Since the major contribution to the sums comes from $L \sim \eta$, one sees that large η implies large L. Here the effects of quantization are small, and in this way the calculation becomes classical.

The particular case $\xi = 0$ is not of too much intrinsic importance, but it does serve to illustrate several typical features of the problem and the importance of more accurate calculations for the general case. Such calculations are in progress.

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Giant Resonance Interpretation of the Nucleon-Nucleus Interaction

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In the consideration of the independent-particle model, a distinction can be made between the spacing D of the levels of the whole nucleus and the spacing d of the levels of individual nucleons. Except in the immediate neighborhood of the normal state of closed-shell nuclei, $d \gg D$. In the "giant-resonance" interpretation considered here, the deviations from the independent-particle model are strong enough to mix many states of the whole nucleus, but the mixing is restricted to an energy range which is less than the order of d. According to this interpretation, the reduced particle widths of the levels of the compound nucleus are, on the average, anomalously large close to the energy values of those states of the independent-particle model which correspond to an unexcited target nucleus and a virtual level of the incident particle. As a consequence, the nuclear cross sections have a gross structure which is similar to a giant resonance, such as is implied by the complex square well representation of the nucleon-nucleus interaction. The position, width, and height of these maxima in the average cross sections are expressed in terms of the parameters of the independent-particle model and the departure of the actual nuclear potential which are responsible for the inaccuracy of this model. It is shown, however, that the conventional nuclear potential gives far too large values for the widths of the giant resonances (that is, for the imaginary part of the presentative complex square well potential).

I. INTRODUCTION

FESHBACH, Porter, and Weisskopf[1] have shown that a complex square well potential gives an accurate representation of some of the features of the neutron-nuclei interaction data at low and intermediate energies ($\lesssim 3$ Mev), such as the total cross section measurements by Barschall, Nereson, and collaborators and the angular distribution data of Walt and Barschall.[2] It has also been shown by them and by one of us[3] that one implication of such a representation is that the sum of the reduced neutron widths $\gamma_{\lambda n}^2$ per unit energy interval of the levels λ of the compound nucleus has a giant resonance-like dependence on the real energies E_λ of these levels. This sum plays a decisive part in the theoretical development and is referred to there as the strength function $s_n(E_\lambda) = \langle \gamma_{\lambda n}^2 \rangle_{\mathrm{Av}} / D$, where D is the mean spacing of the E_λ. The maxima of the giant resonances are associated with the positions E_p of the levels p of the real part of the representative potential, and their widths W_p are related to twice the imaginary part. It is presumed that the real part of the potential is essentially that potential which determines the configuration assignments in the shell-model theory, while the imaginary part is considered as representing the departures from this theory which are expected to be important at the higher excitation energies involved in scattering and reaction phenomena. Although it is beyond the scope of the complex potential representation to specify the properties of the individual resonance

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] Feshbach, Porter, and Weisskopf, Phys. Rev. **90**, 166 (1953); **96**, 448 (1954); R. K. Adair, Phys. Rev. **94**, 737 (1954). The first attempt to interpret the long-range fluctuations of the neutron cross sections by means of a simple potential is due to K. W. Ford and D. Bohm, Phys. Rev. **79**, 745 (1950). A similar model was used for the explanation of the high-energy cross sections by Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949). In fact, the early explanations of the large neutron cross sections by Amaldi, D'Agostino, Fermi, Pontecorvo, Rasetti, and Sègre [Proc. Roy. Soc. (London) **A149**, 522 (1935)], by H. A. Bethe [Phys. Rev. **47**, 747 (1935)], by G. Beck and L. H. Horsley [Phys. Rev. **47**, 510 (1935)] by F. Perrin and W. M. Elsasser [J. phys. radium **6**, 194 (1935)], were all based on a similar model. However, Feshbach, Porter, and Weisskopf were the first ones to recognize that the cross section obtained from the simple potential is not the actual cross section but only its average over many resonance levels, and they were the first ones who thoroughly explored the consequences of their model.

[2] H. H. Barschall, Phys. Rev. **86**, 431 (1952); Am. J. Phys. **22**, 517 (1954); N. Nereson and S. Darden, Phys. Rev. **89**, 775 (1953); **94**, 1678 (1954); Walt, Becker, Okazaki, and Fields, Phys. Rev. **89**, 1271 (1953); Okazaki, Darden, and Walton, Phys. Rev. **93**, 461 (1954); M. Walt and H. H. Barschall, Phys. Rev. **93**, 1062 (1954). See also the early work of Fields, Russell, Sachs, and Wattenberg, Phys. Rev. **71**, 508 (1947).

