

indicated, that this level is reached by  $L$ -electron capture.

(4) Therefore the ground state of  $\text{Bi}^{207}$  is less than 90 keV above the 2.34-MeV level. This is also in agreement with the work of Alburger and Sunyar.<sup>20</sup>

(5) The number of coincidences of 1.07-MeV gamma rays with x-rays indicates the level at 1.64 MeV is delayed, as would be expected from the measured half-life of 0.84 seconds.

(6) The angular correlation measurement of the 1.77–0.570 MeV gamma-ray cascade is consistent with spin 9/2 for the 2.34-MeV level and

$$\delta^2 = 0.0338_{-0.0039}^{+0.0036}.$$

### VII. DISCUSSION

The spins predicted by the shell model for excited states of a single neutron "hole" in the shell closing at 126 neutrons are  $p_{1/2}$ ,  $p_{3/2}$ ,  $f_{5/2}$ ,  $h_{9/2}$ , and  $i_{13/2}$ . Since the 82nd proton of  $\text{Pb}^{207}$  closes a shell, this nucleus presents one of the most ideal cases among all the nuclei of the conditions which would be expected to show single-particle characteristics. Thus the assignment of  $p_{1/2}$ ,  $f_{5/2}$ , and  $i_{13/2}$  is quite natural for the first three states observed in the  $\text{Bi}^{207}$  decay. A state at 0.870 MeV, which has been found by electron decay, has been assigned

as  $p_{3/2}$ . In addition, the intensities of the transitions to the various states are not unreasonable. The half-life of  $\text{Bi}^{207}$  has recently been measured as  $8.0 \pm 0.6$  years.<sup>24</sup> If this value is taken together with the energy fixed by the pure  $L$ -capture to the 2.34-MeV state, the comparative half-life for the transition to the 0.570-MeV state is given by  $\log_{10} ft = 11.8$ . The  $\text{Bi}^{209}$  ground state spin has been measured as 9/2,<sup>25</sup> and the assignment of the 83rd proton in  $\text{Bi}^{207}$  as  $h_{9/2}$  is reasonable. Thus, this transition is probably second-forbidden and its comparative half-life agrees with this assignment. The transition to the isomeric state is first-forbidden based on the spin and parity assignments.  $\log_{10} ft = 9.6$  for this transition seems to be slightly high for such an assignment but not abnormally so. The transition to the 2.34-MeV level should be slowed because of the lowered probability of  $L$ -capture, and the 9.6 percent intensity of the transition may not be unexpected.

The authors would like to express their thanks to Dr. E. C. Campbell and Dr. M. E. Rose for many informative discussions and to Dr. P. R. Bell and Dr. F. K. McGowan for their interest and assistance in many phases of this experiment.

<sup>24</sup> Cheng, Rudolfo, Pool, and Kundu, *Bull. Am. Phys. Soc.* **29**, No. 7, 16 (1954).

<sup>25</sup> P. F. S. Klinkenberg, *Revs. Modern Phys.* **24**, 63 (1952).

## Internal Conversion in the $L$ and $M$ Subshells\*†

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The relative internal conversion of electric and magnetic transitions in the  $L$  and  $M$  subshells has been calculated nonrelativistically with neglect of screening. In the approximations employed, the relative conversion coefficients are functions of the parameter  $Z^2/\hbar\omega$ , where  $Z$  is the atomic number, and  $\hbar\omega$  the nuclear transition energy. For the  $L$  shell, these quantities have been evaluated over the entire range of practical interest, while for simplicity, the  $M$  shell has been considered only for vanishing electron momentum. Results are given for transitions of multipole order 1 through 5.

Comparison with existing relativistic calculations and experimental data indicates the essential validity of the results obtained, and allows semiempirical generalizations to be made. It is found that dipole transitions, especially magnetic, convert strongly in the  $S$  subshells. With increasing multipole order, subshells of successively higher orbital angular momenta are preferred. Electric transitions exhibit comparable conversion in the relativistic doublets,  $j = l \pm \frac{1}{2}$ , while magnetic transitions convert almost entirely in the  $j = l + \frac{1}{2}$  component.

The usefulness of these properties in the identification of the character of low-energy transitions in heavy elements is indicated.

