# *T-F* Emission: Experimental Measurement of the Average Electron Current Density from Tungsten*

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An experimental study of temperature-and-field emission is described in which the average electron current density from the tungsten monocrystal was measured at several fields and temperatures in the ranges $2.5\times10^7<F<7\times10^7$ v/cm and $300°<T<2000°$K, respectively. The observed agreement between measured and calculated values supports the wave mechanical theory over larger combined ranges of the variables than previously reported, and further confirms that a high cathode temperature is an arc-initiating factor.

## INTRODUCTION

A PREVIOUS paper[1] presented calculated values of the electron current density emitted from metals in the presence of both a high temperature $T$ and a large surface electric field $F$. Under such conditions the emission process has been called "temperature-and-field emission" (abbreviated $T$–$F$ emission), because current density was found to be sensitive to the values of both variables over their experimentally useful range. $T$–$F$ emission is a process which is distinct from both field emission (large $F$ and small $T$) and thermal emission (large $T$ and small $F$).

This paper presents an experimental study of $T$–$F$ emission in which the average electron current density from a tungsten monocrystal was measured at several fields and temperatures in the ranges $2.5\times10^7<F<7\times10^7$ v/cm and $300°<T<2000°$K, respectively. The experimental values are found to be in good agreement with those calculated by the methods of reference 1.

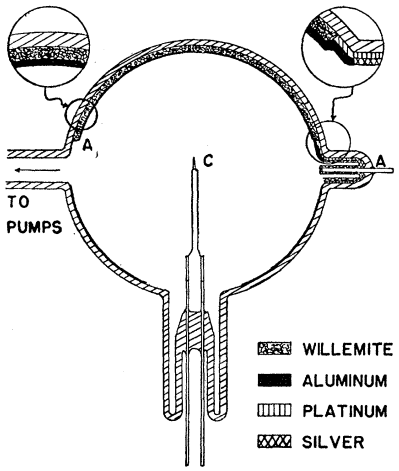


FIG. 1. Experimental electron projection tube: (*C*) needle shaped tungsten cathode; (*A*) aluminum-backed-willemite anode; anode-cathode spacing 4 cm. For details of anode construction see reference 7.

The increase of $T$–$F$ emission current density with temperature, over the initial field current value, is consistent with the increase in current density observed just prior to the transition between field emission and the vacuum arc.[2] This result adds further support to the previous conclusion that a high cathode temperature is an arc-initiating factor.[1–4]

The present work extends the quantitative test of the wave mechanical emission theory to combined high values of temperature and field; previous quantitative tests at high fields have been restricted to low cathode temperatures.[3,5]

## EXPERIMENTAL METHOD

For the present experiment, it was necessary simultaneously to measure or determine values of current density, electric field, work function and temperature at a metal cathode during the $T$–$F$ emission of electrons from its surface. An experimental method was used which involved a straightforward modification of established field emission techniques,[2,3] the modification including primarily a means for maintaining and measuring a desired cathode temperature.

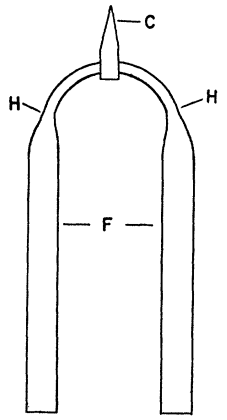


FIG. 2. Cathode assembly including: (*C*) needle shaped tungsten cathode; (*F*) support filament; (*H–H*), heated portion of support filament. For details of construction see reference 8.

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[1] W. W. Dolan and W. P. Dyke, Phys. Rev. **95**, 327 (1954).

[2] Dyke, Trolan, Martin, and Barbour, Phys. Rev. **91**, 1043 (1953).

[3] W. P. Dyke and J. K. Trolan, Phys. Rev. **89**, 799 (1953).

[4] Dolan, Dyke, and Trolan, Phys. Rev. **91**, 1054 (1953).

[5] R. H. Haefer, Z. Physik **116**, 604 (1940).

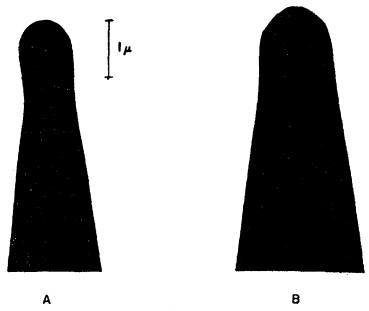


FIG. 3. Electron micrographs of needle shaped tungsten cathodes: (A), the cathode from which the data in Fig. 4 were obtained; (B) a "built-up" cathode which resulted from a 2-second application of an electric field of $3.4\times10^7$ v/cm and a temperature of 1900°K.