[3] R. G. Thomas, Phys. Rev. **97**, 224 (1955).

levels (that is, the fluctuations), it does give the averages over many such levels of the total cross section and the cross section for compound-nucleus formation, the latter being interpreted as the sum of the contributions to reaction, elastic and inelastic scattering processes which proceed through the intermediary of a compound nucleus. In the interpretation of Feshbach, Porter, and Weisskopf these averages are found to reflect the giant resonance nature of the strength function s_n.

In view of the repeated success of the complex potential representation,[4] it is natural to seek a deeper understanding of it, that is to determine what it is a substitution for; one would like to be able to relate the imaginary part of the potential to the internucleon interactions, the density of nucleons, etc., and to determine whether other representations exist which may even be more satisfactory. One would also like to know if the complex potential and other such representations involve any implications concerning the decay of the compound nucleus; that is, if it is possible to distinguish between the different processes, such as (n,n'), (n,p), (n,α) which comprise the cross section for compound-nucleus formation. These matters are investigated here by decomposing the wave functions for the levels of the compound nucleus into a sum of the products ψ_ν of the stationary-state wave functions of the residual nucleus and the wave functions for a single neutron (or proton) moving in an appropriate average *real* potential of the residual nucleus.

The model which forms the basis of the following giant resonance interpretation may be considered as intermediate between the independent-particle model and the uniform model. In order to explain its character, it is necessary to define, within the independent-particle model, two types of level spacings: the spacing D of the levels of the whole nucleus and the spacing $d(\gg D)$ of the individual particle levels. Although the former is at least approximately the same order of magnitude as the actual level spacing, i.e., less than a few thousand volts whenever one can speak of average cross sections, the latter is greater than several Mev even in the heaviest elements. The independent-particle model is accurate whenever the matrix elements of the Hamiltonian which connect the various combined states ψ_ν are small compared with D. In this case the real, characteristic functions X_λ of the Hamiltonian contain essentially only one ψ_ν, the admixture of adjacent ψ_ν being small. In the opposite case of the uniform model, the X_λ will contain many ψ_ν with roughly equal coefficients, including those ψ_ν which differ in the excitation of a single nucleon, the independent-particle energy of which differ by d or more. With such complete mixing of the ψ_ν, one will find hardly any trace left of the independent-particle aspect of the nuclear behavior, and this is the state of affairs which is usually considered as implied by "Bohr's picture of the nucleus."[5] For the interpretation of these data we will therefore consider a model which is intermediate between the independent-particle and uniform models.[6] In this intermediate model the spread W in energies E_λ of those X_λ containing an appreciable amount of a particular ψ_ν will satisfy the condition $D \ll W \ll d$, and the spread will be of the general form of a giant resonance.

II. DECOMPOSITION OF THE WAVE FUNCTIONS FOR THE COMPOUND NUCLEUS

The wave functions X_λ of the compound nucleus satisfy the wave equation

$$HX_\lambda = E_\lambda X_\lambda, \tag{1}$$

when certain fixed boundary conditions are imposed at the nuclear surface.[7,8] These wave functions are decomposed in the form[6,7]

$$X_\lambda = \sum_{cp} C_{\lambda;cp} \psi_c(\mathbf{r}_1, \mathbf{r}_2, \cdots \mathbf{r}_A) u_p(\mathbf{r}_n), \tag{2}$$

where the ψ_c are the wave functions of the stationary states c of the residual nucleus of A particles. The $u_p(\mathbf{r}_n)$ are the wave functions of the extra neutron n moving in the average potential of the residual nucleus. They are of the form

$$u_p(\mathbf{r}_n) = Y_p(\Omega_n) R_p(r_n), \tag{2a}$$

where $Y_p(\Omega_n)$ are spherical harmonics, and the $R_p(r_n)$ are radial functions which satisfy the boundary conditions imposed on the X_λ. The subscript p designates one of the single-particle states of the average potential, such as $1s$, $2s$, $1p$, etc. Since the $\psi_c u_p$, as well as the X_λ, form a complete set of *ortho*-normal functions, the expansion coefficients $C_{\lambda;cp}$ are the components of an orthogonal matrix with the usual properties,

$$\sum_{cp} C_{\lambda;cp} C_{\lambda';cp} = \delta_{\lambda\lambda'}, \tag{3a}$$

$$\sum_\lambda C_{\lambda;cp} C_{\lambda;c'p'} = \delta_{cc'} \delta_{pp'}. \tag{3b}$$