### INTRODUCTION

INTERNAL-conversion electron spectroscopy is a powerful tool for the study of nuclear structure. The classification of various nuclear transitions in terms

of their relative conversion in the  $K$  and  $L$  atomic shells has been used extensively for this purpose, and has been discussed in detail elsewhere.<sup>1</sup> In heavy elements ( $Z > 50$ ), however, the low energy of the transitions encountered and the high binding energy of the  $K$  shell frequently make the observation of  $K$ -conversion electrons difficult or impossible. Under these

\* Preliminary report given by E. L. Church and J. E. Monahan, *Phys. Rev.* **94**, 762(A) (1954).

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‡ On loan from Frankford Arsenal, Philadelphia, Pennsylvania.

<sup>1</sup> M. Goldhaber and A. W. Sunyar, *Phys. Rev.* **83**, 906 (1951).

conditions, a knowledge of the conversion properties of transitions of different types and multiplicities in the  $L$  and  $M$  shells is indispensable for the resolution of the decay schemes of these nuclei. The presently available exact calculations of internal-conversion coefficients are inadequate for this purpose.

The present paper presents the results of purely nonrelativistic calculations of the internal-conversion properties of electric and magnetic transitions in the  $L$  and  $M$  shells. Attention has been confined to the evaluation of the relative conversion coefficients in the various subshells because of their direct experimental significance, and the probability that they are less sensitive to the approximations employed than the absolute values. In the approximations considered, these ratios are found to be functions of the parameter  $Z^2/\hbar\omega$ , where  $Z$  is the atomic number, and  $\hbar\omega$  the nuclear transition energy.

No attempt has been made to justify the many assumptions underlying the present calculations. Rather, the qualitative validity of the results obtained have been demonstrated by comparison with existing relativistic calculations for the  $L$  shell,<sup>2,3</sup> and with experimental data for heavy elements for both the  $L$  and  $M$  shells. Eventual comparison of the present results with the exact calculations of Rose *et al.*,<sup>3</sup> for the  $L$  shell, should provide a further means for estimating the validity of these approximations and offer additional indirect support for the present predictions regarding the  $M$  subshells.

### RESULTS

Internal-conversion coefficients for electric transitions have been calculated completely nonrelativistically by using hydrogenic Schrödinger wave functions and considering only the leading term in the expansion of the multipole field. For electric radiations this represents the lowest-order approximation leading to physically reasonable results. The present calculations represent an extension of the work of Hebb and Nelson,<sup>4</sup> who computed absolute  $K$ -shell and total  $L$ -shell conversion coefficients under the same conditions.

The results obtained for the  $L$  shell are presented in Fig. 1, where the ratio of  $P$  subshell ( $L_{II}+L_{III}$ ) to  $S$  subshell ( $L_I$ ) conversion is shown. In the present approximations the  $L_{II}$  to  $L_{III}$  conversion is always in the ratio of their statistical weights (1:2). The relative conversion coefficients for  $E2$ – $E5$  transitions become infinite for  $Z^2/\hbar\omega=200$ , 100, 66, and 50, respectively, due to the vanishing of the  $L_I$  conversion coefficient at these points. The relative conversion in both the  $L$  and  $M$  shells expected at their respective nonrelativistic thresholds is shown in Fig. 2. In the latter case,  $S$  represents the  $M_I$  subshell,  $P$  conversion is  $M_{II}+M_{III}$  in the ratio of 1:2, and  $D$  is  $M_{IV}+M_V$  in the constant ratio of 2:3.

<sup>2</sup> Gellman, Griffith, and Stanley, Phys. Rev. **85**, 944 (1952).

<sup>3</sup> Rose, Goertzel, and Swift (prepublication notes, 1954).

<sup>4</sup> M. H. Hebb and E. Nelson, Phys. Rev. **58**, 486 (1940).

