Correction to the Exponential Dependence of Neutron Transmissions*

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If the energy spread of the neutron source used in a transmission type of measurement of total neutron cross sections is large compared with the mean spacing of the resonance levels of the sample, these levels may effect a deviation from the simple exponential dependence on sample thickness. The correction to this dependence is derived in the case where capture, inelastic scattering, and Doppler broadening may be neglected. The derivation does not depend upon any special assumption regarding the distributions of the widths and spacings of the levels other than that they have suitable averages, and the correction is found to be proportional to the ratio of these averages, as one would expect. It is noted that by measurement of this ratio for certain elements, it may be possible to distinguish between the predictions of the strong-coupling and complex square-well representations of the neutron-nucleus interaction.

If the energy spread of the neutron source used in a transmission type of measurement of total neutron cross sections[1] is large compared with the mean spacing D of the resonance levels of the sample, these levels will effect a deviation from the simple exponential dependence on sample thickness t.[2] Thus, the transmission

$$T=\int f(E)\exp[-\sigma(E)Nt]dE, \tag{1}$$

where the normalized function $f(E)$ represents the source spectrum in terms of the neutron energy E, will deviate from $\bar{T}=\exp(-\bar{\sigma}Nt)$, where $\bar{\sigma}=\int f(E)\sigma(E)dE$ is the average of the total cross section $\sigma(E)$, by a factor such as $(1+\alpha)$:

$$T=\bar{T}(1+\alpha); \tag{2}$$

$\alpha=\alpha(T)$ and N is the atomic density. This correction may be large at low energies where elastic scattering is the dominant process, the cross section of which varies from 0 to $4\pi\lambda\!\!\!^-{}^2$ through resonances and thus may depart considerably from the average.

At low energies where only the s-wave interaction is effective and in the case of a monoisotopic sample of zero spin, the total cross section may be expressed in terms of a single collision function $U(E)$ according to[3]

$$\sigma(E)=\pi\lambda\!\!\!^-{}^2[2-U(E)-U(E)^*]. \tag{3}$$

The correction term $\alpha(T)$ may be expressed theoretically in terms of the average $\bar{U}$ of $U(E)$ by substitution of (3) into (1) and evaluation of the averages of the individual terms arising in a power series expansion of the exponential. It is assumed that the neutron energies are low enough so that inelastic scattering and reactions do not occur and high enough so that capture is negligible, in which case $|U(E)|=1$. It is also assumed that

$$\int f(E)U(E)^n=\bar{U}^n \tag{4}$$

for positive integers n. It has been shown that $\bar{U}$ is obtained by substitution of the Stieltjes transform of the strength function $s(E_\lambda)=\langle\gamma_{\lambda n}{}^2\rangle_{\text{Av}}/D$ for the R function in the theoretical expression for $U(E)$.[3] Equation (4) constitutes a generalization of Eq. (43) of reference 3, and the arguments given in connection with the latter equation may also be used to justify (4). It is also convenient to use the expansion $a^n+b^n=\sum_q C_q{}^n\times(a+b)^{n-2q}(ab)^q$, in which the q sum extends from 0 to $\frac{1}{2}n$ if n is even and to $\frac{1}{2}(n-1)$ if n is odd, with the coefficients $C_q{}^n=(-1)^q n(n-q-1)!/q!(n-2q)!$. The result is that

$$\alpha(T)=Qc^2\exp(-c\bar{F})\times\sum_{n=2}^{\infty}\xi_n, \tag{5}$$

where

$$c=\pi\lambda\!\!\!^-{}^2Nt,\quad Q=1-|\bar{U}|^2,$$

$$\bar{F}=\bar{U}+\bar{U}^*=2-(\bar{\sigma}/\pi\lambda\!\!\!^-{}^2),$$

$$\xi_n=\frac{c^{n-2}}{n!}\left[\sum_{r=1}\binom{n}{r}P_r(Q)\times\sum_{q=0}C_q{}^{n-2r}\bar{F}^{n-2(r+q)}(1-Q)^q+\binom{n}{\frac{1}{2}n}P_{\frac{1}{2}n}(Q)\cdot\frac{1+(-1)^n}{2}\right],$$

$$P_r(Q)=r+\sum_{t=1}^{r-1}(-1)^t\binom{r}{t+1}Q^t.$$

The upper limit of the r sum of ξ_n is $\frac{1}{2}n-1$ if n is even and $\frac{1}{2}(n-1)$ if n is odd, and that of the q sum is $\frac{1}{2}(n-2r)$ if n is even and $\frac{1}{2}(n-2r-1)$ if n is odd. The quantity $Q(\leq 1)$ is significant as the cross section for compound nucleus formation divided by $\pi\lambda\!\!\!^-{}^2$; if the average of the neutron widths $\Gamma_{\lambda n}$ is small compared with D, Q is equal to $2\pi\langle\Gamma_{\lambda n}\rangle_{\text{Av}}/D$. Since the first term $\xi_2=1$, it is evident that $\alpha(T)$ is proportional to t^2, to Q, and inversely to E, for small values of t, as one would expect.