In order to obtain the partial widths with respect to the emission of protons and heavier particles, other

[4] Ostrofsky, Breit, and Johnson, Phys. Rev. **49**, 22 (1936); H. A. Bethe, Phys. Rev. **57**, 1125 (1940); M. Goldhaber and E. Teller, Phys. Rev. **74**, 1046 (1948); H. Steinwedel and J. H. D. Jensen, Z. Naturforsch. **52**, 413 (1950); B. Freeman and J. McHale, Phys. Rev. **89**, 223 (1953); N. C. Francis and K. M. Watson, Am. J. Phys. **21**, 659 (1953).

[5] N. Bohr, Nature **137**, 344 (1936); Science **86**, 161 (1937); E. Wigner and G. Breit, Phys. Rev. **49**, 642 (1936), Phys. Rev. **49**, 519 (1936). These last two articles, which were simultaneous with the first one, were often misunderstood to propose a somewhat different model—actually the model which underlies the present article.

[6] A brief account of some aspects of the present work was given at the New York Meeting of the National Academy of Sciences, Nov. 9, 1954 (unpublished); E. P. Wigner, Science **120**, 790 (1954). See also J. M. C. Scott, Phil. Mag. **45**, 1322 (1954), who proposes the same picture.

[7] T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952). The development in this section is largely a repetition of the arguments given in connection with Eq. (31) of this reference.

[8] J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), Chap. VIII and X; R. G. Sachs, *Nuclear Theory* (Addison-Wesley Press, Cambridge, 1953), pp. 290–304.

decompositions of the X_λ must be used instead of (2) in which the coordinates of the extra proton (or heavier particle) play the role of $\mathbf{r}_n$.

It is well to note that (2) is not antisymmetrized, i.e., that it neglects exchange. Inclusion of exchange effects, that is antisymmetrization of (2), would render all of the calculations more cumbersome and would introduce correction terms into almost all of the formulas. However, it is believed that the essence of the proposed interpretation would not be affected by these terms.

The functions $\psi_c u_p$ satisfy the equation

$$H_0\psi_c u_p = E_{cp}\psi_c u_p, \tag{4}$$

where H_0 contains the whole Hamiltonian of the residual nucleus, the kinetic energy of the extra neutron and its average potential $\bar{V}(\mathbf{r}_n)$ in the field of the residual nucleus. The difference $V=H-H_0$ is the departure of the actual potential, which is experienced by the extra neutron, from the average potential

$$V=\sum_{i=1}^{A}\mathcal{V}(\mathbf{r}_i,\mathbf{r}_n)-\bar{V}(\mathbf{r}_n). \tag{5}$$

The quantities $\mathcal{V}(\mathbf{r}_i,\mathbf{r}_n)$ are the potentials which act between the neutron n and the ith particle of the residual nucleus, and the average potential is

$$\bar{V}(\mathbf{r}_n)=\sum_{i=1}^{A}\int\psi_c^*\mathcal{V}(\mathbf{r}_i,\mathbf{r}_n)\psi_c d\tau, \tag{5a}$$

where the integration is over the coordinates τ of the residual nucleus. It will be evident from the estimate of Sec. V that this potential is probably not very dependent on c, the state of excitation of the residual nucleus. However, should it be so dependent, the ψ_c in (5a) shall be considered as the normal state ψ_0. It follows from (5), (5a), and the normalization of ψ_c that

$$V_{cp;\,cp}\equiv(\psi_c u_p, V\psi_c u_p)=0. \tag{6}$$

Thus, the expectation value of $H-H_0$ vanishes for the $\psi_c u_p$ [p arbitrary, $c=0$ unless (5a) is independent of c].