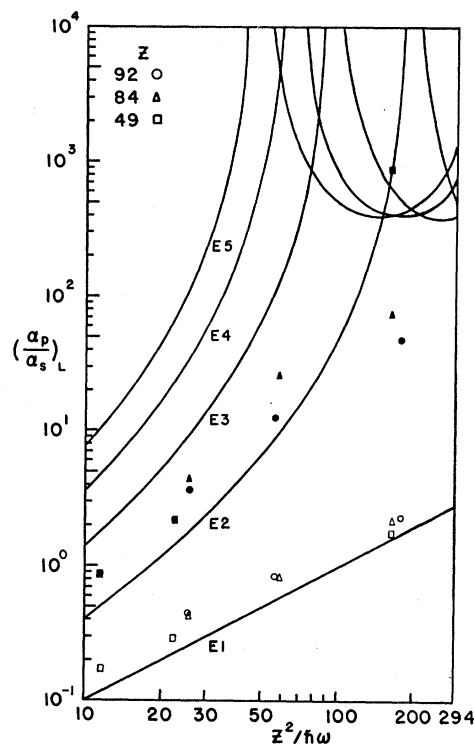


Fig. 1. Relative internal conversion of electric transitions in the  $L$  subshells. Here  $S$  represents the  $L_I$  subshell,  $P$  is  $L_{II}+L_{III}$ ,  $Z$  is the atomic number, and  $\hbar\omega$  the nuclear transition energy in kev. The solid lines are the results of the present nonrelativistic calculation. The individual points have been derived from the relativistic calculations of Gellman *et al.* for  $E1$  (open points) and  $E2$  radiations (solid points). Screening has been neglected in all cases.

The internal conversion of magnetic transitions has been calculated in the Pauli approximation under the same conditions as for electric radiations. The present calculations parallel those of Berestetskii,<sup>5</sup> who com-

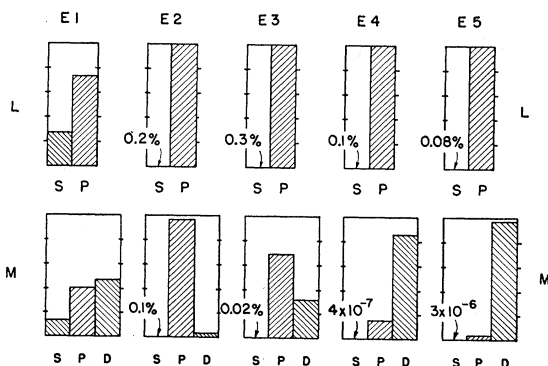


Fig. 2. Relative internal conversion of electric transitions in the  $L$  and  $M$  subshells at their respective thresholds in the nonrelativistic limit. As in Fig. 1,  $S$ ,  $P$ , and  $D$  refer to the orbital quantum numbers of the atomic subshells. The threshold values of  $Z^2/\hbar\omega$  are 294 and 662, respectively.

<sup>5</sup> V. B. Berestetskii, Zhur. Eksptl'. i Teoret. Fiz. **18**, 1070 (1948), (see NSA-6598, TT-255).

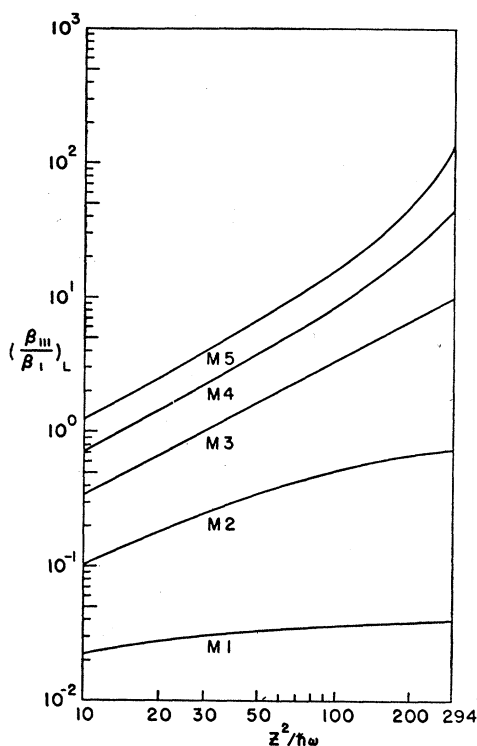


FIG. 3. Relative internal conversion of magnetic transitions in the  $L_{III}$  and  $L_I$  subshells in the Pauli approximation. The abscissa is that of Fig. 1.

puted absolute conversion coefficients for the  $K$  and  $L$  shells, and those of Tralli and Lowen,<sup>6</sup> who evaluated the relative  $L$ -conversion coefficients for  $Z^2/\hbar\omega < 35$ .<sup>7</sup> The present results for the  $L$  shell are given in Figs. 3 and 4. In contrast with electric transitions, there is now a strong distinction between  $L_{II}$  and  $L_{III}$  conversion.

