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Scattering of Mesons by Light Nuclei

L. S. KISSLINGER*
Indiana University, Bloomington, Indiana
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An improved optical model is used to study the scattering of mesons by light nuclei, C^{12} in particular. It is found that the differential cross section can be fit reasonably well if a nucleus is used which has a diffuse edge; the Gaussian shape seems to work well for C^{12}.

I. INTRODUCTION

An optical model is one that describes the scattering of an incident particle by a nucleus by means of an interaction potential which is independent of the coordinates of the individual nucleons, and which depends only on the coordinate of the incident particle with respect to the nucleus as a whole. In the conventional optical model[1] the nucleus is considered to be a continuous medium with an index of refraction n, with the incident particle having a wave number k_0 outside and nk_0 inside the nucleus.

The scattering of particles by complex nuclei has been treated by Watson[2] and Francis and Watson[3] where it was shown that the multiple scattering formalism for elastic scattering leads to an optical model to order $1/A$, where A is the number of scatterers. In II, it was demonstrated how an approximate interaction potential can be obtained from the equations describing the multiple scattering, this potential having the simplicity of the conventional optical model. This model seems more general than the older optical model in that both the form and the magnitude of the potential reflect the scattering of the incident particle by the individual nucleons bound in the nucleus. As will be seen later, the resulting potential is no longer restricted to have the same form as the nucleon distribution, an inherent characteristic of the older optical potential.

A good region in which to test this model is that of light nuclei, where the effect of the diffuse edge is certainly important and the description of the older optical model is not correct. In particular the scattering of mesons by nuclei should provide a convenient example, as the meson-nucleon scattering is now rather well known in the 100-Mev range, and there is some experimental investigation of meson-nuclear scattering.

One experiment, in which the scattering of 62-Mev mesons by carbon was measured,[4] was analyzed by Peaslee.[5] Peaslee, neglecting the multiple scattering and absorption, was able to fit the differential cross section, which is more than twice as large at 180° than at 90°. On the other hand the older optical model failed to predict this large backward scattering.[4] Now the meson-nucleon differential cross section is quite highly peaked in the backward direction, a fact which enabled Peaslee to obtain his result. It was hoped that this improved optical model would be able to retain this feature for light nuclei.

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[1] Fernback, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).
[2] K. M. Watson, Phys. Rev. **89**, 575 (1953). Referred to hereafter as I.
[3] N. C. Francis and K. M. Watson, Phys. Rev. **92**, 291 (1953). Referred to hereafter as II.
[4] Byfield, Kessler, and Lederman, Phys. Rev. **86**, 17 (1952).
[5] D. C. Peaslee, Phys. Rev. **87**, 862 (1952).

II. THE POTENTIAL

A. Derivation of the Potential

Francis and Watson define a wave matrix for elastic scattering, Ω_c, which is obtained from the wave matrix by keeping only those matrix elements which leave the nucleus with a final energy equal to the initial energy. In analogy to the definition of the ordinary scattering operator T, they define T_c such that $\Omega_c = 1 + (1/a)T_c$, where $a = E_a + i\eta - H_0$; E_a being the total energy before collision, and H_0 the nuclear Hamiltonian plus the kinetic energy operator of the meson. Then they investigate the existence of a potential V_c, which is related to T_c as the ordinary interaction potential is related to T, i.e.,

$$T_c = V_c + V_c(i/a)T_c. \tag{1}$$

Starting with an interaction potential composed of the sum of interactions of the incident particle with the nucleons, they show by taking the coherent part of the multiple scattering equations of I that to order $1/A$ there does exist a solution of (1) of the form $V_c = T_c + v_1 + \Delta$; where $T_c = \sum_{\alpha=1}^{A} \langle t_\alpha \rangle$, $\langle t_\alpha \rangle$ being the coherent part of t_α, the scattering operator for the αth nucleon bound in the nucleus, v_1 depends on nuclear correlations (allowing several collisions to leave the nucleus in its initial state), and Δ gives the contribution of true absorption. In the present calculation such nuclear correlations were neglected, leaving $v_1 = 0$. Also it is expected that the contribution of true absorption is small for the lighter nuclei, so the Born approximation was used for this part of the potential. Then

$$V_c = T_c. \tag{2}$$

The meson-nucleon scattering amplitude in the energy range of interest is given by the $l=0$ and $l=1$ contribution.