In the theory of nuclear reactions the reduced widths for the states X_λ with respect to disintegrations into a nucleon and the states c (channels) of the residual nucleus play an important role. They are denoted by $\gamma_{\lambda c}^2$, where[9]

$$\gamma_{\lambda c}=(a\hbar^2/2M)^{\frac{1}{2}}\int\psi_c^*Y_p(\Omega_n)^*X_\lambda d\tau d\Omega_n. \tag{7}$$

Here the radial coordinate of the neutron, which is a variable of X_λ, is to be set equal to the radius a of the boundary of the internal region, so that (7) is actually a surface integral over this boundary. According to (7) and (2), the reduced-width amplitudes may be expressed as

$$\gamma_{\lambda c}=\sum\nolimits_p \gamma_{\lambda;\,cp}, \tag{7a}$$

where

$$\gamma_{\lambda;\,cp}=(a\hbar^2/2M)^{\frac{1}{2}}C_{\lambda;\,cp}R_p(a). \tag{7b}$$

The reduced widths appear then as a double sum over the single-particle levels p. In order to simplify matters, either one or both of two assumptions can be made: (a) The signs of the $C_{\lambda;\,cp}$ referring to different p are considered as likely to be positive as negative so that the cross terms can be disregarded in the consideration of the most probable value,[7]

$$\gamma_{\lambda c}^2\approx\sum\nolimits_p \gamma_{\lambda;\,cp}^2. \tag{8a}$$

(b) It may be assumed that, for each λ, one of the terms in (7a) is much larger than the others, in which case the sum in (7a) can be replaced by a single term, and therefore

$$\gamma_{\lambda c}^2\approx\gamma_{\lambda;\,cp}^2. \tag{8b}$$

Assumption (b) will be valid if the coefficients $C_{\lambda;\,cp}$ of (2) are small when the energy E_λ of the compound state X_λ differs by more than a certain amount W from the energy E_{cp} of the independent-particle state $\psi_c u_p$, provided that W is small compared with the spacing d of the individual-particle levels. At the same time, W can be, and usually will be, large compared with the spacing D of the levels E_λ of the compound nucleus, or the spacing of the levels of the residual nucleus. Hence, for given c and λ, only one p will give an appreciable $C_{\lambda;\,cp}$ but, conversely, for a given c and p, many levels λ will have a substantial $C_{\lambda;\,cp}$. (Similarly, for given λ and p, many c will have appreciable $C_{\lambda;\,cp}$.) The states X_λ in the expansion (2) of which, for a given c, the same p have appreciable $C_{\lambda;\,cp}$ will be said to form the p-group of states with respect to the channel c. The energies E_λ of these states will be within an interval the width of which is of the order W. Assumption (b) and hence Eq. (8b) are therefore appropriate for the intermediate-coupling case where the widths of the giant resonances are much less than the spacings between the single-particle levels, but larger than the spacings D of the E_λ.

It is convenient to introduce the single-particle reduced widths,

$$\zeta_p^2=(a\hbar^2/2M)R_p(a)^2, \tag{9}$$

which are expected to be of the order of magnitude[7] of $\hbar^2/Ma^2$. The ζ_p^2 are the reduced widths which appear in the R-function expansion for the average single-particle potential (5a), and the E_p of (4) are the corresponding level positions. Sum rules for the reduced-width amplitudes, similar to the orthogonality relations (3), may be expressed in terms of the ζ_p^2:

$$\sum\nolimits_{cp}\gamma_{\lambda;\,cp}\gamma_{\lambda';\,cp}\approx\langle\zeta_p^2\rangle_{\text{Av}}\delta_{\lambda\lambda'}, \tag{10a}$$

$$\sum\nolimits_{\lambda}\gamma_{\lambda;\,cp}\gamma_{\lambda;\,c'p'}=\zeta_p^2\delta_{cc'}\delta_{pp'}. \tag{10b}$$

[9] The reduced widths used here have the dimensions of energy, as in reference 3 and in Chap. VIII of Blatt and Weisskopf (reference 8), rather than energy-times-distance, as in reference 7 and in Sachs (reference 8). They differ from the latter by a factor a. Note the factor $a^{\frac{1}{2}}$ in (7).