As pointed out by Berestetskii<sup>5</sup> and Drell,<sup>8</sup> the radial matrix elements for magnetic transitions properly consist of both "volume" terms, similar to those appearing for electric transitions of the next highest multipole order, and "surface" terms, which vanish in the electric case. The magnitudes of the surface terms are found to be considerably greater than those of the volume terms, although they contribute only in consecutive odd subshells (I, III, V,  $\dots$ , etc.), where the total number of such subshells equals the multipole order of the transition considered. This effect is particularly noticeable for  $M1$  radiations, which are known to convert almost exclusively in the I subshells.

The relative internal conversion of  $M1$  transitions in the five  $M$  subshells is shown in Fig. 5. In this case the total volume contribution in any subshell is found to be less than 10 percent of the total conversion, in

<sup>6</sup> N. Tralli and I. S. Lowen, Phys. Rev. **76**, 1541 (1949); N. Tralli (private communication, 1953).

<sup>7</sup> Because of their inclusion of the effects of screening for  $Z=35$ , Tralli and Lowen's abscissa is 1.29 times that of Fig. 3.

<sup>8</sup> S. D. Drell, Phys. Rev. **75**, 132 (1949).

agreement with results for the  $L$  shell. In the latter case, however, the contribution of these terms is found to decrease rapidly with increasing multipole order. For simplicity, therefore, only the surface terms have been used in the calculation of the relative  $M$ -subshell conversion coefficients of higher magnetic multipoles. These results, along with the analogous but complete results for the  $L$  shell, are shown in Fig. 6.

#### MIXED TRANSITIONS

The transition of a nucleus from a state of spin  $I_i$  to one of spin  $I_f$  may proceed by means of transitions with multipole orders equal to any of the values  $|I_i - I_f| \dots I_i + I_f$ . Although the transition of the lowest multipolarity is usually strongly favored, evidence for low-energy  $M1 + E2$ ,  $M3 + E4$ , and possibly  $E1 + M2$  mixtures, have been detected in heavy elements. It is of interest, therefore, to determine the conversion properties of mixed transitions in terms of the pure transitions considered above.

In 1936 Casimir<sup>9</sup> showed that internal conversion is an incoherent process in the sense that

$$\gamma = \sum \gamma_i, \quad e_j = \sum \gamma_i \alpha_{ij},$$

where  $\gamma_i$  is the unconverted gamma-ray intensity of the  $i$ th type, and  $\alpha_{ij}$  its absolute conversion coefficient in the  $j$ th shell or subshell. For the important case of the

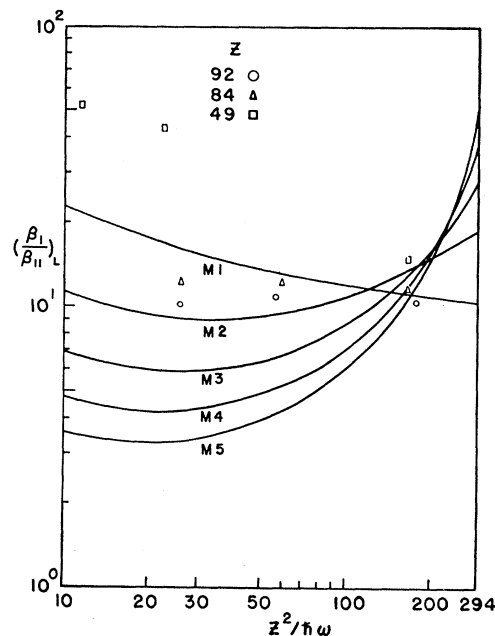


FIG. 4. Relative internal conversion of magnetic transitions in the  $L_I$  and  $L_{II}$  subshells as computed in the Pauli approximation. The individual points represent the results of the relativistic calculations of Gellman *et al.* for magnetic-dipole radiation. The remaining notation is that of Fig. 1.

