New Relativistic Two-Body Equation

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INFORMATION on properties of coupled fields may be obtained by considering equations satisfied by quantities of the type

$$\langle \mathrm{out}|O(x)|\mathrm{in}\rangle \tag{1}$$

[$O(x)$ Heisenberg operator, $|\mathrm{in}\rangle$ state of incoming particles, $|\mathrm{out}\rangle$ state of outgoing particles].

The main difference between such quantities and "covariant wave functions" previously used[1-4] is that (1) contains only one Heisenberg operator and several creation and annihilation operators for outgoing and incoming particles.

As an example, let us consider two Dirac fields $\psi_P(x)$ and $\psi_N(x)$, interacting through a pseudo-scalar field $A(x)$, corresponding to particles of rest-mass μ. The interaction term in the Lagrangian is then

$$L_I = gA(x)[\bar{\psi}\gamma\psi_P(x)+\bar{\psi}\gamma\psi_N(x)].$$

We introduce the quantity of type (1)

$$\varphi(x,y)=\langle\bar{0}|\psi_N{}^{\mathrm{out}}(x)\psi_P(y)|P\rangle,$$

where $|\bar{0}\rangle$ is the true vacuum, and $|P\rangle$ is an eigenstate of the total energy-momentum four-vector of the interacting fields, corresponding to eigenvalues P_μ. It is easy to show that

$$\varphi(x,y)=\varphi(x-y)e^{iPy},$$

where $\varphi(x)=\langle\bar{0}|\psi_N{}^{\mathrm{out}}(x)\psi_P(0)|P\rangle$.

The equation satisfied by $\varphi(x)$ will be nonlinear. The interaction of lowest order in g gives, however (as is most easily seen by starting from the Bethe-Salpeter equation), a linear equation,

$$(P_\mu\gamma^\mu - p_\mu\gamma^\mu+m)_P\chi(p)=\frac{g^2m_N}{(2\pi)^3}\times\int\frac{Q_N(p)\gamma_N\gamma_P\chi(q)d\sigma(q)}{(p+q)^2+\mu^2} \tag{2}$$

where

$\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=-2g^{\mu\nu}$, $g^{44}=-1$, $Q_N(p)=(m-p_\mu\gamma^\mu)_N/2m_N$, $\chi(p)$ is defined by

$$\varphi(x)=(2\pi)^{-\frac{3}{2}}\int\exp[ipx]\chi(p)d\sigma(p),$$

and

$$d\sigma(p)=d^3p/E_p{}^N,\quad E_p{}^N=(\sum_{i=1}^{3}p_i{}^2+m_N{}^2)^{\frac{1}{2}}.$$

Using nonrelativistic notation, $\psi(-\mathbf{p})=\chi(p)/E_p{}^N$, and working in the center-of-mass system ($P^1=P^2=P^3=0$, $P^4=M$), we obtain

$$(\boldsymbol{\alpha}\mathbf{p}+m\beta+E_p{}^N-M)_P\psi(\mathbf{p})=\frac{g^2}{(2\pi)^3}\times\int\frac{P_N(-\mathbf{p})\beta_N\beta_P\gamma_N\gamma_P P_N(-\mathbf{q})\psi(\mathbf{q})}{(\mathbf{p}-\mathbf{q})^2+(E_p{}^N-E_q{}^N)^2+\mu^2}d^3q, \tag{3}$$

where $P_N(\mathbf{p})=(E_p{}^N+\boldsymbol{\alpha}\mathbf{p}+m\beta)_N/2E_p{}^N$.

Equation (3) may be checked by applying it, suitably modified, to the problem of the hydrogen atom. It then reads

$$(\boldsymbol{\alpha}\mathbf{p}+m\beta+E_p{}^P)\psi(\mathbf{p})=-\frac{e^2}{2\pi^2}\times\int\frac{P_P(-\mathbf{p})(\boldsymbol{\alpha}_P\boldsymbol{\alpha}_e-1)P_P(-\mathbf{q})}{(\mathbf{p}-\mathbf{q})^2+(E_p{}^P-E_q{}^P)^2}\psi(\mathbf{q})d^3q,$$

where the subscripts e and P indicate electron and proton quantities, respectively.

The first term of a development with respect to m_e/m_P gives, in the coordinate space, the familiar Dirac equation for a particle in a Coulomb field.

The advantages of Eq. (2) are: first, it contains the eigenvalue parameter M linearly; second, it is manifestly covariant; third, it can be handled, in principle, by the same methods as the usual one-body Dirac equation; fourth, it can readily be extended to many-body problems.

