(p,a) and (p,ab) Reactions at 100 Mev*

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Nuclear reactions at energies of the order of 100 Mev and higher are considered as being divided into two independent steps, a knock-on process followed by nucleon evaporation. The effect of the knock-on process is evaluated by a method similar to that of Goldberger, while the evaporation process is treated in the usual way. Cross sections for several (p,n), $(p,2n)$, (p,pn), and $(p,2p)$ reactions on intermediate nuclei are calculated and compared with experimental results.

I. INTRODUCTION

WITH the development of high-energy accelerators which permitted studies of nuclear reactions in the region of 100 Mev and above, it became evident that the compound nucleus theory[1,2] could not be extended to these energies. At high energies this theory predicts a wide spread of reaction products with maximum yields concentrated several mass numbers from the parent nucleus. Bombardment by various particles at energies above 100 Mev produced the expected wide spread of products but the highest yields were found to be those in the immediate neighborhood of the parent nucleus.[3–7]

A proposal advanced by Serber[8] suggested that the first step of a reaction at these energies be regarded in terms of collisions between the incident particle and individual nuclear particles. If these collisions could be described by the corresponding free particle collision cross sections then the mean free path of the incident particle in nuclear matter would be comparable to the nuclear radius and would be increased further by the restriction placed on momentum transfers by the degeneracy of nuclear matter. Furthermore, since the incident particle could transfer, on the average, only about one-quarter of its energy, it might leave the nucleus after a single collision retaining most of its original energy, or it might initiate a cascade inside the nucleus which might result in one or more high-energy particles being immediately ejected. In either case, the nucleus would be left with a wide spread of excitation energies which it could lose by evaporating additional particles. This model was developed in greater detail by Goldberger[9] and later by Bernardini[10] *et al.*, and has been used by several investigators to explain high-energy spallation reactions.[3–6]

In this paper this model has been applied in a more quantitative manner to a number of (p,n), $(p,2n)$, (p,pn), and $(p,2p)$ reactions at 100 Mev and the results compared with experimental data.

II. CALCULATION OF CROSS SECTIONS

A nuclear reaction in the 100-Mev region was considered as taking place in two steps. First, the incident particle passes through the nucleus, interacting with it by a series of individual nucleon-nucleon collisions and initiating a nuclear cascade which results in the immediate ejection of one or more high-energy particles and leaves the nucleus in an excited state. Second, this excited nucleus may lose energy by emitting heavy particles or photons. Reactions of the type (p,n), $(p,2n)$, (p,pn), and $(p,2p)$ may occur in one or more of the ways indicated in Table I. A represents the target nucleus, C^* the excited residual nucleus, and B the product nucleus which may differ from C^* only in its excitation energy. There are two additional processes not listed, namely, the complete capture of the incident proton followed by nucleon evaporation and the initial ejection of a deuteron followed by gamma emission. The first of these, complete capture of a 100-Mev proton, should give a negligible contribution to these reactions. Complete capture of such a particle with such a high energy by a nucleus of intermediate mass is unlikely, and if it is captured the probability of a compound nucleus with such an excitation energy evaporating only two nucleons is very small. The second process, initial ejection of a deuteron, is not included although experimental data indicate that it may be appreciable.[11,12]

TABLE I. Ways in which reactions of the type (p,n), $(p,2n)$, (p,pn), and $(p,2p)$ may occur.

