TABLE I. A reanalysis of Helfer's data from five binary star systems by which Freundlich's red-shift interpretation was tested. The results in column 7 are obtained upon the basis of formula (5′). For the minimum and middle shifts the values obtained by means of the point-source model do not differ appreciably from the more exact values given in column 7; for the maximum shift the difference is appreciable and is indicated in parentheses.

1 Star name	2 Eff. temp. °K	3 Mean radius in units of the solar radius	4 Radial distance between stars in units of their radius	5 Observed semi-amplitudes expressed in km/sec	6 Calculated fixed shift of each star due to own radiation field, km/sec	7 Calculated min middle and max shift in a half-period, due to radiation field of companion km/sec
TT Aur.	18 000	4.3	2.7	197; 246	188	36 ; 55 ; 102 (259)
WW Aur.	8000	2.1	6.0	116; 135	3.6	0.3; 0.5; 0.9 (5.3)
TX Her.	10 000	1.6	6.7	121; 140	6.7	0.5; 0.8; 1.3 (10)
Z Vul.	18 000	4.3	3.5	96; 214	188	28 ; 42 ; 81 (268)
Y Cyg.	25 000	5.9	4.8	245; 241	960	101 ; 156 ; 304 (1405)

$\pi/2$, and $\pi-\sin^{-1}(1/\alpha)]$, in the light of one component due to the radiation field of its companion. It is seen that, while the variations in shift are now within the semiamplitudes of the radial velocity curves, they are still, except for two of the systems, considerably outside of observational error.

In view of the reduced magnitude of the stellar-statistical K-term which emerges from Weaver's recent analysis,[1] there is no longer any possibility of correlating the solar shift values with those found in O and B stars—that is if one adheres to the temperature fourth power (radiation density) dependence, or any reasonable modification thereof. This is illustrated forcibly again by the large values in column 6 of Table I.

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Entropy of Radiation

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In view of the recent publication of a derivation of the expression for the entropy flux of radiation, in terms of its spectral specific intensity, a brief comparative study of various known methods for obtaining the entropy formula is presented.

IN a recent article P. Rosen presents an interesting discussion of the question of how to obtain an expression for the entropy flux of electromagnetic radiation of specified intensity.[1] The very problem itself implies, in its general formulation, a well-defined entropy concept for nonequilibrium radiation. Rosen's derivation of the above-mentioned expression starts with the ordinary statistical definition of entropy, in this case for a system of photons (bosons) in terms of their density in phase space, followed by the substitution of this density by the equivalent expression in terms of the specific intensity of the corresponding radiation.

The explicit solution to the problem may be expressed in "differential" form as a relation connecting K_ν, the specific radiation (energy) intensity of linearly polarized light of frequency ν, and L_ν, the corresponding specific entropy intensity. K_ν is defined in such a manner that $K_\nu d\nu d\sigma \cos\theta d\Omega$ represents radiation energy pr. sec (power) in the frequency band $d\nu$ which passes through an element of area, $d\sigma$, and inside an element of solid angle, $d\Omega$, in a direction which makes an angle θ with the normal to the area. L_ν is defined correspondingly. The desired relation turns out to be the following:

$$L_\nu=(k\nu^2/c^2)[(1+x)\ln(1+x)-x\ln x],\quad x\equiv c^2K_\nu/h\nu^3, \qquad (1)$$

where k = Boltzmann's constant, h = Planck's constant, and c = velocity of light.

In passing to present his elegant derivation of what amounts to this formula, Rosen makes a statement to the effect that while Planck has previously derived a functional relation, $L_\nu=\nu^2 f(K_\nu/\nu^3)$, no explicit expression was given by the latter[2] for $f(K_\nu/\nu^3)$.

In the opinion of the present author, however, the explicit relationship for L_ν in terms of K_ν was indeed known to Planck, even prior to the advent of the Bose-Einstein statistics proper.[3] It seems worth while, therefore, to present a brief comparison of various methods of proving the entropy formula (1).