$$a_s=(3/k)[(\eta_{++}E_{\frac{3}{2}}+\eta_{-+}E_{\frac{1}{2}})T_{\frac{3}{2}}+(\eta_{+-}E_{\frac{3}{2}}+\eta_{--}E_{\frac{1}{2}})T_{\frac{1}{2}}]+(1/k)[\eta_+T_{\frac{3}{2}}+\eta_-T_{\frac{1}{2}}],$$

where $\eta=e^{i\delta}\sin\delta$; η_+, η_- correspond to $l=0$, isotopic spin $\frac{3}{2}$ and $\frac{1}{2}$ respectively, while those scattering amplitudes with two indices correspond to $l=1$ with the first index giving the isotopic spin and the second the angular momentum (+ stands for $\frac{3}{2}$ and − for $\frac{1}{2}$). $E_{\frac{3}{2}}$ and $E_{\frac{1}{2}}$ are the projection operators for $l=1$, $J=\frac{3}{2}$, and $J=\frac{1}{2}$; $T_{\frac{3}{2}}$ and $T_{\frac{1}{2}}$ are the projection operators for isotopic spin $I=\frac{3}{2}$ and $\frac{1}{2}$ respectively. According to the results of recent experiments[6,7] the meson-nucleon scattering can be fit by using only η_{++}, η_+, and η_-. After a sum over spin and isotopic spin in which the spin dependence disappears because the ground state of C^{12} has both spin and isotopic spin zero, the scattering operator then takes the form

$$\frac{(a_s)}{(2\pi)^2\epsilon_q}=(t_\alpha)=s+t\mathbf{k}\cdot\mathbf{k}', \tag{3}$$

where

$$s=\frac{1}{(2\pi)^2\epsilon_q}(2\eta_++\eta_-),\quad t=\frac{1}{(2\pi)^2\epsilon_q}\frac{4}{3}\eta_{++},$$

$\mathbf{k}$ and $\mathbf{k}'$ are the initial and final wave numbers, and ϵ_q is the energy of the meson. According to the energy dependence of the phase shifts given by both Glicksman and Bethe-de Hoffman, s and t are approximately constants for the energy region 50–125 Mev. For energies larger than this the form (3) breaks down. In the first place, the phase shift δ_{++}, which gives the most important contribution to the scattering, no longer has an energy dependence of k^3, and moreover in this region of higher energy the phase shifts are of such magnitude that η begins to differ considerably from δ. Probably in the carbon nucleus the momenta that occur are of such magnitude as to make the form (3) somewhat inaccurate. In this calculation the small imaginary part of the scattering amplitude was lumped with the imaginary part of the potential coming from pure absorption. This form is then assumed for the energy dependence of the scattering operator with s and t constant, representing an average over π^+ and π^- with sum taken over spin. It might be noted that even with a nucleus of nonzero spin the spin contribution would be of order $1/A$.

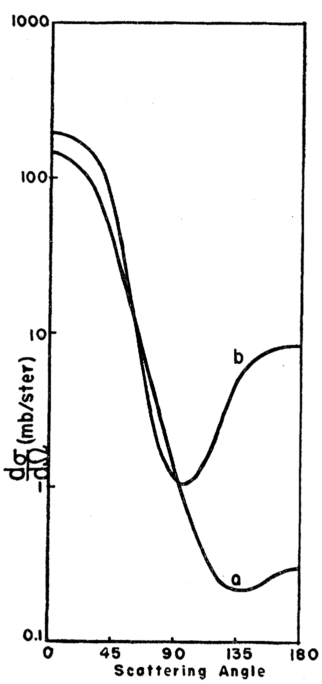

FIG. 1. Elastic differential cross section for uniform distribution (a) with change in slope factor, (b) without change in slope factor.