Knock-on	Evaporation
$p+A\rightarrow C_1^*+p$	$C_1^*\rightarrow B_1+n$
	$C_1^*\rightarrow B_2+p$
$p+A\rightarrow C_2^*+n$	$C_2^*\rightarrow B_1+p$
	$C_2^*\rightarrow B_3+n$
	$C_2^*\rightarrow B_4+\gamma$
$p+A\rightarrow C_3^*+2p$	$C_3^*\rightarrow B_2+\gamma$
$p+A\rightarrow C_4^*+2n$	$C_4^*\rightarrow B_3+\gamma$
$p+A\rightarrow C_5^*+p+n$	$C_5^*\rightarrow B_1+\gamma$

* Supported by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

1 N. Bohr, Nature 137, 344 (1936).
2 V. F. Weisskopf and D. H. Ewing, Phys. Rev. 57, 472 (1940).
3 M. Lindner and I. Perlman, Phys. Rev. 78, 499 (1950).
4 Bartell, Helmholtz, Softky, and Stewart, Phys. Rev. 80, 1006 (1950).
5 Rudstam, Stevenson, and Folger, Phys. Rev. 87, 358 (1952).
6 S. G. Rudstam, Phil. Mag. 44, 1131 (1953).
7 L. Marquez, Phys. Rev. 88, 225 (1952).
8 R. Serber, Phys. Rev. 72, 1114 (1947).
9 M. L. Goldberger, Phys. Rev. 74, 1269 (1948).
10 Bernardini, Booth, and Lindenbaum, Phys. Rev. 88, 1017 (1952).
11 J. Hadley and H. York, Phys. Rev. 80, 345 (1950).
12 W. Selove, Phys. Rev. 92, 1328 (1953).

We may write the cross section for forming the final nucleus B as

$$\sigma_B=\sigma_r\sum_j P_j\int_0^{E_{x\max}} N_j(E_x)F_b(E_x)dE_x,$$

where σ_r is the cross section for the incident particle entering the nucleus, P_j is the probability of a given knock-on process which can lead to the final nucleus B, $N_j(E_x)$ is the normalized distribution of excitation energy E_x in the residual nucleus after the knock-on process has occurred, and $F_b(E_x)$ is the probability of the residual nucleus forming the final nucleus B by evaporating a particle or a photon. The sum is over only those knock-on processes which can eventually result in nucleus B. For these calculations σ_r was assumed to be the geometrical cross section, P_j and $N_j(E_x)$ were calculated by the method described in references 9 and 10, and $F_b(E_x)$ was calculated from statistical theory as formulated by Weisskopf.[2]

A. Knock-On Calculations

The nucleus is represented by two noninteracting, degenerate Fermi gases of neutrons and protons in a square potential well. The maximum Fermi momentum is

$$p_{\max}=(3\pi^2\hbar^3\rho)^{\frac{1}{3}},$$

where ρ is the density of neutrons or protons. If the nuclear radius is given by

$$R=1.4A^{\frac{1}{3}}\times10^{-13}\text{ cm},$$

the corresponding maximum Fermi energy is 22 Mev. The average binding energy is assumed to be 8 Mev, giving a total well depth of 30 Mev. In addition there is a 7-Mev Coulomb barrier for protons which, for the knock-on calculations, is assumed to be impenetrable.

The usual Fermi momentum distribution was used. Below the maximum that is given by

$$\frac{dn}{dp}=\frac{4\pi}{(2\pi\hbar)^3}\Omega p^2,$$

where Ω is the nuclear volume. The interpretation of recent scattering and meson production experiments (see for example Wilcox[13] and Block *et al.*[14]) indicate a Gaussian rather than a Fermi momentum distribution. However, since most of those experiments have been done for light elements where the gas model would be expected to be least valid, it was felt that the Fermi distribution still represented the best choice for heavy and intermediate nuclei.

It is assumed that the interaction of particles inside the well can be described by the interaction of the corresponding free particles and that the other nucleons exert no influence other than to provide the potential well, the initial momentum distribution of the nuclear particles, and to prohibit collisions into already filled states according to the Pauli exclusion principle. It is further assumed that during the initial cascade all particles not directly involved remain unchanged in energy.

The Monte Carlo method was used for the calculations. This involves following in detail, collision by collision, the progress of a number of nucleons through the nucleus until they either reach the edge and escape or their energy falls below the barrier and they are captured. Whenever there is a choice of one of a number of equally probable events, the choice is made on a random basis. In its ultimate refinement this process reproduces the natural process and the final results are subject to the same statistical errors as the corresponding experimental data.

