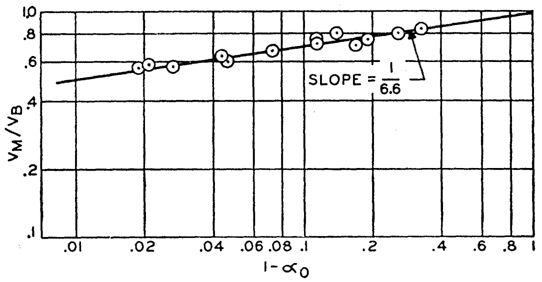

FIG. 5. V_M/V_B *vs* $1-\alpha_0$ for *npn* transistors with 0.5 Ω-cm base layer resistivity.

logarithm of the ratio of the body breakdown voltage to the surface breakdown voltage.

Figures 4 and 5 are typical plots of V_M/V_B *vs* $1-\alpha_0$. The large variation in α_0 is effected by geometrical changes involving the width of the base region and the relative sizes of the emitter and collector. In this manner the n values for a limited range of resistivities of n and p type has been determined. The ranges are restricted by the limitations of the present transistor technology. Table I gives the n values appropriate to step junctions *vs* the resistivity and resistivity type of the high-resistivity side. The *npn* transistors were made by alloying Pb As buttons onto wafers of p-type Ge, while the *pnp* transistors were made by the indium-alloying process on n-type wafers.

It should be noted that, in these measurements of the functional form of the multiplication, purely transistor effects have been minimized. All of the measurements are taken at the point where the alpha of the transistor times the multiplication equals unity or the base current is zero. Therefore there are no transverse majority carrier base currents to change the geometrical distribution of emission from the emitter junction.

The width of the space-charge region and the maximum field in the junction at breakdown are determined from the breakdown voltage as a function of the width constant. Step junctions of both the alloy type and grown-single-crystal type have been investigated for the breakdown point. Figure 6 gives the result *vs* the net number of impurity centers on the high-resistance side of the junction. The number of impurity centers on the high-resistance side of the junction was determined from measurements of capacitance per unit area.

TABLE I. The parameter n as a function of resistivity and resistivity type.

n	High ρ side of step junction
4.7	0.15 Ω-cm, p type
5.5	0.25 Ω-cm, p type
6.6	0.5 Ω-cm, p type
~6	2 Ω-cm, p type
3	0.1 Ω-cm, n type
3.4	0.6 Ω-cm, n type
3	2 Ω-cm, n type

All the junctions, whether grown or alloy or having n or p type on the high-resistivity side, fall on the same line on this log log plot. It is not incompatible with a difference between the ionization rates of holes and electrons that np^+ and pn^+ junctions fall on the same curve. At very high multiplications the number of electrons and holes participating in the avalanche are essentially equal and so the average effective ionization rate would be very close to the average between that for electrons and holes individually. In a junction across which the field is constant as a function of distance, it can be shown rigorously that the breakdown voltage would be the same regardless of whether the initiating particles are electrons or holes. Since the ionization rates are a very rapidly rising function of field strength, the high-field end of a step junction can be approximated by a plateau of constant field and the rest of the junction neglected. Insofar as this approximation is valid, the breakdown voltages of a set of complementary junctions would be the same.

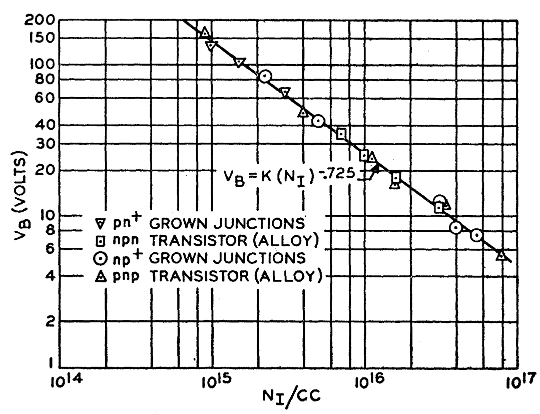

FIG. 6. The breakdown voltage *vs* the net number of impurity centers per cubic centimeter on the high resistivity side of step junctions.

