The value of r_0 used in I was 1.09 A, so the inequality is easily satisfied. Consequently $(\partial p/\partial V)_T<0$, for $T<T_\lambda$, and the system is stable below the λ temperature. If we bear in mind that $(\partial p/\partial V)_T$ is discontinuous at T_λ then this implies that the system undergoes a second-order transition under conditions of constant pressure. The inequality (5) has a simple physical interpretation. It can only be satisfied if the potential is positive for some range of values of r. This simply means that it is the positive or "repulsive" part of the potential that prevents the particles from "condensing" into zero volume and exhibiting a first-order transition.

We can therefore conclude that a system of weakly interacting Bose particles undergoes a second-order transition both under conditions of constant pressure and constant volume. This behavior is qualitatively identical with that shown by liquid helium. These conclusions are, however, based on a treatment of a weakly interacting system and it remains to be shown that they carry over to the physically interesting case of a strongly interacting one.

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Quantum Effects in the Interaction between Free Electrons and Electromagnetic Fields

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An electron beam shot through a transverse rf field may suffer a directional spread. If experimental conditions are suitably chosen, the directional spread may be due only to the quantum dispersion of energy exchange between free electrons and rf field. A simple collector electrode system might allow not only the detection of the directional spread of the electrons, but the presence of a quantum effect might be checked by plotting the collector current *versus* rf field amplitude, the plot for the quantum effect being different from those for classical effects. The results of various theoretical treatments of the effect are briefly compared, both from the point of view of their principal foundations and of the possibility of their experimental verification.

IN recent years several papers[1–4] have been published dealing with that part of the dispersion in energy exchange between free electrons and rf fields which the classical theory cannot predict. The problem has been treated from various points of view and differing results were obtained for the energy spread due to the quantum nature of the interaction.

One method[1,3] consists essentially in the assumption of the independence of the elementary emission and absorption processes, so that statistical methods can be applied in treating them. In this case the magnitude of quantum dispersion of the velocity of the electrons can be estimated by taking the classical energy gain W_+ and energy loss W_- during the accelerating and decelerating periods and interpreting them as the average absorption and emission of $N_+=W_+/\hbar\omega$ and $N_-=W_-/\hbar\omega$ photons, respectively, with a standard deviation of

$$\Delta W=\hbar\omega(N_++N_-)^{\frac{1}{2}}. \quad (1)$$

Shulman[3] reports that he has found this effect by measuring the energy distribution of electrons shot through a longitudinal rf field in a wave guide. The difficulty of any experiment of this kind lies in the fact that the expected effect of quantum-mechanical origin and the classical energy spread resulting from the emission velocity distribution and from the dependence of transit time on the entrance phase angle of the electrons are of the same order of magnitude. The classical effects, however, can be made negligible in comparison with the quantum effect, if the electrons are shot through a transverse rf field and the transit angle of the electrons is properly chosen. In this case there should be a dispersion in the direction of the electron velocities caused only by the quantum nature of the energy exchange.

For the sake of simplicity we considered[5] a rectangular cavity excited in its TE_{012} mode. The only component of the electric field is, in this case:

$$E_z=E_0\sin(2\pi x/a)\cdot\sin(2\pi y/b)\cdot\sin\omega t,$$

the symbols having their usual meaning. For a given resonance frequency one can always find such a cavity that an electron entering along the x-direction with a given velocity v_0 spends an integral number of periods

[1] L. P. Smith, Phys. Rev. **69**, 195 (1946).
[2] J. C. Ward, Phys. Rev. **80**, 119 (1950).
[3] C. Shulman, Phys. Rev. **82**, 116 (1951); **83**, 4 (1951).
[4] I. R. Senitzky, Phys. Rev. **86**, 595 (1952); **90**, 386 (1953).
[5] P. S. Faragó and G. Marx, Acta Phys. Hung. **4**, 23 (1954).

k within it. It can be easily shown that such an electron (independently of its entrance phase angle) leaves the cavity without changing its direction, and the parallel shift of the beam cannot be detected for reasonable rf amplitudes and frequencies above 10 kMc/sec. If the initial velocity of the electrons is $v=v_0(1\pm\epsilon)$ ($\epsilon\ll 1$), a directional spread of the order of magnitude

$$\Delta\varphi_c\sim(2\pi eE_0/v_0m\omega)\epsilon$$

occurs.