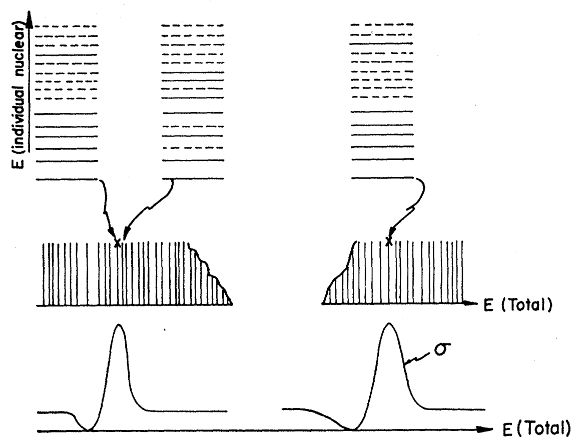

FIG. 1. The extreme independent-particle picture. The top of the figure represents the energy levels of the individual nucleons; the full lines are occupied levels, the broken lines unoccupied. The energy levels of the compound nucleus (middle of the figure) which are marked with a cross correspond to the target nucleus in the normal state, and an incident particle. Of all of the levels of the compound nucleus, only these affect the cross section (bottom part of the figure).

In the approximation (8a), the first of these with $\lambda=\lambda'$ corresponds to the familiar channel sum rule, while the second is related to the general level sum rule, Eq. (23a) of reference 7. The latter rule follows by summing over all p, p', with the result that the left side gives $\sum_\lambda \gamma_{\lambda c}\gamma_{\lambda c'}$ while the right side vanishes if $c\neq c'$ and is infinite if $c=c'$. We shall be particularly concerned with (10b) when $p=p'$, $c=c'$,

$$\sum_\lambda \gamma_{\lambda;\,cp}{}^2=\zeta_p{}^2, \tag{10b'}$$

a relation which shows that the sum of the reduced widths for a particular p-group is equal to the single-particle reduced width for that group.[10]

Before developing the more quantitative implications of the decomposition (2), some of the qualitative ones will be described and the reasons for considering it in the first place will be made apparent. In the extreme form of the independent-particle model, the level λ_0 of the compound nucleus, which is formed as a result of the interaction of the neutron with the residual nucleus in its normal state, would have only one nonvanishing cp contribution in (2), namely $0p$, associated with a particular single-particle state p. There is therefore only one nonvanishing reduced width $\gamma_{\lambda_0 0}{}^2$ for that state, whose magnitude is approximately the single-particle reduced width $\hbar^2/Ma^2$. Only elastic scattering can occur, and its expected behavior for such an extreme picture is as illustrated in Fig. 1.[11] There may be many adjacent levels in the spectrum of the compound nucleus, but these are characterized, in a similar manner, as being combined states of the extra neutron and excited states of the residual nucleus. The singular dependence of the $\gamma_{\lambda 0}{}^2$ on the E_λ is shown in Fig. 2 as the independent-particle model extreme.

When the departure potential V of (5) is introduced, a mixing of the pure independent-particle model states may be expected to occur. The reduced width $\gamma_{\lambda_0 0}{}^2$ will be diminished from its original value, and the state X_{λ_0} will acquire $\psi_c u_p$ terms associated with excited states of the residual nucleus, while the neighboring X_λ will acquire nonvanishing components from the normal state. In view of the sum rule (10b′), the $\gamma_{\lambda_0 0}{}^2$ magnitude of the original X_{λ_0} will be shared among the adjacent states resulting in a giant resonance of the $\gamma_{\lambda 0}{}^2$, as illustrated in Fig. 2 for the intermediate coupling model.[12] Inelastic scattering will occur, and elastic scattering will exhibit resonances at all the states of the compound nucleus, although these will be strongest in the vicinity of the original independent-particle model level. It may be emphasized here that in this model it is the $\gamma_{\lambda 0}{}^2$, rather than the D^{-1}, which is anomalously large at the Barschall maxima.

If the departure potential is allowed to increase, the width W of the spectrum of levels of the compound nucleus over which the original $\gamma_{\lambda_0 0}{}^2$ is appreciably shared will also increase. Ultimately, this width may become as large as the spacing d of the single-particle levels. The uniform model of Fig. 2 is approached, and the giant resonances of $\gamma_{\lambda 0}{}^2$ and of the average cross

