recombination level, it has a much greater probability of going to the valence band (p_0C_p/N_t) than returning to the conduction band (n_1C_n/N_t). Nevertheless to check against any possible unforeseen variations with temperature in the diffusion coefficient, the lifetime was also measured by a conductivity modulation technique, with substantially the same results. The conductivity modulation decays due to recombination of both electrons and holes, which do not necessarily have the same steady-state lifetime if the charge in the traps cannot be neglected. However, if it is presumed that the holes in the traps rapidly come to equilibrium with the holes in the valence band, as seems to be the case for the upper copper level, then both electrons and holes subsequently decay with the lifetime of the electrons.

The apparent exponential increase of C_n with increasing temperature is regarded as somewhat surprising. It is the result that would be anticipated if only the electrons of energy greater than a certain critical energy above the bottom of the conduction band were allowed to enter the upper copper levels. This energy as obtained from the slopes of Fig. 2 is approximately 0.22 ev.

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Ferromagnetic Resonance in Ferroxdure

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Ferromagnetic resonance experiments have been performed on Ferroxdure at various microwave frequencies from 9000 Mc/sec to 50 000 Mc/sec. With no externally applied field a natural resonance occurs near 50 000 Mc/sec. With magnetic field applied perpendicular to the hexagonal axis, a double resonance occurs for frequencies below 50 000 Mc/sec, while with a field applied along the hexagonal axis a single resonance occurs above 50 000 Mc/sec. A theory is developed to explain the above results.

FERROMAGNETIC resonance experiments have been performed on Ferroxdure at various microwave frequencies. Ferroxdure[1] is a ceramic permanent magnet material with high anisotropy having a hexagonal crystal structure and having essentially the composition $BaFe_{12}O_{19}$.

THEORY

The ferromagnetic resonance frequencies of a hexagonal crystal with high anisotropy along the hexagonal axis can be calculated as follows:

The anisotropy energy is given by $U=K\sin^2\theta$, where θ is the angle of the magnetization with respect to the hexagonal axis. Higher order terms in $\sin\theta$ as well as anisotropy within the basal plane can be neglected for Ferroxdure at room temperature. The torque due to the anisotropy is given by

$$T=-\frac{\partial U}{\partial\theta}=-\frac{\partial U}{\partial M_{xy}}\frac{\partial M_{xy}}{\partial\theta}=-\frac{2K}{M^2}M_{xy}M_z, \qquad (1)$$

or vectorially

$$\mathbf{T}=+(2K/M^2)\mathbf{M}\times\mathbf{M}_z=\mathbf{M}\times\mathbf{H}_A,$$

where $\mathbf{M}$ is the magnetization vector, $\mathbf{M}_z$ and $\mathbf{M}_{xy}$ are evident from Fig. 1, and $\mathbf{H}_A$ is an equivalent anisotropy field which would produce the same torque. Evidently,

$$\mathbf{H}_A=(2K/M^2)\mathbf{M}_z=A\mathbf{M}_z, \qquad (2)$$

so that the equivalent anisotropy field is along the hexagonal axis and is proportional to the magnetization along that axis.

In the standard equation of motion[2] for the magnetization $d\mathbf{M}/dt=\gamma\mathbf{M}\times\mathbf{H}$, one must take into account

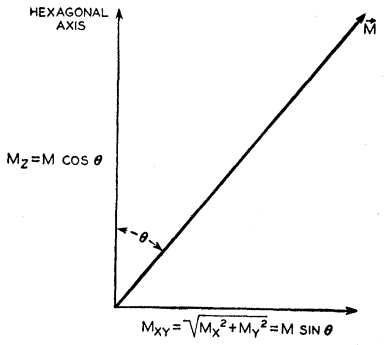

FIG. 1. Vectors used in calculating ferromagnetic resonance frequencies of Ferroxdure.

[1] Went, Rathenau, Gorter, and Van Oosterhaut, Philips Tech. Rev. **13**, 194–208 (1952).

[2] C. Kittel, *Introduction to Solid State Physics* (John Wiley and Sons, Inc., New York, 1953), p. 155.