In a forthcoming paper, Eq. (2) will be discussed in more detail, and higher-order equations for $\varphi(x)$ will be considered.

[1] E. E. Salpeter and H. A. Bethe, Phys. Rev. **84**, 1232 (1951).
[2] M. Gell-Mann and F. Low, Phys. Rev. **84**, 350 (1951).
[3] W. Macke, Z. Naturforsch. **8a**, 599 (1953).
[4] S. M. Dancoff, Phys. Rev. **88**, 382 (1950).

Rotation of Polarization Vector and Depolarization in p-p Scattering*†

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IT has been pointed out by Wolfenstein[1] that triple-scattering experiments can give information beyond that obtainable with simple double-scattering polarization measurements. The difference between these varieties of experiments can be stated as follows: in a typical double-scattering experiment a polarized beam is incident on a hydrogen target and the intensities of the scattered protons in various directions are measured, whereas in a triple-scattering experiment the change in the state of polarization caused by the scattering on hydrogen is measured.

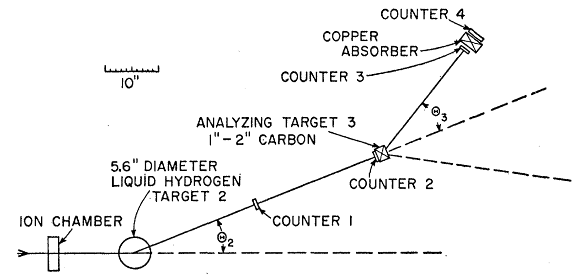

FIG. 1. Scale drawing of targets and counters for measurement of the depolarization parameter D. Target 1, a beryllium target inside the cyclotron, is not shown in this figure.

The two simplest independent cases are those in which the triple-scattering experiments are performed as indicated schematically in Figs. 1 and 2. In the configuration represented by Fig. 1 all the scattering processes occur in the same plane and, in an oversimplified schematization, one measures the probability of flipping the spin in the collision occurring at target 2. Targets 1 and 3 act as a polarizer and analyzer, respectively. The asymmetry e_{3n} observed after the scattering on target 3 is connected with a coefficient D (for depolarization) defined by Wolfenstein as

$$e_{3n}=P_3(P_2+DP_1)/(1+P_1P_2), \tag{1}$$

where P_i is the polarization generated by scattering an unpolarized beam on target i.

Figure 2 represents the triple-scattering experiment in which the second scattering plane is perpendicular to the first. The scattering on target 2 rotates the spin in a complicated way and by analyzing the polarization by scattering on target 3 we find the component of the spin P_s in the direction n_3 perpendicular to the plane π''. The asymmetry after the scattering on target 3 is

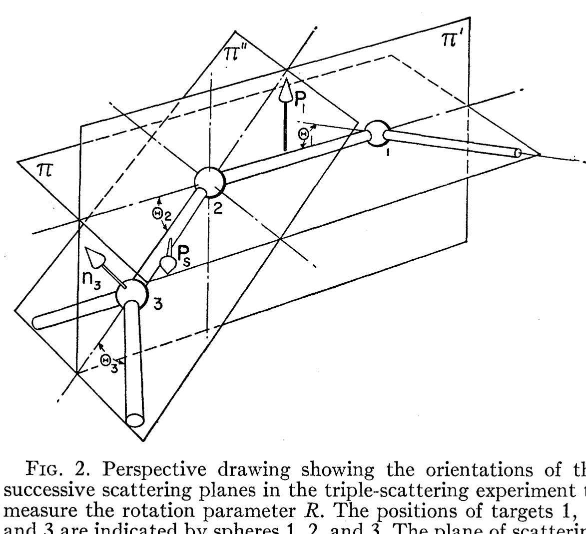

FIG. 2. Perspective drawing showing the orientations of the successive scattering planes in the triple-scattering experiment to measure the rotation parameter R. The positions of targets 1, 2, and 3 are indicated by spheres 1, 2, and 3. The plane of scattering at target 2 (indicated π') is perpendicular to the plane of scattering at target $1(\pi)$. The plane of scattering at target $3(\pi'')$ is perpendicular to the plane π'. The figure is not to scale.