The equivalence in breakdown properties between alloy and grown junction indicates that whatever strains or imperfections the alloying process introduces near the junction ordinarily do not affect the body breakdown process. The slope of the breakdown voltage plot is approximately -0.725 whereas it should have been -1.0 if the Zener theory of breakdown held. Since there is no tendency toward this latter slope even at the highest impurity concentrations, which correspond to the highest electric fields, it is considered that this plot gives strong evidence that the internal field emission has not been observed in this experiment. The lowest breakdown voltage observed corresponds to a maximum electric field of 320 000 volts/cm. Therefore internal field emission does not occur below this field strength in germanium.

A serious experimental problem in this type of investigation is the differentiation between breakdown on the surface of the semiconductor and in the body of the junction. The measurement of the multiplication below breakdown is one method which has already been mentioned. However, there is considerable experimental evidence that many surface breakdowns are also multiplicative. But, in experiments like those discussed above for the determination of the parameter n, very few of the carriers coming from the emitter would be collected by such a surface breakdown region. In the case of isolated junctions, measurements of the resistance after breakdown is a good indication. Breakdowns occurring near the surface in regions of small cross section have considerably higher series resistance than body breakdowns. Therefore pulse measurements of voltage at high currents can be used to separate the two. For ordinary junctions there will always be a small positive resistance in the breakdown region because of the body resistance of the semiconductor on both sides of the junction and because of the effect of the space charge of the carriers in the high field region with increasing current.

The quantity $2n/W_B$ is determined from the breakdown and multiplication data for different junctions in the limited range of resistivities accessible. These values are plotted against the maximum field in the junction at breakdown, E_{MB}, in Fig. 7. From curves through these experimental points, the values of n and $2n/W_B$ can be estimated very closely for two complementary junctions. (These have the same E_{MB}.) Two sets of complementary junctions were chosen. Set 1 has a maximum electric field at breakdown of 237 kv/cm, $n_\alpha \simeq 6$ and $n_\beta \simeq 3$. Set 2 has an E_{MB} of 260 kv/cm, $n_\alpha \simeq 4.5$ and $n_\beta \simeq 3$.

For the first set of junctions, the equation to be solved is

$$H''-2yH=0.$$

The solution of this equation is

$$\exp\left(-\tfrac{1}{2}W_1{}^2\int_0^{E_M}[\alpha_i(E)-\beta_i(E)]dE\right)=-\frac{H'(y)}{H(y)}$$

$$=\frac{1-y+\frac{2}{3}y^3-(2/15)y^5+(1/18)y^6\cdots}{1-y+\frac{1}{3}y^3-\frac{1}{6}y^4+(1/45)y^6-(1/126)y^7\cdots}.$$

For the second set of junctions the solution of the appropriate equation is

$$\exp\left(-\tfrac{1}{2}W_1{}^2\int_0^{E_M}[\alpha_i(E)-\beta_i(E)]dE\right)=-\frac{H'(y)}{H(y)}$$

$$=\frac{1-y^{3/2}+\frac{3}{5}y^{5/2}-(15/100)y^4+(9/175)y^5\cdots}{1-y+\frac{2}{5}y^{5/2}-(6/35)y^{7/2}+(3/100)y^5-(3/350)y^7}.$$

With the use of these functions, the values of $\alpha_i(E_M)$ and $\beta_i(E_M)$ were determined for (E_M/E_{MB}) equal to 0.95, 0.9, 0.8, and 0.7 in both cases. The series are highly convergent for these values since $y=(E_M/E_{MB})^{2n\beta}$. The results of these calculations are plotted in Fig. 8. The electron ionization rate curve is about a factor of two below the curve for holes in this region. Points obtained from the two sets of junctions are in excellent agreement with each other and are therefore consistent with the assumption that the ionization rate per unit path length is primarily only a function of field strength.