<sup>9</sup> H. B. G. Casimir, *On the Interaction between Atomic Nuclei and Electrons* (De Erven F. Bohn, Haarlem, 1936).

mixing of two components, a mixing ratio,  $\delta^2$ , may be conveniently defined as the ratio of the corresponding *unconverted* gamma-ray intensities. This definition is advantageous since it is directly related to the corresponding ratio of nuclear matrix elements. The mixing ratio may then be expressed in terms of the observed conversion coefficient of the mixed transition and those of the corresponding pure components. One finds

$$\delta_{12}^2 \equiv \frac{\gamma_1}{\gamma_2} = \frac{\alpha_{0j} - \alpha_{2j}}{\alpha_{1j} - \alpha_{0j}},$$

where  $\alpha_{0j}$  is the observed conversion coefficient in the *j*th shell. Similarly,  $\delta^2$  may be expressed in terms of the relative conversion coefficients in any two shells or subshells, *j* and *k*, according to

$$\delta_{12}^2 = \frac{\alpha_{2k} R_0 - R_2}{\alpha_{1k} R_1 - R_0}, \quad \text{where } R_i \equiv \frac{\alpha_{ij}}{\alpha_{ik}}.$$

The agreement between the values of the mixing ratios derived from absolute and/or relative conversion-coefficient measurements offers a check on the consistent interpretation of mixed transitions. Because of their very different conversion properties, this technique is particularly sensitive in the analysis of *M1*+*E2* mixtures.

#### COMPARISON WITH EXISTING CALCULATIONS

The above results for relative *L*-conversion coefficients may be compared directly with those of the relativistic unscreened calculations of Gellman *et al.*<sup>2</sup> These authors have computed the absolute conversion coefficients of *E1*, *E2*, and *M1* radiations in the three *L* subshells for three atomic numbers and three transition energies. Their results are plotted as individual points in Figs. 1 and 4.

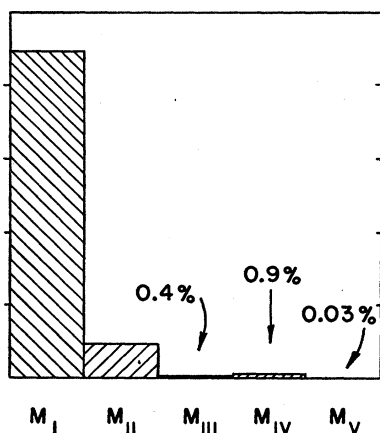


FIG. 5. Relative internal conversion of magnetic-dipole transitions in the *M* subshells. The indicated results have been computed in the Pauli approximation for vanishing electron momentum.

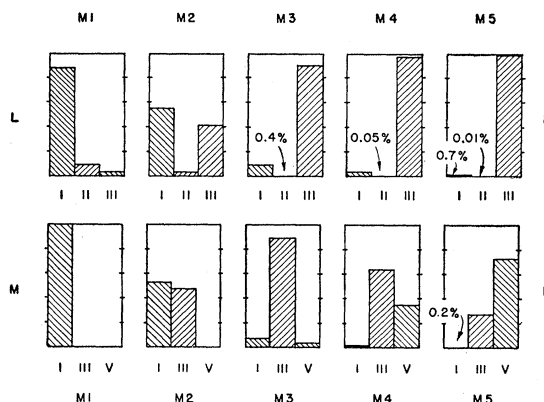


FIG. 6. Relative internal conversion of magnetic transitions in the *L* and *M* subshells in the Pauli approximation. Results are given at their respective nonrelativistic thresholds, in analogy with Fig. 2. The indicated results for the *M* shell include only the contributions of surface terms which vanish for even subshells. The error thereby introduced in any subshell is less than 10 percent of the total conversion.

For *E1* transitions the agreement between the two calculations is striking—the relative *P* to *S* conversion being very nearly proportional to the nonrelativistic estimate over the entire range of *Z* and energy considered. As expected, the relative conversion of *E2* transitions in the *P* subshells is considerably enhanced over dipole transitions, although breakdown of the  $Z^2/\hbar\omega$  dependence is clearly evident for higher values of atomic number. The strong *Z* dependence of the relative internal-conversion coefficients in the region of the nonrelativistic infinity is related to the inclusion of higher terms in the multipole expansion in the complete calculation. The relative conversion in the *L*<sub>III</sub> and *L*<sub>II</sub> subshells is found to be comparable for both *E1* and *E2* transitions, although distinctly less than the nonrelativistic value of 2. As might have been expected, this deviation is greatest for high-energy transitions in heavy elements.