In coordinate space

$$(z|T_c\varphi)=\frac{1}{(2\pi)^3}\int\exp(i\mathbf{q}'\cdot\mathbf{z})(q'|T_c|q)\times\exp(i\mathbf{q}\cdot\mathbf{z}')\varphi(z')d^3qd^3q'd^3t'. \tag{4}$$

From I, Eq. (25)

$$(q'|T_c|q)=A\int\langle(q'|t_\alpha|q)\rangle\times\exp[-i(\mathbf{q}'-\mathbf{q})\cdot\mathbf{z}_\alpha]\rho(z_\alpha)d^3z_\alpha, \tag{5}$$

where $\rho(z_\alpha)$ is the nucleon distribution normalized so $\int\rho(z)dV=1$. Using (2), (3), and (5), (4) becomes

$$\begin{aligned}(z|v_c\varphi)&=\frac{A}{(2\pi)^3}\int(s+t\mathbf{q}\cdot\mathbf{q}')\varphi(z')\exp[i\mathbf{q}'\cdot(\mathbf{z}-\mathbf{z}_\alpha)]\\&\quad\times\exp[-i\mathbf{q}\cdot(\mathbf{z}'-\mathbf{z}_\alpha)]d^3qd^3q'd^3z'd^3z_\alpha\\&=\frac{A}{(2\pi)^3}\int\{s\varphi(z')\rho(z_\alpha)\exp[i\mathbf{q}'\cdot(\mathbf{z}-\mathbf{z}_\alpha)]\\&\quad\times\exp[-i\mathbf{q}\cdot(\mathbf{z}'-\mathbf{z}_\alpha)]+t\rho(z_\alpha)\varphi(z')\nabla_z\\&\quad\times\exp[i\mathbf{q}'\cdot(\mathbf{z}-\mathbf{z}_\alpha)]\cdot\nabla_{z'}\exp[-i\mathbf{q}\cdot(\mathbf{z}'-\mathbf{z}_\alpha)]\}\\&\quad\times d^3qd^3q'd^3z'd^3z_\alpha\\&=-B\nabla\cdot(\rho\nabla\varphi)+B'\rho\varphi\\&=-B\rho\nabla^2\varphi-B\nabla\rho\cdot\nabla\varphi+B'\rho\varphi,\end{aligned} \tag{6}$$

where $B=(2\pi)^3At$ and $B'=(2\pi)^3As$. Since $B'>0$ the last term in (6) is repulsive corresponding to the repulsive s-wave contribution from the meson-nucleon scattering. The other two terms arise from the p-wave contribution to the meson-nucleon scattering and are essentially different in character. The first term is always attractive, and is equivalent to a mass change inside the nucleus. The second term, depending on the gradient of the nuclear shape, is essentially a surface term. For large nuclei, which according to current

[6] Maurice Glicksman, Phys. Rev. 94, 1335 (1954).
[7] H. A. Bethe and F. de Hoffmann, Phys. Rev. 95, 1100 (1954).

models have a rather constant density in the center, this term contributes only in the small region at the surface. However, in light nuclei, where the entire nucleus is surface, this term is quite important. In fact the strong l-dependence of this part of the potential will be seen to be the property which enables us to fit the back-scattering of mesons from light nuclei.

B. Uniform Distribution

If we use the Schrödinger equation with a uniform distribution, this potential differs from the form of the old optical model essentially only in that it contains the gradient term, which is nonzero only at the boundary. To estimate the effect of this term for a square well, the radial Schrödinger equation with $\rho=\rho(r)$,

$$\frac{1}{2\epsilon_q}\frac{d^2R}{dr^2}+\left[\frac{1}{2\epsilon_q}+B\rho(r)\right]\frac{2}{r}\frac{dR}{dr}+B\frac{d}{dr}\left[\rho(r)\frac{dR}{dr}\right]+\left\{B'\rho(r)-\frac{l(l+1)}{r^2}\left[\frac{1}{2\epsilon_q}+B\rho(r)\right]\right\}R+ER=0,$$

is examined. Assuming the nuclear density goes to zero in a distance 2δ about the assumed nuclear radius, the equation was integrated through this region, giving in the limit as δ goes to zero:

$$(dR/dr)^+=(1+2\epsilon_qB\rho_u)(dR/dr)^-, \qquad (7)$$

where $(dR/dr)^+$ and $(dR/dr)^-$ are the outside and inside derivative respectively at $r=R$. The phase shifts are found by matching logarithmic derivatives at the boundary, including the factor given by (7). Figure 1 shows the differential cross section for C^{12} with and without this change in slope factor, where the phase shifts of Bodansky *et al.*[8] have been used to determine B and B'. ($2\epsilon_qB\rho_u=-0.32$, $2\epsilon_qB'\rho_ur_0^2=0.15$.)