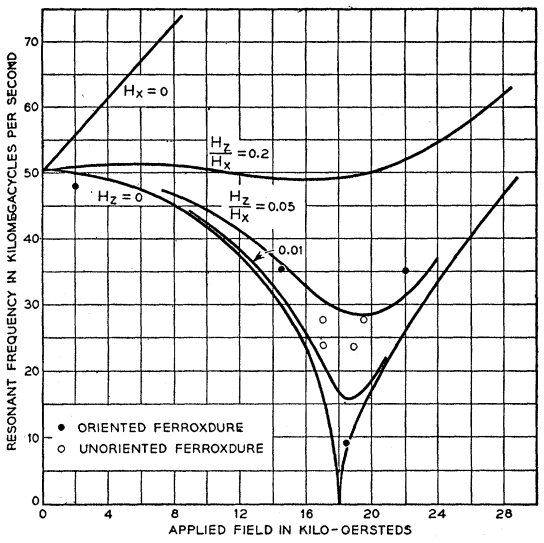

FIG. 2. Theoretical curves and experimental results of ferromagnetic resonance frequency *versus* applied field for Ferroxdure.

both the aforementioned equivalent anisotropy field as well as the externally applied field. Thus, for small variations of **M** about its equilibrium position, assumed to be in the xz plane, one obtains

$$\begin{aligned} j\omega\delta M_x &= \gamma[\delta M_y(H_z+AM_z)], \\ j\omega\delta M_y &= \gamma[-\delta M_x(H_z+AM_z)+\delta M_z(H_x-AM_x)], \\ j\omega\delta M_z &= -\gamma\delta M_y H_x, \end{aligned} \tag{3}$$

where δM_i indicates a small variation in M_i about its equilibrium value. It should be noted from the second of the above Eqs. (3), that although the anisotropy field is in the z direction, the equivalent field along the x axis is affected by the anisotropy for variations in M_z. Solving the secular determinant of the above equations one obtains the resonant frequencies given by

$$(\omega/\gamma)^2=H_x(H_x-AM_x)+(H_z+AM_z)^2. \tag{4}$$

The equilibrium values of M_x and M_z are determined from

$$(H_z+AM_z)/H_x=M_z/M_x. \tag{5}$$

For the special case where the applied magnetic field is perpendicular to the hexagonal axis so that $H_z=0$, Eq. (4) reduces to

$$(\omega/\gamma)^2=A^2M^2-H_{\text{app}}^2 \quad \text{for} \quad H_{\text{app}}<AM,$$

and

$$(\omega/\gamma)^2=H_{\text{app}}[H_{\text{app}}-AM] \quad \text{for} \quad H_{\text{app}}>AM. \tag{6}$$

If the applied field is along the hexagonal axis so that $H_x=0$, then the resonant frequency is given by

$$(\omega/\gamma)=H_{\text{app}}+AM. \tag{7}$$

If neither H_x nor H_z is equal to zero, one must combine Eqs. (4) and (5) in order to obtain the resonant frequencies as a function of applied field. In the previous discussion, demagnetizing factors due to sample shape or poles formed at domain walls have been neglected.[3]

Figure 2 shows the theoretical curves of resonant frequency *versus* applied field for the applied field along the hexagonal axis, perpendicular to the hexagonal axis, and for $H_z/H_x=0.01$, 0.05, and 0.2. These curves are drawn for a material whose maximum anisotropy field, AM, is equal to 18 000 oersteds. It is to be noted that with magnetic field applied perpendicular to the hexagonal axis a double resonance is predicted for frequencies below 50 000 Mc/sec, while no resonance is to be observed for fields along the hexagonal axis at frequencies below 50 000 Mc/sec. If the field is not applied exactly perpendicular to the hexagonal axis, the resonant frequencies are substantially affected at fields near 18 000 oersteds, as seen from the curves for $H_z/H_x=0.05$ and 0.01.

EXPERIMENTAL RESULTS

The above theory is directly applicable to single crystal material. Since single crystal Ferroxdure was not available to us, our experiments at 9200 Mc/sec and at 35 300 Mc/sec were performed with oriented polycrystalline Ferroxdure, having the hexagonal axis of the crystallites oriented along one direction. The results were in agreement with theory in that no resonances at these frequencies were observed with field applied along the hexagonal axis, double resonances were observed with field applied perpendicular to the hexagonal axis, while with zero applied field a resonance near 50 000 Mc/sec was observed. The results obtained with the oriented material, as well as those obtained at 23 600 Mc/sec and 27 800 Mc/sec with unoriented material are shown in Fig. 2 and are to be compared with the theoretical curves. The above theory cannot, of course, be expected to apply to the experiments with unoriented material since for this material the magnetic field is automatically applied in all directions with respect to the hexagonal axis. Therefore, some sort of averaging process is required in order to determine the resonant fields. Even for the oriented sample an averaging process is required since uniform orientation of the crystallites over the spherical sample was impossible to obtain. Within these experimental limitations our results are in fair agreement with the theory.

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[3] These corrections were discussed by J. Smit at the Conference on Ferrimagnetism held at the U. S. Naval Ordnance Laboratory on October 11–12, 1954.