In the case of *M1* transitions, the relative conversion in the *L*<sub>I</sub> and *L*<sub>II</sub> subshells as shown in Fig. 4 is again strongly *Z* dependent. However, reasonable agreement between the two calculations is obtained for high atomic numbers. On the other hand, the relative *L*<sub>III</sub> conversion predicted by the complete calculation is generally an order of magnitude less than the nonrelativistic estimate, especially for high *Z*. These values have not been included in Fig. 3. §

Before comparison can be made between the present results and experiment, or the forthcoming calculations of Rose *et al.*, the possible effects of screening must be estimated. These effects may be taken into account to first order by replacing *Z* by *Z*− $\sigma$ , where  $\sigma=4.2$  is Slater's screening constant for the *L* shell.<sup>10</sup> For heavy

§ Note added in proof.—Recent results of Rose for *Z*=85 indicate that Gellman's *L*<sub>III</sub> conversion coefficients for *M1* radiation may be too small by factors of 2–5 in this region.

<sup>10</sup> J. C. Slater, Phys. Rev. **36**, 57 (1930).

elements this distinction is negligible in the present degree of approximation. In the  $M$  shell, however, the effects of screening should be considerably more important, as indicated by the larger screening constants of 11 for the  $S$  and  $P$  subshells, and 20 for the  $D$  electrons. In general, this distinction might be expected to reduce the conversion in the  $M_V$  subshell relative to  $M_{III}$  and  $M_I$ , while leaving the latter relatively unchanged.

Rose *et al.*<sup>3</sup> are in the process of computing absolute  $L$ -shell conversion coefficients using screened relativistic wave functions. At present, however, reasonably complete data are available only for  $Z=85$ . For low multipole orders these results are in agreement with those of Gellman *et al.*, as discussed above. For higher electric transitions, the  $P/S$  conversion ratios behave very similarly to the nonrelativistic predictions, although the  $L_I$  conversion is generally greater than that indicated in Fig. 1. This is usually of little practical importance, however, because of the overwhelming conversion in the  $L_{II}$  and  $L_{III}$  subshells. The relative conversion in these two subshells is found to be comparable, as already noted for dipole and quadrupole transitions. The  $L_{II}$  subshell appears to assume greater importance with increasing multipole order and transition energy.

As noted for  $M1$  transitions, the exact  $L_I/L_{II}$  conversion ratios for  $Z=85$  are in agreement with the nonrelativistic estimates. This is also found to be the case for higher magnetic transitions over the region considered. As for dipole transitions, the nonrelativistic theory consistently overestimates the importance of the  $L_{III}$  subshell for higher transitions. However, this difference decreases with decreasing energy, until the threshold values are in quantitative agreement.

Drell<sup>11</sup> has considered the conversion properties of  $E0$  transitions (i.e.,  $0\pm \rightarrow 0\pm$ ). Such transitions are strictly forbidden for single gamma emission, although they can proceed directly by internal conversion. Their relative conversion properties should be very similar to  $M1$  transitions in that  $S$  electrons are strongly favored, leading to high  $K/L$  ratios and predominant  $L_I$  conversion. However, such transitions may be readily identified by their high (infinite) conversion coefficients and their long lifetimes. The analogous  $M0$  transitions (i.e.,  $0\pm \rightarrow 0\mp$ ) are strictly forbidden for all first-order processes, and do not lead to discrete conversion "lines."

#### COMPARISON WITH EXPERIMENTAL DATA<sup>12</sup>

Experimental data on relative conversion properties in the  $L$  subshells are usually available only for low-

<sup>11</sup> S. D. Drell, Oak Ridge National Laboratory Report ORNL-792 (unpublished).

<sup>12</sup> Those experimental data on  $Np^{237}$ ,  $Bi^{210}$ , and  $U^{234}$  listed without reference have been obtained with a  $180^\circ$  focusing beta-ray spectrograph having a permanent magnetic field of  $\sim 100$  gauss [E. L. Church (unpublished)].

energy transitions in heavy elements. In the rare-earth region these transitions fall within the range  $Z^2/\hbar\omega = 20-100$ , while for the very heavy elements, transitions with  $Z^2/\hbar\omega > 100$  are commonly observed. For the  $M$  subshells, data are generally only available in the latter region, where the corresponding conversion-electron lines may be experimentally resolved.