Just the presence in the potential of the term depending on the derivative of the nuclear shape indicates the square distribution is not useful. However, a better test of this distribution is found in a heavier nucleus, where more partial waves enter into the problem and the effect of the discontinuity of the potential is more clearly illustrated. Since the effect of absorption is the major one in this region, the application of (6) is clearly incorrect as far as the description of meson scattering by a heavy nucleus is concerned; however, to investigate the properties of this potential the following calculations were done. First a phase-shift analysis was made using a uniform distribution with a radius of $R_0=6.93\times10^{-13}$ cm, corresponding to Pb and $r_0=1.17\times10^{-13}$ cm. The other nuclear distribution used was uniform in the center and dropped linearly to zero in one meson Compton wavelength with a nuclear volume equal to that obtained with the uniform distribution, a form approximating the distribution f_{10} given by Hill and Ford.[9] This gives a region of constant potential $0\leq r\leq 6.19\times10^{-13}$, and in the region $6.19\times10^{-13}\leq r\leq 7.59\times10^{-13}$,

$$2\epsilon_qV=\{2\epsilon_q\rho_0/[1+2\epsilon_q\rho_0B(5.42-0.714r)]\}[0.714Bd/dr+(2\epsilon_qEB+B')(5.42-0.714r)].$$

In the scattering by large nuclei it is expected that the tendency for back-scattering will not be observed, as is predicted by the present calculation with the diffuse edge. The uniform distribution, however results in the phase shift for $l=6$ to be about twice the size of that for $l=5$; that is the partial wave corresponding to the impact parameter approximately that of the radius of the nucleus is strongly affected by the square edge. As seen in Fig. 2, this effect is most noticeable in the backward direction where there is interference between alternate waves. By this it is seen that the uniform distribution is quite a poor approximation for the elastic scattering from even a large nucleus when this potential is used.

C. Gaussian Distribution

There is some evidence that the nuclear distribution of C^{12} resembles a Gaussian[10] $\rho_G=\rho_0\exp(-r^2/\sigma^2)$. ρ_0 and σ were determined by setting $\int\rho dv=1$ and making $\langle r^2\rangle_{Av}=\int r^2\rho(r)dv$ the same for this distribution as the uniform case. This gave $\sigma=1.45r_0$ and $\rho_0=1/(\sigma\sqrt{\pi})^3$

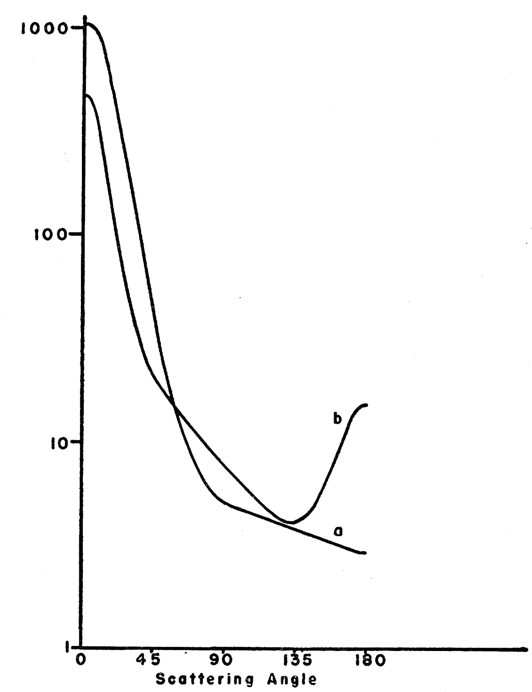

FIG. 2. Elastic differential cross section for large nucleus using (a) diffuse edge and (b) uniform distribution.

[8] Bodansky, Sachs, and Steinberger, Phys. Rev. 91, 467 (1953).

[9] David L. Hill and Kenneth W. Ford, Phys. Rev. 94, 1617 (1954).

[10] E. M. Henley, Phys. Rev. 85, 204 (1952).