The actual mechanics of the calculation were very similar to those described by Bernardini *et al.*[10] The same approximation of a planar nucleus was employed. The nucleus was replaced by a circle of the same diameter oriented so that one diameter was parallel to the incident beam. Chords were drawn parallel to the direction of the incident beam dividing the circle into ten sections which when rotated about the axis of the beam gave a number of concentric rings with equal projected frontal areas. An equal number of particles was assumed to be incident on each of the sections.

After the incident proton entered the nucleus the first problem was to determine where it made its first collision with a target nucleon. No consideration of the effect of the exclusion principle or of the nature of the target nucleon was introduced at this point.

The mean free path of a particle in nuclear matter may be expressed as

$$\lambda=1/\bar{\sigma}(\epsilon)\rho_t,$$

where $\bar{\sigma}(\epsilon)$ is the average total cross section for nucleon-nucleon collisions as a function of energy and ρ_t is the total nucleon density. The average total cross section was determined from experimental values[15,16] for the free particle cross sections suitably weighted to account for the relative number of the two types of particles. Since there is no direct experimental determination of the *n*-*n* cross section, it was estimated from a *n*-*d*, *n*-*p* substraction by using the data of DeJuren and Knable[17] and was assumed to be isotropic. All cross sections were assumed to have a $1/E$ dependence.

The path of the incident particle was divided into units equal to $\frac{1}{10}$ of the mean free path. The probability of a collision occurring in the nth interval is

$$P(n)=\exp(-n/10)[1-\exp\tfrac{1}{10}].$$

[13] J. M. Wilcox, University of California Radiation Laboratory Report UCRL-2540, 1953 (unpublished).
[14] Block, Passman, and Havens, Phys. Rev. **88**, 1239 (1952).
[15] Birge, Kruse, and Ramsey, Phys. Rev. **83**, 274 (1951).
[16] Hadley, Kelly, Leith, Segré, Wiegand, and York, Phys. Rev. **75**, 351 (1949).
[17] J. DeJuren and N. Knable, Phys. Rev. **77**, 606 (1950).

The intervals were then weighted accordingly and a selection of the interval in which a collision occurred was made by consulting a table of random numbers.

Next a decision was made as to the nature of the target nucleon, i.e., whether it was a neutron or a proton. Their relative probabilities may be determined from the ratio of $N_n\sigma_{pn}$ to $N_p\sigma_{pp}$ (or $N_p\sigma_{pn}$ to $N_n\sigma_{nn}$ if a neutron is being followed) and a choice again made by consulting a table of random numbers.

Having determined the nature of the target particle, it was necessary to next determine its momentum. The Fermi momentum sphere was compressed into a two-dimensional circle with its polar axis parallel to the path of the incident particle and divided into a number of areas such that when the circle was rotated 180° about its polar axis these areas swept equal volumes. There was a total of 640 such areas. A choice was made at random.

The two-nucleon system was transferred into its center-of-mass system and a choice of the scattering angle was made. The cross section for *p-p* scattering has been found to be essentially isotropic.[15] A similar angular dependence was assumed for the *n-n* cross section. The *n-p* cross section has been found to be symmetric around 90° with peaks in the forward and backward directions.[16] In order to simplify the calculations and still retain the *n-p* anisotropy, it was assumed that the angular dependence at 95 Mev was retained at all energies. This, of course, gives a much greater angular dependence at lower energies than is actually the case, but this effect is reduced by the Pauli exclusion principle which restricts small momentum transfers. The scattering angles were weighted according to the above angular dependencies and to the corresponding three dimensional solid angle and the choice again made by consulting a table of random numbers. If both final momentum vectors fell outside the Fermi momentum sphere the collision was allowed. If not, another choice for a collision was made and the

TABLE II. Results of Monte Carlo calculations for 200 incident protons of which 30 pass through the nucleus without collision.