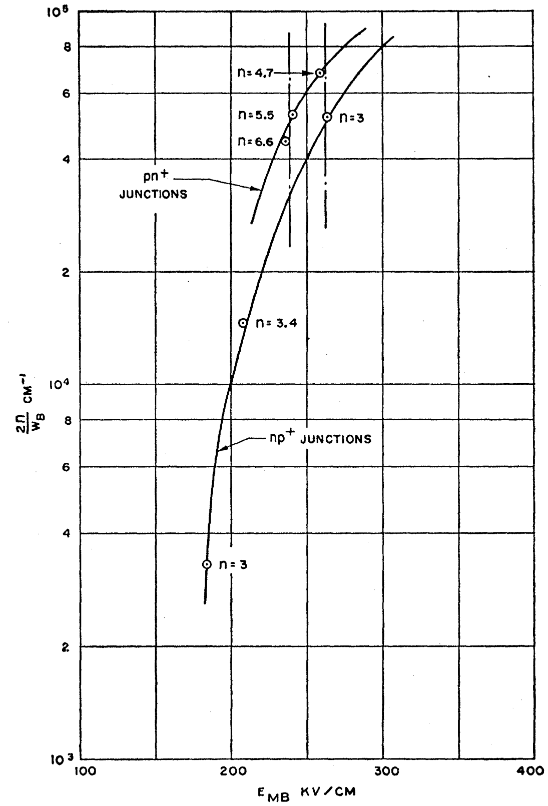

FIG. 7. The quantity $2n/W_B$ *vs* the maximum field in the step junction at breakdown. The dashed lines represent the two sets of complementary junctions chosen for the ionization rate calculations.

A theoretical curve calculated by Wolff for an assumed mean free path between electron (or hole)—phonon collisions of 130A and a threshold for electron-hole pair production of 1.5 ev is also given in Fig. 8. This curve corresponds almost exactly with the experimental ionization rate curve for holes while the electron curve is parallel and slightly below. Such a small difference between the electron and hole ionization rate corresponds to only about a 5 to 10% positive change in the value of the ratio of energy threshold to mean free path in going from holes to electrons. This could be made up of changes in either or both of the relevant parameters. The threshold for pair production could certainly be different for electrons and holes since the band shapes are known to be different.[9] There could also be some variation in the mean free path due to a

[9] C. Kittel, Physica **20**, 829 (1954).

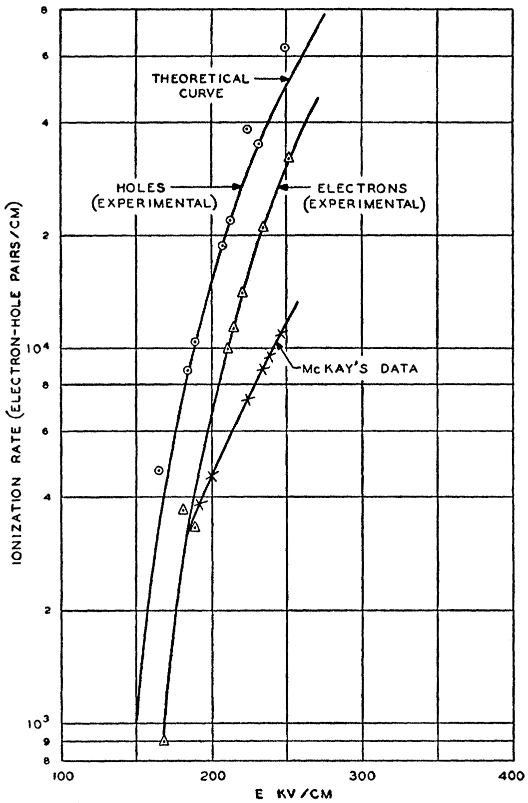

FIG. 8. Ionization rate *vs* electric field strength for Ge. ⊙ calculated from the experiment for holes. △ calculated from the experiment for electrons. × obtained by McKay from his experiment on a linear graded Ge junction. The experimental curve for holes was drawn to coincide with the theoretical curve.

difference in the coupling of electrons and holes to the optical phonon field.