$=2.97\rho_u$. In order to avoid some of the inaccuracies present in the potential (6) the problem was set up in momentum space where the Schrödinger equation takes the form

$$(\epsilon_q - W_k)\varphi_k(q) = \frac{2\pi A}{\epsilon_q}\int [\alpha(q,q') + \beta(q,q')\mathbf{q}\cdot\mathbf{q}'\rho(q-q')\varphi(q')d^3q'],$$

where

$$\langle(q|t_\alpha|q')\rangle = \frac{2\pi}{\epsilon_q}[\alpha(q,q')+\beta(q,q')\mathbf{q}\cdot\mathbf{q}'].$$

α and β now take the place of s and t which appeared in (3). Their form is chosen to fit the experimental data for elastic scattering.[6] The same form is used for the off-energy-shell scattering, not a very good approximation but the natural one to make with no other experimental evidence. Also, because of the rapid decrease of the magnitude of the distribution away from the most probable value, the scattering near the energy shell will be most important. Upon breaking the equation into component waves, the resulting one-dimensional equation is

$$(\epsilon_q - W_k)\varphi_l(q) = -\int_0^\infty K_l(q,q')\varphi_l(q')dq',$$

where

$$K_l(q,q') = -\frac{(2\pi)^2}{\epsilon_q}\int_{-1}^{1} P_l(\mu) \times[\alpha(q,q')+\beta(q,q')qq'\mu]\exp[-\tfrac{1}{4}\sigma^2|\mathbf{q}-\mathbf{q}'|^2]d\mu. \quad (8)$$

There will also be two imaginary contributions to the potential, one coming from the imaginary part of the scattering amplitude, a term $-iv_\pi/2\lambda_s$, where v_π is the velocity of the incident meson and λ_s is the mean free path for scattering in the nucleus, and the other given by $-iv_\pi/2\lambda_a$, where λ_a is the mean free path for absorption. Using λ_a as given by Francis and Watson,[11]

$$\frac{1}{\lambda_a} = 4\pi\frac{Z}{V_A}\left(\frac{q}{mc}\right)^2\frac{1}{v_\pi}[1.2\times10^{-27}\text{ cm}^2], \quad (9)$$

and taking λ_s as an average of scattering from protons and neutrons

$$\frac{1}{\lambda_s} = \frac{1}{2}\frac{A}{V_A}\frac{(\sigma_p+\sigma_N)}{2} = \frac{A}{V_A}16.6\left(\frac{q}{k}\right)^4\times10^{-27}\text{ cm}^2, \quad (10)$$

where σ_p and σ_N are obtained from the curves of Anderson *et al.*,[12] and the factor $\frac{1}{2}$ is an estimate of the exclusion principle effect. Because of the approximations going into the derivation of Eq. (8) it did not seem worthy of solution by lengthy numerical methods. A variational principle in momentum space[13] was used to get an approximate solution. The trial wave function used was of the form of the first Born approximation.

To check the accuracy of this method, the $l=0$ phase shift for the potential

$$V = \frac{1.5}{2\epsilon_q}\frac{1}{r_0^2}\exp[-(r/\sigma)^2]$$

was found in coordinate space and in momentum space by use of the Kohn variational principle with a trial wave function obtained in the same manner as the one used in solving (8). The Born approximation result is identical, but the phase shift found by the variational principle was about twice the size as the phase shift found by integrating the differential equation in coordinate space. The source of the discrepancy is probably the poor choice of trial wave function. A comparison of the first Born approximation wave function in momentum space and the Fourier transform of the integrated wave function in coordinate space indicates the high momentum components of the wave function are considerably underestimated by the Born approximation, a fact which would considerably alter the integrals used in the variational expression.

Because of the uncertainty in the result of the variational principle calculation the problem was set up in coordinate space for comparison, using the potential (6) with the gaussian density. As mentioned before this potential is not correct for a deep well, a situation obviously present near the center of the nucleus when a gaussian distribution is assumed. However the form of the potential seems to have some measure of validity, although the numbers B and B' would not be correctly given by the experimental data on elastic scattering. Choosing the magnitude of B, the magnitude of the cross section in the forward direction and the forward to backward ratio of scattering could be fit by using a ratio of B'/B which is approximately that predicted by experiment. Again the imaginary part of the potential was taken in the Born approximation. The resulting potential is $(\rho=r/\sigma)$

$$2\epsilon_q V = (1+2\epsilon_q\rho_0\exp(-\rho^2)^{-1}\left[2\epsilon_q B\rho_0\exp(-\rho^2)\frac{2\rho}{\sigma^2}\frac{d}{d\rho} + 2\epsilon_q(2\epsilon_q EB+B')\rho_0\exp(-\rho^2)\right] + Ai\exp(-\rho^2). \quad (11)$$