As there is no component of the field parallel to v_0, the energy dispersion results only from the deviation Δv_z of the velocity component $v_z=dz/dt$, and thus produces a dispersion in the direction of the velocity. The calculations are straightforward and give for the directional dispersion:

$$\Delta\varphi_q\sim(e^{\frac{1}{2}}/m^{\frac{3}{4}}v_0)(k\hbar/\omega)^{\frac{1}{4}}E_0^{\frac{1}{2}}.$$

Consequently the ratio of the quantum dispersion in angle to the classical dispersion is

$$\Delta\varphi_q/\Delta\varphi_c\sim(k\hbar m\omega^3)^{\frac{1}{4}}(2\pi^2e)^{-\frac{1}{2}}E_0^{-\frac{1}{2}}\epsilon^{-1},$$

where k, an integer, is the number of periods spent by the electrons in the rf field. Taking for example 10-kev electrons, $\omega=2\pi\times 10^{10}$ cps, $k=10$, and $\epsilon=10^{-3}$, we obtain

$$\Delta\varphi_q\sim 10^{-3}E_0^{\frac{1}{2}}\text{ rad},\quad\text{and}\quad\Delta\varphi_q/\Delta\varphi_c\sim 10^4E_0^{-\frac{1}{2}},$$

where E_0 is measured in volts/meter.

It is obviously desirable to have a very well focused beam and a long drift space when carrying out the experiment. However, with the aid of a simple collector system the expected effect can be studied quantitatively without making extremely high demands on focusing.[5] The current density in an electron beam produced by a conventional electron gun as a function of the distance from the beam axis is given by the Gaussian distribution law. Within the limits of our approximate estimate, the directional distribution due to the quantum effect may be assumed to follow the same law. Thus the resulting distribution is also of the same kind, and the ratio of the beam currents outside a circle of radius R with and without the rf field in the cavity is

$$I/I_0=1+2(R+r_0)^2(L/r_0)\Delta\varphi,$$

where L is the length of the drift space and r_0 is the distance from the beam axis to the point where the current density reduces to $1/e$ times its value on the axis.

As the directional spread $\Delta\varphi$ is proportional to E_0 in the classical case, and it is proportional to $E_0^{\frac{1}{2}}$ if the quantum nature of the interaction is dominant, the two effects can be distinguished without ambiguity if I/I_0 is plotted *versus* E_0.

Another method, followed by Senitzky[4,6] and others, consists in the application of quantum electrodynamics to the oscillations maintained in a resonant cavity. These considerations do not yield any dispersion of the kind given by Eq. (1). If the electron spends an integral number of periods in the cavity, the energy gain computed classically and quantum electrodynamically is zero simultaneously.[4] This means that the elementary emission and absorption acts should not be considered as independent processes. The velocity distribution obtained by Senitzky in the first approximation of the perturbation theory [Eq. (31) of reference 6] contains as a result of the quantization of the field only a vacuum fluctuation term, the other terms resulting from the wave mechanical treatment of the electrons.[7] It may be noted that the equations can be solved exactly for certain modes of the cavity (the electric field must be independent of the coordinate measured along the path of the electron). The results are in agreement with the first approximation obtained by Senitzky: the dispersion of the energy absorbed by the electrons during their transit time is independent of the field intensity and the number of periods spent within the cavity, if the effects characteristic of the wave-mechanical treatment of the electrons (spreading out of the wave packet) are neglected.

An experimental check of the velocity distribution given by Senitzky seems also to be more promising if a direction spread caused by a transverse rf field is sought. By following his method of calculation for a transverse rf field, the velocity spread is found to be

$$\begin{aligned}\langle v_z^2\rangle-\langle v_z\rangle^2&=\langle\Delta v_z^2\rangle-\langle\Delta v_z\rangle^2\\&=(\hbar^2/2m^2b^2)(e^2E_0^2u_0^2/v_0^2\omega_0)A^2\\&\quad+\tfrac{1}{2}b^2(e^2E_0^2u_0^2/mv_0^2)B^2,\qquad(2)\end{aligned}$$

instead of Eq. (31) of reference 6, if one neglects the field fluctuation term. If the number of periods spent by the electrons in the rf field is odd, then $B=0$. Thus by decreasing the quantity b, the velocity spread can be increased to an appreciable value. In spite of this favorable situation the experimental results could not be interpreted without ambiguity. With $\Delta r=b/\sqrt{2}$ and $\Delta v=\hbar/\sqrt{2}mb$, we get from Eq. (2) an expression for the resulting velocity spread which depends on Δr and Δv, the initial spread of coordinates and of velocities respectively, but does not contain Planck's constant. This means that the spreads of coordinates and velocities due to technical reasons (which are obviously greater than those of quantum mechanical origin) contribute to the directional spread in the same manner, and consequently the two kinds of effects cannot be distinguished.

Experiments based upon the above considerations are in progress to study in detail the validity of the relation (1), which is the consequence of the assumption of the independent character of the elementary processes. Our results will be published in the Acta Physica Hungarica at a later date.

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[6] I. R. Senitzky, Phys. Rev. **95**, 904 (1954).

[7] It is curious that according to this result the zero-point fluctuation of the field can be increased by decreasing the volume of the cavity.