As previously mentioned, Rosen bases his method on

[1] P. Rosen, Phys. Rev. 96, 555 (1954).

[2] M. Planck, *The Theory of Heat* (The MacMillan Company, New York, 1949).

[3] Bose, Z. Physik 26, 178 (1924).

the statistical extension of the entropy concept to non-equilibrium states of a system of bosons. Long ago Planck[4] derived formula (1) by an admittedly less satisfactory method which, on the other hand, in part bears a formal resemblance to the more modern one based on Bose-Einstein statistics. The introduction of the concept of entropy of radiation dates back toWien.[5] Planck has shown, furthermore, that reasoning along the lines of ordinary thermodynamics and classical electrodynamics leads, firstly, to what is a generalized form of Wien's displacement law, namely, the functional relation previously indicated:

$$L_\nu = (\nu^2/c^2)F(c^2K_\nu/\nu^3), \tag{2}$$

and, secondly, to the following relations connecting, in the stationary state for oscillators exposed to radiation, the mean vibrational energy, U, and the mean entropy, S, of a linear, harmonic oscillator (resonator) of frequency ν, with the field quantities K_ν and L_ν:

$$U = (c^2/\nu^2)K_\nu, \quad S = (c^2/\nu^2)L_\nu. \tag{3}$$

Equations (2) and (3) imply

$$S = F(U/\nu). \tag{4}$$

The statistical definition of entropy is then introduced in order to obtain Eq. (4), and thereby Eq. (2), in explicit form. In distributing a given amount of energy on a system of identical resonators Planck[4] actually introduces energy quanta, $h\nu$. The statistical problem is then mathematically equivalent to that for bosons, a fact which is also apparent from the resultant expression:

$$S = k\left[\left(1+\frac{U}{h\nu}\right)\ln\left(1+\frac{U}{h\nu}\right)-\frac{U}{h\nu}\ln\frac{U}{h\nu}\right], \tag{5}$$

when this is being compared with the statistical expression for the entropy of a system of bosons.[1] It is readily seen, furthermore, that the formula (5) renders Eq. (2) identical with Eq. (1). It may, perhaps, be considered a matter of convenience whether the explicit form of the function F be derived in this way rather than by considering directly the relation (2) for the radiation itself.

A different method of deducing the entropy formula (1) is one which makes explicit use of the extension of the temperature concept. The first attempt to assign a definite temperature to monochromatic radiation seems to have been made by Wiedemann,[6] but in a rather limited way. More generally, the temperature may be defined thermodynamically, i.e., by putting

$$dL_\nu = dK_\nu/T, \quad \text{or} \quad \partial L_\nu/\partial K_\nu = 1/T. \tag{6}$$

Again, T and K_ν are interlinked through the celebrated Planck formula:

$$K_\nu = \frac{h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}, \tag{7}$$

which has been derived in a variety of more or less satisfactory ways during the first quarter of this century. This is the general relation between K_ν and T because in the equilibrium state (cavity radiation of temperature T) radiation of each frequency must be thought of as having the same temperature, equal to that of the cavity wall, as there is no internal or external dissipation of energy.

When Eq. (7) is solved with respect to $1/T$, and the result inserted in Eq. (6), integration of the latter leads once more to the same (absolute) entropy formula (1).[7]

[4] See M. Planck, *Theorie der Wärmestrahlung* (Johann Ambrosius Barth, Leipzig, 1906), first edition, p. 148ff.
[5] W. Wien, Ann. Physik **52**, 132 (1894).

[6] E. Wiedemann, Ann. Physik **34**, 446 (1888).
[7] M. Planck, *Theorie der Wärmestrahlung* (Johann Ambrosius Barth, Leipzig, 1913), second and subsequent editions. See also R. Clark Jones, J. Opt. Soc. Am. **43**, 138 (1953).