Electron emission was obtained from a tungsten single crystal in an electron projection tube.[6] With the aid of the observed emission pattern it was possible to detect and maintain conditions to be described under which the cathode surface was clean and free from geometric distortion. The work function for the clean tungsten surface is well known; the value used here, which is the average over all crystal faces, is $\phi=4.5$ ev. The projection tube is shown in Fig. 1; it consists of a needle-shaped tungsten cathode $C$ mounted on a support filament, and an aluminum-backed willemite anode $A$ on which the emission pattern was viewed. Fabrication of the particular tube design in Fig. 1, has been described elsewhere.[7] The tube was sealed off at an ion gauge pressure of $10^{-9}$ mm of Hg and further evacuated by the use of tantalum getters (not shown), resulting in a residual pressure of $10^{-12}$ mm of Hg for chemically active gases. The evacuation procedure has been described previously.[3] Figure 2 shows an enlarged schematic drawing of the cathode $C$ and support filament $F$. The cathode was tungsten, chosen for its mechanical strength, high melting point, and favorable electrical properties. That choice also permitted correlation of the present data with earlier experiments in field emission from cold tungsten. Fabrication of the cathode has been described.[8]

In order to provide electric field values up to $7\times10^7$ v/cm with reasonable values of the applied potential and electrode spacing, a needle-shaped cathode was used. The tungsten single crystal cathode approximated a cone of vertex angle 16 degrees with a hemispherical tip of radius $r=5\times10^{-5}$ cm [Fig. 3(A)]. The cathode projected 0.4 mm beyond the support filament $F$, Fig. 2. The electron-emitting area was restricted to a portion of the hemispherical tip; a method for calculating the value of the emitting area has been discussed.[8]

[6] E. W. Mueller, Z. Physik **106**, 541 (1937).
[7] Dyke, Trolan, Dolan, and Grundhauser, J. Appl. Phys. **25**, 106 (1954).
[8] Dyke, Trolan, Dolan, and Barnes, J. Appl. Phys. **24**, 570 (1953).

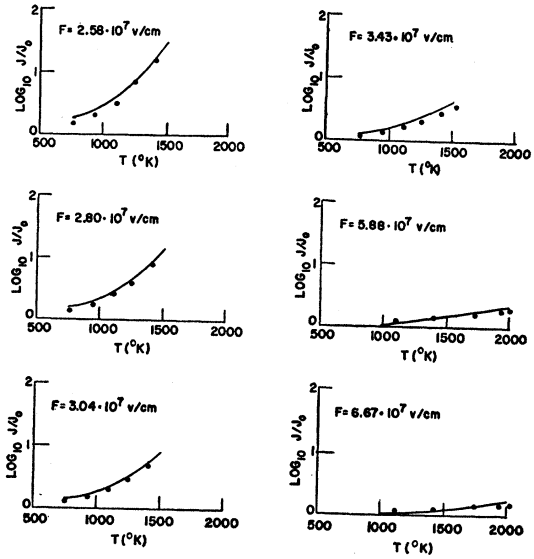


FIG. 4. Graphs relating $\log_{10}J/J_0$ to temperature $T$ in °K for several values of the electric field. $J_0$ is the average value of the field current density from the cold cathode; $J$ is the average value of the $T$–$F$ current density from the heated cathode at the same field. Circles represent experimental data; solid curves show values calculated by the methods of reference 1.

Thermal energy was supplied to the emitter by conduction from the filament $F$ (Fig. 2) which was maintained at a desired temperature by passing through it an electric current from an external source. Filament temperatures were measured in two ways; first, by use of a Leeds and Northrup optical pyrometer with a $1\frac{1}{2}$-inch lens, which provided adequate optical resolution of the heated section of the filament $HH$, and second, by comparing measured and calculated values of thermal emission obtained from the same filament section. The cathode tip temperature was calculated from the measured filament temperature after correction for radiation loss by a method similar to that used by Mueller.[9]

The total electron emission current from the cathode, the applied voltage, the emission pattern photograph, and the cathode geometry were recorded by methods previously described.[2,3,7,8] From these data were calculated values for the cathode electric field[8] and the average cathode current density,[7] both of which will be useful in Fig. 4. Calculation of the latter by the indicated methods required that thermal emission from the emitter shank and support filament be negligible compared with $T$–$F$ emission from the hemispherical emitting tip. This was achieved by restricting the temperature to values below 2000°K and minimizing the area of the heated portion of the cathode surface to the section $HH$, Fig. 2. For the latter purpose, the section $HH$ was reduced from its original diameter of 10 mils to a diameter of 3 mils by electrolytic etch[8] in a concentrated solution of NaOH.