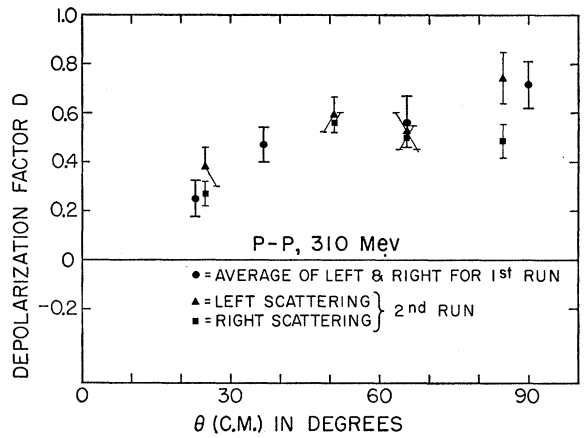

FIG. 3. Depolarization factor D plotted against center-of-mass scattering angle θ for proton-proton scattering at 310 Mev.

given by

$$e_{3s}=P_1P_3R, \tag{2}$$

which defines the rotation parameter R.

The experiments have been performed by use of the polarized beam of the 184-inch Berkeley cyclotron and will be described in a later paper. The average energy of the protons incident on target 2 was 310 Mev in the laboratory system and their polarization was 0.74.

Figures 3 and 4 give graphs of D and R as obtained in our measurements. The differential cross section I_0, the polarization P, R, and D are connected with the elements of the p-p scattering matrix as indicated by Wolfenstein[1] [Eqs. (3) and (5)].

The information obtained from our investigation should be sufficient to determine the phase shifts of the various partial waves up to and including F waves. Expressions for the observable quantities in terms of the phase shifts have been obtained by Stapp[2] and the numerical calculation with an electronic computing machine has been initiated.

The two varieties of triple-scattering experiments

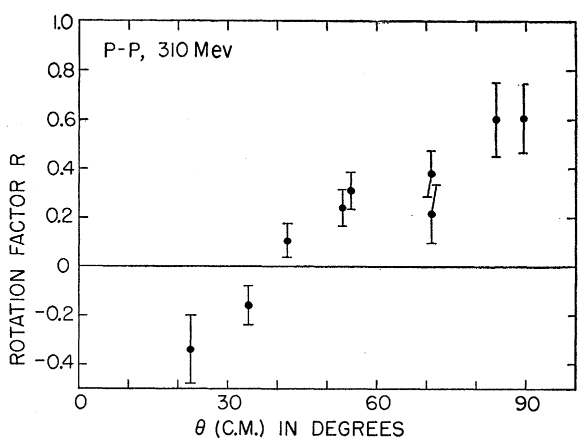

FIG. 4. Rotation factor R plotted against center-of-mass scattering angle θ for proton-proton scattering at 310 Mev.

discussed here have in common the property that, (a) the beam incident on the second target is polarized perpendicular to the direction of incidence, and (b) the direction of analysis of the polarization is perpendicular to the scattering direction. Other independent triple-scattering experiments would involve auxiliary magnetic fields that would alter in a known fashion the relative orientation of the directions of the beam and of the polarization.

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[1] L. Wolfenstein, Phys. Rev. 96, 1654 (1954).

[2] H. P. Stapp, University of California Radiation Laboratory Report, UCRL-2825 (unpublished).

Scattering of High-Energy Neutrons and Johnson-Teller Type Nuclei

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THE model of Johnson and Teller,[1] which deals with the neutron and proton distributions in nuclei, requires direct experimental evidence, which should be possible by using suitable probe particles. Courant[2] offered the first method to determine its reality from the interactions of high-energy pions with Pb nuclei. His proposal, however, has not been verified experimentally.

This report gives evidence for the Johnson-Teller (J-T) type model from the analysis of the total cross sections for scattering of high-energy neutrons (275–410 Mev). The experimental data in these regions do not agree with the Fernbach-Serber-Taylor[3] (F-S-T) theory as discussed by Nedzel,[4] who suggested that the difficulties originate from the assumption of uniform nuclear densities in the usual models. The author has extended the optical method of F-S-T to the J-T type non-uniform nuclear model, and derived the corresponding formulas for σ_t, σ_a, and σ_d for the neutron scattering. The division of a nucleus into an inner zone with radius R_1 and an outer zone extending to the whole radius R is the same as that of Courant. We use the usual notation of the optical model, with quantities in the inner and outer zones described as K, k_1, and $\bar{K}$, $\bar{k}_1$ respectively. Putting $R_1/R=(Z/N)^{\frac{1}{3}}\equiv\gamma$, we will define

$$\begin{aligned} K&=C(\sigma_{np}+\sigma_{nn})3Z/4\pi(\gamma R)^3,\\ \bar{K}&=C\sigma_{nn}3(N-Z)/4\pi R^3(1-\gamma^3), \end{aligned} \tag{1}$$

whereas in the usual F-S-T theory K is given as $K_F=C(Z\sigma_{np}+N\sigma_{nn})3/4\pi R^3$. C is a common factor (independent of elements) of order unity, which reflects the character of many-body problems and upon the basic structure of the optical model itself. The