The values of 1.5 ev and 130 A for energy threshold and mean free path are to be compared with the values 2.3 ev and 130A obtained for silicon by Wolff[5,10] from McKay's data. These sets of parameters are consistent with each other in that in both cases the energy threshold is slightly above twice the energy gap.

It would be appropriate here to examine the physical reason why such a relatively small difference between the ionization rates for holes and electrons is reflected in such a large difference in the empirical multiplication laws for np^+ and pn^+ junctions. In a pn^+ junction where the initial current is made up of electrons, the average particle ionization rate starts on the electron curve for low voltages or electric fields (and low multiplication) and as the voltage and multiplication increase the average particle ionization rate rises toward the hole curve since holes make up a higher and higher proportion of the particles participating in the avalanche. Therefore the multiplication rises very sharply with increasing voltage. On the other hand, similar arguments would tend to make the rise in multiplication of np^+ junctions far more gradual.

In Fig. 8 are also plotted some ionization rates *vs* electric field obtained by McKay on a linear graded germainium junction by his method. The agreement between the two methods is rather good. The discrepancy is probably due to the neglect of the difference in ionization rates for electrons and holes and the greater experimental difficulties and uncertainties attendant upon McKay's experiment.

CONCLUSIONS

Data on breakdown and multiplication *vs* bias of germanium junctions have been used to obtain ionization rates for both holes and electrons in high fields. The ionization rate for holes is greater than that for electrons by about a factor of two. The variation of ionization rate with field strength is in excellent agreement with the theoretical predictions of Wolff.

A by-product of this agreement with theory is a determination of the mean free path for electron (or hole)—phonon collisions and the energy threshold for pair production in Ge. These are respectively about 130A and 1.5 ev. These numbers individually are subject to considerable uncertainty but their ratio is probably quite good.

The experimental evidence is strong that there is no field emission in germanium up to a field strength of at least 320 000 volts/cm. Therefore it is concluded that all breakdowns (not occurring on the surface) of germanium junctions above about 5 volts are the result of McKay's avalanche process, although the multiplication inherent in such a process was actually measured in only some of the junctions. These did, however, cover the whole range investigated.

ACKNOWLEDGMENTS

This work has been made possible by the cooperation of a great many members of the Bell Telephone Laboratories Staff. Particular acknowledgment is due W. C. Meyer who gathered most of the experimental data and J. J. Ebers, J. L. Moll, J. M. Early, and P. A. Wolff for many enlightening discussions on this subject. Thanks are also due K. G. McKay for making his unpublished data on germanium available and for a critical reading of the final manuscript.

APPENDIX

Equations (9) and (10) of the text are rewritten using the relation $W=\frac{1}{2}W_1{}^2E_M$ which can be derived from (1). Then

$$\frac{2n_\alpha}{E_{MB}}\left(\frac{E_M}{E_{MB}}\right)^{2n_\alpha-1}=\frac{W_1{}^2}{2}\alpha_i(E_M)I(E_M), \qquad (14)$$

[10] P. A. Wolff (personal communication). The value for the mean free path has been corrected from that appearing in reference 5.

and

$$\frac{2n_\beta}{E_{MB}}\left(\frac{E_M}{E_{MB}}\right)^{2n_\beta-1}=\frac{W_1^2}{2}\frac{\beta_i(E_M)}{I(E_M)}, \qquad (15)$$

where

$$I(E)=\exp\left(-\frac{W_1^2}{2}\int_0^E(\alpha_i-\beta_i)dE\right).$$

But

$$\frac{W_1^2}{2}\alpha_i(E_M)I(E_M)=-\frac{dI(E_M)}{dE_M}+\frac{W_1^2}{2}\beta_i(E_M)I(E_M).$$