[9] E. W. Mueller, Z. Physik **126**, 642 (1949).

For two reasons, the present experiments were conducted in a range of current densities ($J<10^7$ amp/cm$^2$) for which space charge effects were negligible.[10]

First, it was then possible to calculate the cathode electric field $F$ from

$$F=\beta V$$

where $\beta$ is a constant computed from potential theory using measured values of the electrode geometrics, and $V$, the applied voltage.[8] With $F$ in v/cm and $V$ in volts, $\beta=3.9\pm0.2\times10^3$ cm$^{-1}$ for the cathode of Fig. 3(A). Second, restriction of current densities to the indicated range avoided space-charge limitation of thermally dependent increases in current density.

It was necessary to obtain the experimental data in a time short compared with that in which geometric distortion of the cathode was observed, since it is known that the hemispherical tungsten cathode tip assumes a polyhedral shape after prolonged exposure simultaneously to high field and high temperature.[11,12] In that process, called "buildup," crystal faces with low indices such as the (110), (211), and (100) become planes of extended area and the intervening surfaces develop small radii of curvature. Buildup is illustrated in Fig. 3(B), a form which resulted during a time of 2 seconds at a field $F=3.4\times10^7$ v/cm and a temperature $T=1900$°K. Buildup undesirably intensifies electric field and current density and makes their measurement difficult at those cathode surfaces whose radii of curvature are decreased.

Buildup was avoided in the present work by use of two techniques. At the higher values of $T$ and $F$, the electric field was applied for short time intervals by use of electronic pulse techniques; in this case the data were obtained during microsecond intervals on a single-pulse basis, both current and voltage being recorded as oscillographs by methods previously described.[2,3] At the lower fields, where the reduced current yield makes the use of pulse techniques difficult, the temperature was limited to a range in which buildup was negligible; in this case data were recorded in a few seconds using previously described direct-current techniques[3] which were modified only to include a current-metering system with fast response. For steady-state observations, current was determined from the voltage drop across a known resistance in series with the cathode, the voltage being measured by a Hewlett Packard vacuum-tube voltmeter.

Proof that buildup was negligible under the conditions used herein was provided by both emission pattern photographs and electron micrographs of the emitter which were obtained after the most severe usage; appreciable buildup is detectable both from changes in the emission pattern[11] and in the emitter geometry.[12] Additional evidence was provided by the reproducibility of the data. For this purpose, current and voltage were recorded from the cold emitter, then from the heated emitter and finally from the cold emitter, all at the same value of electric field; comparison of initial and final readings determined the reproducibility of the data.

## EXPERIMENTAL RESULTS

The experimental data are presented in Fig. 4 in graphical form relating $\log(J/J_0)$ to cathode tip temperature $T$ in °K for 6 values of the electric field in the range $2.58\times10^7<F<6.67\times10^7$ v/cm. $J$ is the measured value of the $T$–$F$ current density averaged over the emitting portion of the cathode hemispherical tip at the indicated temperature, while $J_0$ is the average value of the field current density from the cold cathode. The experimental data are represented by circles, and the solid curves represent values calculated by the methods of reference 1. While the absolute value of a particular electric field was known within 10%, the relative values of the various fields were known within 1%, which encouraged retention of the third significant figure in the field values used in the calculations on which Fig. 4 were based.

Corrections for the increase in work function with temperature,[13,14] and for the change in cathode electric field due to its geometric change during thermal expansion, were calculated for typical cases chosen from the present data. Since the corrections were comparable in magnitude to experimental error, they were ignored in preparing Fig. 4.

The experimental data for $F<4\times10^7$ v/cm were obtained by direct current techniques and are limited to temperatures less than 1550°K for which buildup was judged negligible. For fields $F>4\times10^7$ v/cm, the field was applied and data were obtained during single microsecond intervals, and no buildup was observed at the highest temperature shown, $T=2000$°K. As a check on the present work, measurements were made by a second method in which cathode current densities were determined by a photometric measurement of the light output from the phosphor screen.[7]

The experimental data quantitatively confirm the ratios of $T$–$F$ and field emission current densities that are predicted in reference 1 for the stated ranges of temperatures and fields. The confirmation includes the predicted trend of decreasing temperature effect at higher fields. These results also appear to be in good agreement with the earlier work of Millikan and Eyring[15] over the more limited range of temperatures and fields reported in that work.