The relative  $L$ -conversion properties of electric-dipole transitions in the rare-earth region<sup>13</sup> are in good agreement with the values interpolated from the results of Gellman *et al.* The  $59.6 \pm 0.3$ -keV  $E1$  transition in  $Np^{237}$  exhibits comparable conversion in all three subshells, with  $L_{II}$  favored. This behavior is unique among the pure transitions, and is also in agreement with the previous speculations.

Mihelich *et al.*<sup>14</sup> have collected considerable experimental data on the  $L$ -conversion properties of various transitions, principally in the rare-earth region. In general,  $E2$  and  $E3$  transitions are found to convert almost exclusively in the  $L_{II}$  and  $L_{III}$  subshells, in agreement with the nonrelativistic predictions shown in Fig. 1. It is found that  $L_{II} \sim L_{III}$  for both types of transitions. Similar results of Swan and Hill<sup>15</sup> for  $E2$  and  $M1+E2$  transitions indicate quantitative agreement with the calculations of Gellman *et al.* Passell<sup>16</sup> has recently collected experimental data on the  $L$ -conversion properties of  $E2$  transitions in the very heavy element region, and has also found good agreement with the relativistic unscreened calculations.

The data of Mihelich indicate that  $M1$  transitions convert most strongly in the  $L_I$  subshell. This observation is in complete accord with the results of the previous calculations. It is interesting to note that Wu *et al.*<sup>17</sup> have found the  $L_I$  and  $L_{II}$  conversion properties of the  $46.7 \pm 0.3$ -keV magnetic-dipole transition in  $Bi^{210}$  to be in agreement with Gellman's results, while the relative conversion in the  $L_{III}$  subshell falls between the relativistic and nonrelativistic estimates.<sup>11</sup>

Data on  $M4$  transitions<sup>14</sup> indicate comparable conversion in the  $L_I$  and  $L_{III}$  subshells, with negligible  $L_{II}$  conversion, in accord with the results shown in Figs. 3 and 4. The observed  $L_{III}/L_I$  conversion ratios are in fair agreement with the nonrelativistic predictions, and are improved by the inclusion of the effects of screening. It should be pointed out that the conversion properties of electric and magnetic isomeric transitions are so vastly different that even crude estimates of their conversion in the various subshells can be used to uniquely determine their character.

<sup>13</sup> E. L. Church and M. Goldhaber (to be published); A. W. Sunyar, Phys. Rev. **90**, 387 (1953).

<sup>14</sup> J. W. Mihelich, Phys. Rev. **87**, 646 (1952); J. W. Mihelich and A. de-Shalit, Phys. Rev. **91**, 78 (1953); **93**, 135 (1954).

<sup>15</sup> J. B. Swan and R. D. Hill, Phys. Rev. **91**, 424 (1953).

<sup>16</sup> T. O. Passell, University of California Radiation Laboratory Report UCRL-2528 (unpublished).

<sup>17</sup> Wu, Boem, and Nagel, Phys. Rev. **91**, 319 (1953).

<sup>11</sup> Note added in proof.—Recent results of Rose for  $Z=85$  are in agreement with the observed  $L_{III}/L_I$  conversion ratio of this transition.

Presently available data on  $M$ -conversion properties are limited to qualitative observations on a few isolated cases. The 59.6-keV electric-dipole transition in  $\text{Np}^{237}$  exhibits detectable conversion in all five  $M$  subshells, with  $M_I$  and  $M_{II}$  predominating. This behavior is in qualitative agreement with the threshold results indicated in Fig. 2. Although a large reduction of the  $M_{IV,V}$  conversion is indicated, the results are still very different from those expected for other transitions. This transition is also observed to convert most strongly in the  $N_I$  and  $O_I$  subshells.