No. particles out	No. of cases	P_j	Average excitation of residual nucleus (Mev)
0	1	0.005	108
n	16	0.08	39
p	25	0.125	
2*n*	8	0.004	
2*p*	21	0.105	34
np	53	0.265	
3	42	0.21	30
4	4	0.02	26
Total	170		

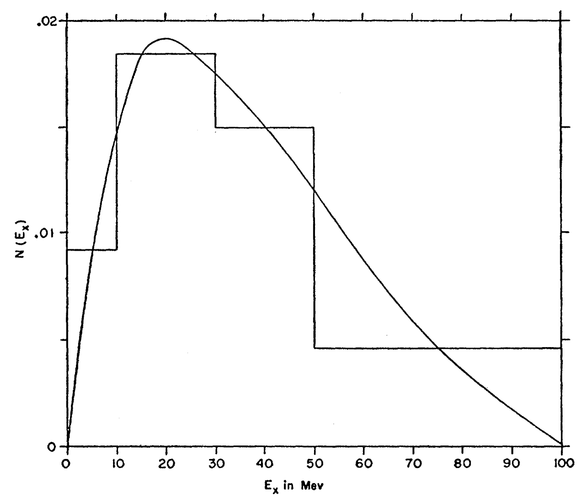

FIG. 1. Distribution of excitation energy in the residual nucleus when a single neutron or proton is ejected in the knock-on process.

calculation was repeated. This process was continued for all nucleons involved in the cascade until they either reached the edge of the nucleus and escaped or their energy fell below the nuclear barrier.

The thermal excitation energy of the residual nucleus was then determined by the conservation of energy, remembering that the incident proton brings in its kinetic energy plus its binding energy and each ejected nucleon carries off its kinetic energy plus its binding energy.

These calculations were performed for a nucleus of mass 64. A total of 200 incident protons with 100-Mev energy were followed. The values of P_j are given in Table II. The distribution of excitation energy in the residual nuclei are given in Figs. 1 and 2. Although ideally there should be a different distribution for each type of knock-on process, they are separated into two groups here in order to improve statistics. The smooth curves drawn through the histograms are the distributions used in the calculation of the reaction cross sections.

B. Evaporation Calculations

In the reactions being considered we are concerned only with those cases where the excited nucleus resulting from the knock-on process evaporates only one particle or loses its energy by gamma emission. The probability of an excited nucleus evaporating only one particle is

$$F_b(E_x)=\frac{\int_0^{E_x-E_b} I_b(\epsilon)\pi_\gamma(E_x-E_b-\epsilon)d\epsilon}{\sum_i \int_0^{E_x-E_i} I_i(\epsilon)d\epsilon}.$$

The probability of decaying by emission of a photon is

$$F_\gamma(E_x)=\pi_\gamma(E_x).$$

$I_b(\epsilon)$ is the energy spectrum of the x evaporated particle, ϵ is its energy, E_x is the excitation energy, E_b is the binding energy of particle b to the original nucleus, and $\pi_\gamma(E_x-E_b-\epsilon)$ is the probability of the product nucleus decaying by gamma emission.

$$\pi_\gamma=\Gamma_\gamma/\textstyle\sum_b \Gamma_b,$$

where Γ is defined as a level width. The level widths were calculated by using the energy dependence suggested by Weisskopf[2] and adjusting the constants to fit the observed ratio of neutron capture cross section to total neutron cross section for Cu^{65} at 0.025 Mev.[18] Γ_γ becomes very small a few kev above the neutron binding energy. Thus if the neutron binding energy to the final nucleus is smaller than the proton binding energy, the lower limit of the integral in the numerator of the expression for F_b may be replaced by $E_x-E_b-E_n$, where E_n is the neutron binding energy of the product nucleus, and term π_γ dropped. If the proton binding energy is less than the neutron binding energy, π_γ must be retained since Γ_γ is much greater than Γ_p until the excitation is $\sim$2 Mev above the proton binding energy because of the Coulomb barrier.