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1 H. H. Barschall, Am. J. Phys. 18, 535 (1950).
2 Darden, Okazaki, and Becker, Bull. Am. Phys. Soc. 30, No. 1 (1955).
3 R. G. Thomas, Phys. Rev. 97, 224 (1955).

In general there will be contributions $\sigma_i(E)$ to $\sigma(E)$ from various partial waves, channel spin states, and isotopes of atomic densities N_i, so that $T=\int f(E)\prod_i \times\exp[-\sigma_i(E)N_it]dE$ where $\sigma_i(E)=\pi\lambda^2\omega_i[2-U_i(E)-U_i(E)^*]$, ω_i being the statistical factor for the ith interaction. Since the resonance levels associated with the various interactions i are presumably uncorrelated, it is reasonable to assume that[4] $T=\prod_i T_i$, where $T_i=\int f(E)\exp[-\sigma_i(E)N_it]dE$, in which case $T=\bar{T}(1+\alpha)$, where $\bar{T}=\prod_i\bar{T}_i$ and $1+\alpha=\prod_i(1+\alpha_i)$. In view of the proportionality for small t, of α_i to $N_i^2\omega_i^2$ and α to $\sum_i\alpha_i$, $\alpha(T)$ will be smaller the larger the number of participating isotopes and spin states.

The effect of capture and inelastic scattering, which was not considered in the derivation, is to broaden the resonances and to reduce their effective maxima, thus diminishing the α_i. In fact at high energies where the total level widths are much larger than the spacings, the $U_i(E)$ and their associated $\sigma_i(E)$ are essentially constant. The α_i will also be reduced by the Doppler effect. For this effect to be negligible it is necessary that the $\Gamma_{\lambda n}$ be much greater than the Doppler width $\Delta=2(EkT/A)^{\frac{1}{2}}$ of the sample element of mass number A.

The complex square well (c.s.w.) has been found to provide an accurate representation of certain aspects of the neutron-nuclei interactions at low energies.[5] The Stieltjes transform of the strength function corresponding to this model is simply the reciprocal of the product of the nuclear radius with the logarithmic derivative at the radius of the radial part of the c.s.w. wave function.[3] A prediction of this model is that the strength $s(E_\lambda)$ of the s-wave resonances, and hence the Q_i, are considerably larger at the "giant resonances" of the c.s.w. than the strong-coupling theory (s.c.t.) value and considerably smaller between these resonances.[3,5] With a 42-Mev well depth, the c.s.w. resonances for s neutrons are predicted to appear at atomic weights 11, 55, and 150.[5] Indeed, by surveying individual resonance levels of elements with $A>100$, some indication of the predicted maxima of $s(E_\lambda)$ is found in the neighborhood of $A=160$.[6] It would be desirable to verify this indication by the alternative procedure of determining the Q_i, and thus $s(E_\lambda)$, from the measurement of α for these elements. Unfortunately, as also indicated by the survey, $\Gamma_{\lambda n}\lesssim\Delta$ for the elements in this neighborhood, and therefore the α_i would be subject to an appreciable Doppler reduction unless the sample were cooled to extremely low temperatures. However, in the vicinity of the predicted giant s resonance at $A=55$, the individual resonance level spacings are much larger,[7] indicating that $\Gamma_{\lambda n}\gg\Delta$. With a 42-Mev well depth and $\zeta=0.03$ as the imaginary-part parameter, it is predicted that for (monoisotopic) Mn, $\alpha(\frac{1}{4})=0.27$ [compared with $\alpha(\frac{3}{4})=0.012$], which may be distinguishable from the predictions of the s.c.t. of $\alpha(\frac{1}{4})=0.22$ [compared with $\alpha(\frac{3}{4})=0.014$]. Bismuth, being magic, may have a level spacing considerably larger than that of the normal isotopes of the heavy elements,[8] and it is therefore likely that $\Gamma_{\lambda n}\gg\Delta$. It would be of interest to determine its Q by a transmission measurement because Bi falls between the s resonances of the c.s.w. and is therefore predicted to have a small Q and associated α: $\alpha(\frac{1}{4})=0.019$ [compared with $\alpha(\frac{3}{4})=4\times10^{-4}$]; while the correction predicted by the s.c.t. is relatively larger: $\alpha(\frac{1}{4})=0.085$ [compared with $\alpha(\frac{3}{4})=2\times10^{-3}$].

The possible existence of the resonance transmission correction was suggested by H. H. Barschall; the writer is also grateful to him for valuable suggestions and discussions on this subject.

[4] We have not succeeded in proving this assumption, and therefore the results may not be entirely correct.

[5] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

[6] Carter, Harvey, Hughes, and Pilcher, Phys. Rev. **96**, 113 (1954).

[7] *Neutron Cross Sections*, Atomic Energy Commission Report AECU-2040 (Technical Information Division, Department of Commerce, Washington, D. C. 1952), and supplements thereto.

[8] Hughes, Garth, and Levin, Phys. Rev. **91**, 1423 (1953); J. H. Gibbons and Henry W. Newson, Phys. Rev. **91**, 209(A) (1953).