Using the real part of this potential the $l=0, 1$ phase shifts were found by numerical integration of the Schrödinger equation. The contribution of the higher waves was found by using the Born approximation amplitude with the $l=0, 1$ partial waves subtracted out. The results in Fig. 3 are for $2\epsilon_q B\rho_0 = -0.61$ and $2\epsilon_q B'\rho_0 r_0^2 = 0.3$; $A=0.5\sigma$. A more exact treatment would have included the imaginary part of the potential in the

[11] Norman C. Francis and Kenneth M. Watson, Am. J. Phys. 21, 659 (1953).

[12] Anderson, Fermi, Lang, and Nagle, Phys. Rev. 85, 936 (1952).

[13] Walter Kohn, Phys. Rev. 84, 495 (1951).

$l=0$, 1 phase shift calculation, but the front to back ratio of the cross section is rather insensitive to this.

III. DISCUSSION

The optical potential (6) is seen to have the ability to predict the large backward scattering in light nuclei, while indicating that the elastic scattering will be monotonically decreasing for heavy nuclei. It is the strong l dependence of the surface term which produces this effect. Near the origin the spherical Bessel function has the well-known form $\rho^l/1\cdot 3\cdots(2l+1)$, a form not too dissimilar to that of the wave functions obtained by numerical integration. Assuming this asymptotic expansion the real part of the potential with the gaussian distribution near the origin is of the form $(l\neq 0)$;

$$2\epsilon_q V_l=(1+2\epsilon_q\rho_0)^{-1}\left[\frac{4\epsilon_q B\rho_0}{\sigma^2}l+(2\epsilon EB+B')2\epsilon\rho_0\right].$$

In this region the surface term is seen to be of increasing importance with increasing l-values. In particular the derivative of the $l=0$ wave function is zero near the origin and is negative for a considerable region where the density is largest, resulting in a potential of smaller magnitude. On the other hand the surface term adds to the negative potential for $l=1$, increasing the phase shifts. Now in describing the scattering of 62-Mev mesons by a light nucleus only a few partial waves are important, in fact only four phase shifts were found large enough to effect the cross section in the carbon scattering. Since the surface term produces a marked decrease of the s-wave scattering with respect to the p-wave scattering, with these two partial waves being certainly the most important, the scattering is greatly increased in the backward direction. In heavier nuclei this effect is almost completely lost, for the surface term is zero in the region where the $l=0$ wave function has a negative derivative, indeed this term only contributes in the small region of the diffuse edge.

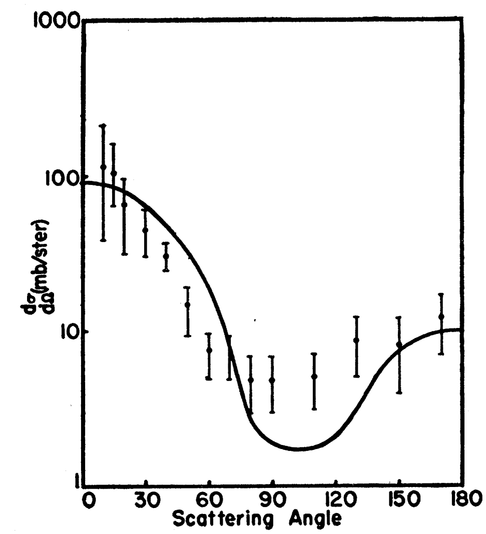

FIG. 3. Differential cross section for the scattering of 62-Mev pions on carbon with a Gaussian distribution. The experimental points are obtained by Byfield *et al.* (reference 4) by an averaging over π^+ and π^-, with the Coulomb scattering of a uniformly-charged sphere subtracted out.

Since the scattering produced by a uniform distribution is so different from that resulting from a distribution with a diffuse edge, it is suggested that this potential could be used to investigate nuclear shapes. A phenomenological treatment in the region where absorption is not too strong might be successful, but the treatment of the Coulomb effect and of the imaginary parts of the potential would have to be improved. For heavier nuclei it is doubtful that a comparison with experiment would select between the diffuse edge used and other shapes which recently have been suggested.

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