The energy distribution of the evaporated particles is given by Weisskopf and Ewing[2] as

$$I_b(\epsilon)=A\epsilon\sigma(\epsilon)\omega(E_x-E_b-\epsilon),$$

where $\sigma(\epsilon)$ is the cross section for the reverse reaction, A is a constant, $\omega(E_x-E_b-\epsilon)$ is the level density of the final nucleus, and ϵ is the energy of the particle

$$\omega=C\exp 2[a(E_x-E_b-\epsilon)]^{\frac{1}{2}}.$$

The constants C and a used were those given by Blatt and Weisskopf[19] for odd-mass nuclei.

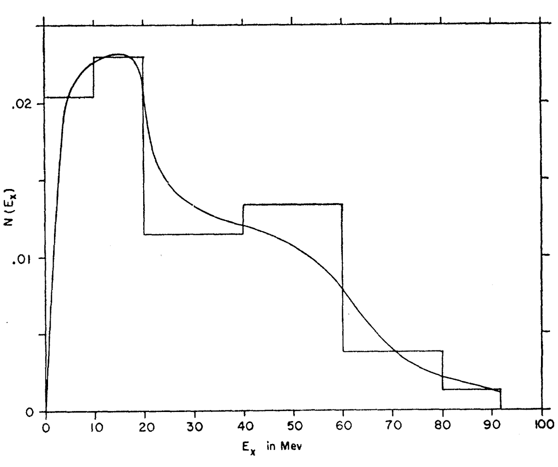

FIG. 2. Distribution of excitation energy in the residual nucleus when two nucleons are ejected in the knock-on process.

[18] Neutron Total Cross Sections, U. S. Atomic Energy Commission Report AECU-2040 (Technical Information Division, Department of Commerce, Washington, D. C., 1952).
[19] J. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).

TABLE III. Comparison of calculated and experimental cross sections at 25 Mev for various values of C_{oo}/C_{ee} ($R=1.5A^{\frac{1}{3}}\times10^{13}$ cm).

C_{oo}/C_{ee} Reaction	σ in millibarns Calculated 4	8	12	Experimental
$Cu^{63}(p,pn)Cu^{62}$	307	445	520	570
$Cu^{63}(p,2n)Zn^{62}$	165	131	104	138
$Cu^{65}(p,pn)Cu^{64}$	328	422	477	525
$Ga^{69}(p,pn)Ga^{68}$	438	544	604	485
$Ga^{69}(p,2n)Ge^{68}$	423	365	324	440
$As^{75}(p,pn)As^{74}$	250	328	370	202

All the $p,2n$ reactions and all except one of the p,pn reactions considered lead to odd-mass nuclei. It has been suggested[2] that

$$C_{oo}=4C_{ee},$$

where the subscripts refer to odd-odd and even-even nuclei. However, calculations at 25 Mev have indicated that this ratio between C_{oo} and C_{ee} is not correct for Cu^{62} and Zn^{62}.[20,21] In order to determine the best values of C_{oo}/C_{ee}, cross sections calculated with various values of this ratio are compared with experimental data at 25 Mev where both reactions are at their maximum and compound nucleus formation is still expected to be the principle mechanism for reaction. (Table III.) A particularly good way of determining this ratio is from a comparison of the cross sections for (p,pn) and $(p,2n)$ reactions from the same parent nucleus. Two such situations are available. The ratio of the cross sections of $Cu^{63}(p,pn)Cu^{62}$ and $Cu^{63}(p,2n)Zn^{62}$ may be determined quite accurately without the usual errors in beta counting since Zn^{62} may be counted by the radiation of its Cu^{62} daughter. The experimental value for this ratio is four at 25 Mev. A previous calculation[20] indicated that if the compound nucleus theory is valid at 25 Mev, C_{oo}/C_{ee} must be $\simeq$28. The results reported in Table III using different values for the constant a, and including the effect of photons and alpha particles gives a value of $C_{oo}/C_{ee}\simeq10$.