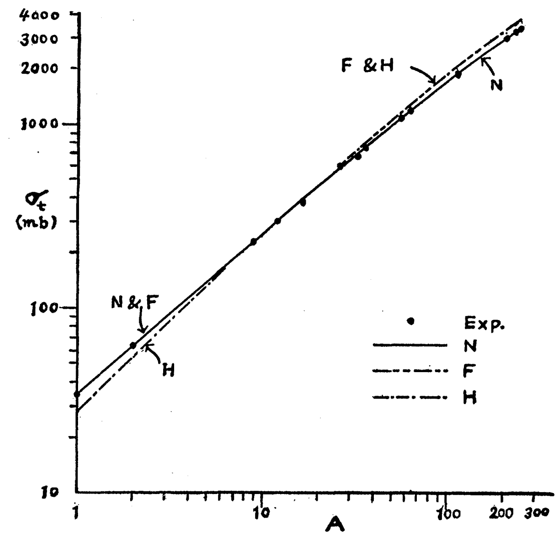

FIG. 1. σ_t *vs* mass number A (best fit). The curves N, F, and H are based respectively on our formulas (2)–(2d), the Fernbach-Serber-Taylor theory (uniform), and Heckrotte's theory (parabolic distribution). $R=r_0A^{\frac{1}{3}}\times10^{-13}$ cm, where $r_0=1.36$ for the curve N and F. The values of K (in 10^{13} cm^{-1}) for the curve $N(K_N)$ are given in Table I, and for the curve F the value of $K(K_F)$ is taken to be 0.33. For the curve H Heckrotte's values were used: $r_0=1.6$, $\bar{K}_0=0.32$, and $k_1=0$. The dots are the experimental values (reference 4).

fact that for incident neutrons $K/\bar{K}=(34+24)$ mb/24 mb$=2.4\approx2$ permit us to determine the following analytical formulas.

$$\sigma_t=\pi R^2\{2-T_a-T_b+(1-\gamma^2)^2(T_c+T_d)/2\}, \tag{2}$$

where the outer zone gives the term T_b and the core gives the other terms. In the limit of $\gamma\rightarrow1$ (uniform distribution), T_a gives the well-known F-S-T formula, while the other terms vanish. σ_a and σ_d will not be considered here. The complete expressions for the terms in (2) are too bulky to be listed here. High-energy neutron scattering experiments show that the total cross sections are nearly constant for the energies considered; hence, we may put $k_1=0$ in our present analysis. With this simplification, the formulas are as follows:

$$\begin{aligned} T_a=4(KR)^{-2}\{&[1+\tfrac{1}{2}(1-\gamma^2)^{\frac{1}{2}}KR]\\ &\times\exp[-\tfrac{1}{2}(1-\gamma^2)^{\frac{1}{2}}KR]-[1+\tfrac{1}{2}(1+\gamma)KR]\\ &\times\exp[-\tfrac{1}{2}(1+\gamma)^{\frac{1}{2}}KR]\}, \end{aligned} \tag{2a}$$

$$\begin{aligned} T_b=16(KR)^{-2}\{1-&[1+\tfrac{1}{2}(1-\gamma^2)^{\frac{1}{2}}KR]\\ &\times\exp[-\tfrac{1}{2}(1-\gamma^2)^{\frac{1}{2}}KR]\}, \end{aligned} \tag{2b}$$

$$\begin{aligned} T_c=&\left\{\frac{1}{1-\gamma^2}-\frac{KR}{2(1-\gamma^2)^{\frac{1}{2}}}\right\}\exp[-\tfrac{1}{2}(1-\gamma^2)^{\frac{1}{2}}KR]\\ &-\left\{\frac{1}{(1+\gamma)^2}-\frac{KR}{2(1+\gamma)}\right\}\exp[-\tfrac{1}{2}(1+\gamma)KR], \end{aligned} \tag{2c}$$

$$T_d=\frac{(KR)^2}{4}\int_{\frac{1}{2}(1-\gamma^2)^{\frac{1}{2}}KR}^{\frac{1}{2}(1+\gamma)KR}(e^{-x}/x)dx. \tag{2d}$$