One result of the present investigation is further to establish a high cathode temperature as an initiating

[10] Barbour, Dolan, Trolan, Martin, and Dyke, Phys. Rev. **92**, 45 (1953).
[11] J. A. Becker, Bell System Tech. J. **30**, 907 (1951).
[12] R. O. Jenkins, *Reports on Progress in Physics* (The Physical Society, London, 1943), Vol. 9, p. 177.
[13] S. Seely, Phys. Rev. **59**, 75 (1941).
[14] J. A. Becker and W. H. Brattain, Phys. Rev. **45**, 694 (1934); A. R. Hutson, Phys. Rev. **98**, 889 (1955).
[15] R. A. Millikan and C. F. Eyring, Phys. Rev. **27**, 51 (1926).

factor for the transition between field emission and the vacuum arc.[1-4] The most direct experimental evidence to that effect was a large increase in current density over the initial field emission value which was observed just prior to arc initiation[2]; the increase can reasonably be attributed to the added effect of high temperature in view of the present results and an earlier calculation[4] of the expected resistive heating of the emitter.

A second result of the present investigation is further to support the validity of the wave mechanical theory which has been used to describe both the field emission[16] and the $T$–$F$ emission[1] processes and to extend the quantitative test of the theory to higher combined values of temperature and field than previously reported.[3,5]


### ACKNOWLEDGMENTS

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[1] A. Sommerfeld and H. Bethe, *Handbuch der Physik* (Verlag Julius Springer, Berlin, 1933), Vol. 24, Part 2, p. 441.

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# Hall Effect and Density of States in Germanium

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The usual assumptions that underlie the analysis of Hall data to obtain activation energy and density of states are based on the simple model of the band structure. They are re-examined here in the light of the known band structure of germanium. In particular, consideration is given to the limits of validity of the assumption that carrier concentration equals $1/Re$. Also, energy levels of electrons bound to donors are investigated and it is shown that the effect of excited states, particularly for Sb-doped germanium, cannot be neglected. The density-of-states mass values obtained from Hall data for $p$- and $n$-type material are considered. In the latter case, the results are shown to favor a band edge at the surface of the Brillouin zone rather than inside.

## INTRODUCTION

ANALYSIS of variation of Hall constant with temperature has been an important source of information on activation energy of impurities and density of states in semiconductors. It has been customary to base such analysis[1] on two principal assumptions:

(1) The reciprocal of the product of Hall constant and charge on the electron equals total carrier concentration.

(2) Variation of carrier concentration, written for an $n$-type semiconductor, is given by

$$\frac{n(N_A+n)}{N_D-N_A-n}=\left(\frac{2\pi m^*kT}{h^2}\right)^{\frac{3}{2}}e^{(E_1-E_c)/kT}, \qquad (1)$$

where the notation is the usual one.[1]

These assumptions are known to be quite reasonable for semiconductors with the band structure usually assumed, the so-called simple model. The fact that neither the conduction band nor the valence band of germanium has this simple structure makes re-examination of these assumptions desirable for these cases. The first assumption, in particular, has been questioned for $p$-germanium because of the existence of two types of holes. In the next section the range of validity of this assumption will be examined. In the succeeding section the necessity for modifications in the equation which constitutes the second assumption will be considered. It will be shown in particular that the effect of excited states, which are not included in Eq. (1), will not be negligible in $n$-germanium. Corrections will therefore be required for activation energies and effective masses obtained from Hall data with the use of (1). In a final section, densities of states calculated from Hall data will be compared with theory. In the case of $n$-type material this is of particular interest because these results supplement the cyclotron resonance results and, at least in principle, permit determination of the number of valleys in the conduction band.

The considerations of this paper will be limited to high-resistivity samples, ones having carrier concentrations less than $10^{15}/cm^3$. One advantage of this is that complicating effects such as variation of activation energy with concentration and temperature should be small. Also, consideration will be limited to samples doped with impurities of small activation energy, of the order of 0.01 ev. For such samples the temperature

[1] See, for example, P. P. Debye and E. M.Conwell, Phys. Rev. 93, 693 (1954).