A similar situation exists for the reactions $Ga^{69}(p,pn)Ga^{68}$ and $Ga^{69}(p,2n)Ge^{68}$. The experimental ratio is 1.1 which is in agreement with $C_{oo}/C_{ee}\simeq4$.

Two additional reactions, $Cu^{65}(p,pn)Cu^{64}$ and $As^{75}(p,pn)As^{74}$, result in odd-mass final nuclei. Comparison of calculated and experimental cross sections at 25 Mev indicate that a choice of $C_{oo}/C_{ee}\simeq10$ for the Cu^{65} reaction and of $C_{oo}/C_{ee}\simeq4$ for the As^{75} reaction yield checks with experimental data which are comparable with those obtained for the Cu^{63} and Ga^{69} reactions.

All the statistical calculations are very sensitive to the values of the binding energies of the various particles. The binding energies used were calculated from

[20] J. W. Meadows, Phys. Rev. **91**, 885 (1953).
[21] S. N. Ghoshal, Phys. Rev. **80**, 939 (1950).

TABLE IV. Binding energies.

	Binding energy in Mev B_n	B_p	B_α
Cu^{62}	8.5	5.3	
Cu^{63}	10.6	5.3	4.6
Cu^{64}	8.0	7.2	
Cu^{65}	9.7	6.9	5.1
Cu^{67}	9.4	8.9	
Zn^{62}		6.2	
Zn^{63}	8.9	6.4	3.6
Zn^{64}	11.5	7.4	3.2
Zn^{65}	8.0	7.4	3.5
Zn^{66}	11.0	8.7	3.2
Zn^{68}	9.6	9.4	4.3
Ga^{64}	10.3[a]	3.7[a]	4.4[a]
Ga^{65}	12.7[a]	4.2	4.5
Ga^{66}	9 1	5.3	
Ga^{67}	11.4	5.7	
Ga^{68}	7.8	5.9	
Ga^{69}	10.5	6.8	4.1
Ge^{68}		6.6[b]	
Ge^{69}	7.5[b]	6.3	2.3
Ge^{70}	13.1	9.0	4.4
As^{74}	10.0	6.8	
As^{75}	10.2	6.6	5.3
Se^{75}	8.0	8.4	4.7
Se^{76}	11.3	9.6	3.7

[a] Calculated from empirical mass formula [N. Metropolis and G. Reitwiesner, U. S. Atomic Energy Commission Report NP-1980, 1950 (unpublished)].
[b] $Ge^{68}-Ga^{68}$ mass difference estimated according to reference 25.

experimental mass[22,23] and radioactivity[24] data. There were a few cases, however, where this was not possible. The mass of Ge^{68} could not be based directly on experimental data as the energy available for its decay to Ga^{68} was not known. Since it decays by K-capture and has a fairly long half-life (250 days), that energy would be expected to be small. An estimate of this energy difference using the method described by Coryell[25] yields $\simeq$0.4 Mev. The nucleus Ga^{64} is unknown. The values used for its proton and neutron binding energies were calculated from the empirical mass formula which gives good agreement with those calculated from experimental data for heavier gallium isotopes. The values of the binding energies used are listed in Table IV.

III. COMPARISON WITH EXPERIMENT AND DISCUSSION

The experimental results for the reactions on Cu^{63} and Cu^{65} have been reported previously.[20] All other cross sections reported here were measured by a similar method, the only difference being in the chemical separation procedures. Enriched isotopes[26] of Zn^{64}, Zn^{66} and Ga^{69} were used. The reactions of Zn^{68} and As^{75} were measured using the natural element. The experimental and calculated values of the reaction cross sections at 100 Mev are listed in Table V.