Electric-quadrupole transitions are observed to convert almost entirely in the  $M_{II}$  and  $M_{III}$  subshells. In the case of the  $43.6 \pm 0.3$ -keV transition in  $\text{U}^{234}$ , one observes  $M_{II} \sim M_{III}$ , with  $M_{IV,V}$  conversion being a negligible fraction of the total. This transition also converts most strongly in the  $N_{II}$ ,  $N_{III}$ , and  $O_{II,III}$  subshells. Mihelich<sup>18</sup> has reported that the  $L$ - and  $M$ -conversion properties of the 32-keV  $E3$  transition in  $\text{Au}^{198}$  are very similar to those expected for an  $E2$  transition, with, however, relatively more conversion observed in the  $M_{IV,V}$  subshells. This behavior is in agreement with the results shown in Fig. 2, which indicate the increasing importance of these subshells with increasing multipole order.

The 46.7-keV  $M1$  transition in  $\text{Bi}^{210}$  exhibits intense conversion in the  $L_I$ ,  $M_I$ ,  $N_I$ , and  $O_I$  subshells, consistent with the general properties of such transitions as previously discussed. The  $M$  subshell spectrum of this transition is in complete qualitative agreement with that shown in Fig. 5. Unfortunately, there are no reliable data on the conversion properties of higher magnetic transitions in the  $M$  and  $N$  subshells.

#### SUMMARY

It follows from the previous discussions that the general behavior of the relative conversion properties of nuclear transitions in the  $L$  subshells are predictable on an essentially nonrelativistic basis. These properties, as well as generalizations to higher shells, are summarized below with reference to low-energy transitions in heavy elements. It is unlikely that the effects of screening will significantly alter these semi-empirical conclusions.

Magnetic-dipole transitions convert most strongly in the  $L_I$  and  $M_I$  subshells, with similar behavior expected for higher shells. Electric-dipole transitions convert to a comparable extent in all three subshells, with  $L_I$  being

definitely favored for low values of  $Z^2/\hbar\omega (< 50)$ . Since they exhibit similarly high  $K/L$  conversion ratios,<sup>19</sup>  $M1$  and  $E1$  transitions may be difficult to distinguish solely on the basis of their relative conversion properties, especially in the rare-earth region.

All electric transitions higher than dipole are characterized by negligible conversion in the  $L_I$  subshell, and comparable conversion in the  $L_{II}$  and  $L_{III}$  subshells. Similar behavior is expected in the  $M$  and possibly higher shells. Since these transitions all exhibit low  $K/L$  ratios,<sup>1</sup> their multipole order may be most easily determined from their lifetimes. In principle, the  $L_{II}/L_{III}$  ratio or the relative importance of the  $M_{IV,V}$  conversion in the  $M$  shell could also be utilized for this purpose.

Higher magnetic transitions are characterized by negligible conversion in the  $L_{II}$  subshell. The ratio  $L_{III}/L_I$  increases with  $Z^2/\hbar\omega$  and with multipole order. The odd  $M$  subshells should be appreciably more intense than the even ones, with  $M_I$ ,  $M_{III}$ , and  $M_V$  dominating successively with increasing multipole order. Similar behavior is expected for higher atomic shells.

Quantitative failure of the simple nonrelativistic predictions arises both from relativistic effects and the neglect of higher terms in the radial expansion of the multipole field. Both lead to deviations from the simple  $Z^2/\hbar\omega$  dependence. In the case of electric transitions, the first directly affects the relative conversion in the relativistic doublets, while the latter gives rise to the observed  $Z$  dependence of the relative  $L_I$  conversion coefficient when the contribution of the first term vanishes. In the case of magnetic transitions, the behavior at high atomic numbers is apparently dominated by the surface terms, which in the Pauli approximation arise only from the leading term in the multipole field. However, deviations from this simple picture are also evident, especially in the observed depression of the  $L_{III}$  subshell for higher-energy transitions.

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<sup>18</sup> J. W. Mihelich (private communication, 1954); J. W. Mihelich, Phys. Rev. **93**, 135 (1954).

<sup>19</sup> The  $K/L$  conversion ratios of  $E1$  transitions may be estimated from the calculations of  $\alpha_K$  by Rose, Goertzel, Spinard, Harr, and Strong [Phys. Rev. **83**, 79 (1951)] or Reitz [Phys. Rev. **77**, 10 (1950)], and  $\alpha_L$  by Gellman, Griffith, and Stanley (see reference 2).