When this relation is substituted into Eq. (14) it becomes

$$\frac{dI(E_M)}{dE_M}-\frac{2n_\beta}{E_{MB}}\left(\frac{E_M}{E_{MB}}\right)^{2n_\beta-1}[I(E_M)]^2 =-\frac{2n_\alpha}{E_{MB}}\left(\frac{E_M}{E_{MB}}\right)^{2n_\alpha-1}. \qquad (16)$$

Now a new variable $y=(E_M/E_{MB})^{2n_\beta}$ is introduced. Equation (16) then simplifies to

$$\frac{dI(y)}{dy}-[I(y)]^2=-\frac{n_\alpha}{n_\beta}y^{(n_\alpha/n_\beta)-1}. \qquad (17)$$

This can be made linear by the substitution $I(y)=-H'(y)/H(y)$. The result is

$$H''-\frac{n_\alpha}{n_\beta}y^{(n_\alpha/n_\beta)-1}H=0. \qquad (18)$$

Equation (18) is a variation of Bessel's equation whose solutions are of the form

$$H(y)=K_1(\surd y)J_{n_\beta/(n_\alpha+n_\beta)}\left[\frac{2i(n_\alpha/n_\beta)^{\frac{1}{2}}}{(n_\alpha/n_\beta)+1}y^{\frac{1}{2}(n_\alpha/n_\beta)+\frac{1}{2}}\right] +K_2(\surd y)J_{-n_\beta/(n_\alpha+n_\beta)}\left[\frac{2i(n_\alpha/n_\beta)^{\frac{1}{2}}}{(n_\alpha/n_\beta)+1}y^{\frac{1}{2}(n_\alpha/n_\beta)+\frac{1}{2}}\right]. \qquad (19)$$

The boundary condition which this solution must obey is that

$$-H'(y)/H(y)=1 \quad \text{when} \quad y=0.$$

Since the highest value of y which is of physical interest is unity, the series expansion for the Bessel function is highly convergent and can be used effectively to evaluate the Bessel functions of fractional order and imaginary argument which appear.

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Gyromagnetic Ratio of Iron at Low Magnetic Intensities

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It is shown that the measured value for the gyromagnetic ratio of pure iron as determined by a direct magneto-mechanical method, undergoes a change for low values of the induced magnetic intensity. The effective value of this ratio extrapolated to zero intensity, checks the value which is theoretically expected from a consideration of recent ferromagnetic resonance experiments. For higher values of the induced magnetic intensity, the gyromagnetic ratio approaches a constant value which checks the value obtained in previous investigations based upon the Einstein-de Haas effect.

INTRODUCTION

FOR a number of years an investigation of the gyromagnetic ratios of the iron-nickel alloy series has been under way at the General Motors Research Laboratories. It was noted during this investigation that the measured value of the gyromagnetic ratio (ρ) for a given alloy always increased for low values of magnetic intensity. Although the shape of the curve of magnetic intensity *vs* gyromagnetic ratio appeared to be different for different concentrations of nickel in iron, there was nevertheless, always an increase in ρ for small induced magnetic intensities. Also when the largest obtained ρ values were plotted against concentration of nickel in iron it was found that all points from 0 percent Fe 100 percent Ni to 90 percent Fe 10 percent Ni fell on a smooth curve. The value for pure iron which had been previously determined was, however, an exception. It was, therefore, decided to undertake an extensive investigation of pure iron varying the induced magnetic intensity, and going down to as low a value of magnetic intensity as was practicable. It was found that a similar effect exists for pure iron also. Furthermore, when the curve of ρ *vs* intensity, was extrapolated to zero intensity, the gyromagnetic ratio was found to have a value which checks the value which is theoretically expected from a consideration of recent work in ferromagnetic resonance. For high values of magnetic intensity the gyromagnetic ratio approaches a constant value, which checks previous work done at the General Motors Research Laboratories, and also the work of other investigators.