Qualitatively, there is agreement in so far as both the experimental and calculated values for the (p,pn) reactions are high compared to the others. Particularly, there is surprisingly good agreement for the $(p,2n)$ and $(p,2p)$ reactions. Similar agreement might be expected for the (p,n) reactions, but this is not found. This may be in part due to the assumption of an average binding energy of 8 Mev for the knock-on calculations. For the isotopes involved in these reactions, which for the most part are neutron poor, the neutron binding energy is about 9 to 10 Mev. The proton binding energy is, on the average, $\sim$3 Mev lower. The statistics of the knock-on calculations were not considered good enough to warrant inclusion of this difference, but qualitatively it would have the effect of shifting the distribution of E_x for single-neutron, double-neutron, and neutron-proton knock-on processes to a correspondingly lower energy. This would raise all the (p,n) cross sections and would be particularly important for the $Zn^{66}(p,n)Ga^{66}$ reaction where this difference in binding energy amounts to 5.7 Mev. Such an effect would be partially self-compensating for the $(p,2n)$ reactions. In the case of the single neutron knock-on process, only values of E_x from $\sim$8 to $\sim$20 Mev contribute greatly to the evaporation of a single additional particle. Since this region is open at both ends, any general downward shift will have little effect. The double-neutron knock-on process contributes little to the $(p,2n)$ cross sections. The same argument concerning the single-neutron knock-on process holds for the (p,pn) reaction, but the inclusion of the differences in the binding energies would be expected to result in an increase in these cross sections due to the large contribution of the proton-neutron knock-on process.

The poor agreement of the (p,pn) cross sections may be in part due to the neglect of deuteron pickup, but it appears unlikely that this would account for more than a small fraction of the observed discrepancy. Hadley and York,[11] when bombarding copper with 90-Mev neutrons, observed a cross section of 52 millibarns for

TABLE V. Experimental and calculated reaction cross sections at 100 Mev.

Reactions	Contribution of individual knock-on processes in millibarns n	p	nn	pp	np	σ in millibarns Calc	Exp
$Cu^{63}(p,n)Zn^{63}$	4					4	7
$Cu^{63}(p,2n)Zn^{62}$	5		5			10	7
$Cu^{63}(p,pn)Cu^{62}$	10	18			28	55	120
$Cu^{65}(p,pn)Cu^{64}$	5	24			33	62	155
$Zn^{64}(p,pn)Zn^{63}$	15	8			34	57	120
$Zn^{66}(p,n)Ga^{66}$	4					4	12
$Zn^{68}(p,2p)Cu^{67}$		2		14		16	14
$Ga^{69}(p,n)Ge^{69}$	4					4	15
$Ga^{69}(p,2n)Ge^{68}$	16		6			21	23
$Ga^{69}(p,pn)Ga^{68}$	3	22			34	59	193
$As^{75}(p,pn)As^{74}$	2	26			39	66	102

[22] Collins, Nier, and Johnson, Phys. Rev. **86**, 408 (1952).
[23] Collins, Johnson, and Nier, Phys. Rev. **94**, 398 (1954).
[24] Hollander, Perlman, and Seaborg, University of California Radiation Laboratory Report UCRL-1928, 1952 (unpublished).
[25] C. D. Coryell, Ann. Rev. Nuc. Sci. **2**, 305 (1953).
[26] Obtained from Oak Ridge National Laboratory, Oak Ridge, Tennessee.

the production of deuterons with energy greater than 27 Mev. However, only a small part of these would leave the nucleus with sufficiently low excitation to contribute to the (p,pn) reaction. Most probably the reasons lie in the effect of the binding energies, the simplifying assumptions made during the calculations, and the poor statistics.

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Effect of the Anomalous Nucleon Magnetic Moment on the π^0 Lifetime*

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The anomalous magnetic moment of the intermediate nucleons has been included in a treatment of the two-photon decay of the π^0. It is found that this inclusion leads to a shorter, rather than a longer, lifetime, thereby failing to reduce the discrepancy between the experimental and previous theoretical values.

THE lowest order perturbation calculation of the transition probability for the π^0 meson decaying into two photons yields, in the case of pseudoscalar mesons with pseudoscalar coupling to the nucleon field, a lifetime of $7\times 10^{-16}(g^2/4\pi\hbar c)^{-1}$ sec. Use of the value of the coupling constant obtained from meson-nucleon scattering experiments,[1] $g^2/4\pi\hbar c\sim 10$, results in a lifetime which is small compared to the experimentally observed mean life of the π^0 of about 10^{-15} sec.[2]

In the present paper the effect of the inclusion of the additional interaction due to the anomalous magnetic moments of the intermediate nucleons is studied. Since with this interaction unrenormalizable divergences appear, a cutoff factor must be introduced. While there is some ambiguity in the method of introduction of this cutoff factor, it is found that the qualitative dependence of the lifetime on the cutoff parameter is not sensitive to the choice of cutoff scheme.

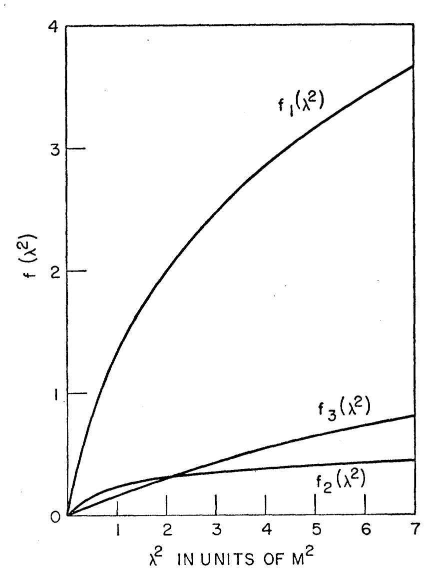

FIG. 1. The dependence of the functions $f_1(\lambda^2)$, $f_2(\lambda^2)$, and $f_3(\lambda^2)$ on the cutoff parameter, λ^2.

The matrix element for the lowest order perturbation calculation of the decay of the π^0 into two photons is, in the notation of Feynman,[3]

$$\mathfrak{M}_0=2A\int \mathrm{Sp}\left\{\frac{1}{p-k_2-M}e_2\frac{1}{p-M}e_1\frac{1}{p+k_1-M}\gamma_5\right\}d^4p,$$

where

$$A=(4\pi)^{\frac{1}{2}}ie^2g/(2\pi)^4.$$

The inclusion of the anomalous magnetic moment interaction gives a matrix element of the form

$$\mathfrak{M}=2A\int \mathrm{Sp}\left\{\frac{1}{p-k_2-M}\left(e_2+\frac{\mu}{2M}e_2k_2G\right)\frac{1}{p-M}\right.$$
$$\left.\times\left(e_1+\frac{\mu}{2M}e_1k_1G\right)\frac{1}{p+k_1-M}\gamma_5\right\}d^4p,$$

where G is the cutoff factor to be introduced. The conventional form of the cutoff factor, $-\lambda^2/(p^2-\lambda^2)$, is chosen but is made symmetric by averaging the results of inserting it into each of the three nucleon propagators, i.e.,

$$G=-\frac{\lambda^2}{3}\left\{\frac{1}{p^2-\lambda^2}+\frac{1}{(p-k_2)^2-\lambda^2}+\frac{1}{(p+k_1)^2-\lambda^2}\right\}.$$

After performing the integrations and neglecting the square of the meson mass as compared to the square of the nucleon mass, it is found that the ratio of $\mathfrak{M}$ to

* This research supported by a grant from the U. S. Atomic Energy Commission.

[1] F. J. Dyson *et al.*, Phys. Rev. **95**, 1644 (1954).
[2] B. M. Anand, Proc. Roy. Soc. (London) **A220**, 183 (1953).
[3] R. P. Feynman, Phys. Rev. **76**, 769 (1949).