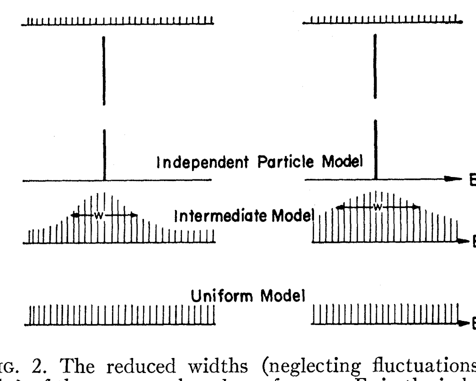

FIG. 2. The reduced widths (neglecting fluctuations) of the levels λ of the compound nucleus of energy E_λ in the independent-particle picture, intermediate coupling, and uniform models. The reduced widths of the levels are represented by the heights of the corresponding lines. In the independent-particle model the reduced widths of all levels vanish, with the exception of those marked with a cross in Fig. 1. In the uniform model, all reduced widths are roughly equal, apart from irregular fluctuations which are not indicated. In the intermediate case, the average widths of the levels are different in different parts of the spectrum: those close to the position of the very broad levels of the independent-particle model are broader than those situated further from these levels. W is the width of the energy spectrum over which the original single-particle reduced width is appreciably shared.

[10] This sum rule has also been noted by A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, 16 (1953). These authors attribute the departure from the independent-particle model to the interaction of the neutron with the surface of the nucleus, an assumption which will not be made here.

[11] These facts were recognized by L. Eisenbud in his doctoral dissertation (Princeton, 1948). See also D. H. Wilkinson and A. M. Lane, Phys. Rev. **97**, 1199 (1955).

[12] The figure does not show the irregular fluctuations of the reduced widths which are expected to be important, as indicated in reference 7 in the section quoted there. See Seidl, Hughes, Palevsky, Levin, Kato, and Sjöstrand, Phys. Rev. **95**, 476 (1954).

section will disappear. This extreme model was generally regarded as the correct one before the measurements of Barschall and collaborators[2] were made. It may also be mentioned that in this extreme the ratio of the reduced widths to the mean spacing D of the levels of the compound nucleus will obey the second sum rule of reference 7. Thus, by replacing the sum in (10b′) over the levels λ of the group p of (8b) by an integration which is extended over the entire width d of the group, and by using the estimate $d\approx(\pi\hbar^2K/Ma)$ for the single-particle level spacing, where K is the wave number characteristic of the neutron motion in the nucleus, one obtains $\langle\gamma_{\lambda 0}^2\rangle_{\mathrm{Av}}/D\approx 1/\pi Ka$.

It was pointed out before that in our model the maxima of γ_λ^2/D are due to anomalously large γ_λ^2 rather than anomalously small D. Another consequence of our model which we wish to point out concerns an expected difference in the behavior of nuclei with $J=0$ on the one hand, and nuclei with $J\neq 0$ on the other. In the former case, there will be, for every u_p, only one λ_0 level. Thus the giant maximum for s neutrons will be simple. Since a p neutron can have two states, the $p_{\frac{1}{2}}$ and $p_{\frac{3}{2}}$ state, the giant maximum for the p neutrons will be double and the same applies for d, f, etc. neutrons.

If the J of the target nucleus is not zero, even an s neutron will give two λ_0 levels; their J values will differ by $\pm\frac{1}{2}$ from the J value of the target nucleus. These states cannot be expected to coincide completely, even in the independent-particle picture. If their distance is smaller than the width W of the giant resonance, the maximum in the average cross section will appear to be broadened. If their distance is larger than W, one may expect two maxima in the average cross section for s neutrons. Similar remarks apply for neutrons with higher angular momenta.

III. STRENGTH FUNCTION

As mentioned in the Introduction, in the intermediate coupling case the strength function $s_c(E_\lambda)=\langle\gamma_{\lambda c}^2\rangle_{\mathrm{Av}}/D$ is expected to manifest a giant resonance behavior. Moreover, it is the strength function $s_0(E_\lambda)$ for the normal state which plays a decisive role in the determination of the behavior of the average total cross section and the cross section for compound-nucleus formation of an incident beam of neutrons. These cross sections are related to the average of the diagonal component of the collision matrix referring to the normal state, and this average depends[3] upon the Stieltjes transform of $s_0(E_\lambda)$,

$$\int s_0(E_\lambda)dE_\lambda/(E_\lambda+\Delta-E-\tfrac{1}{2}i\Gamma), \tag{11}$$

where Γ and Δ are the mean absorption width and shift, respectively. The strength function (and its transform) are therefore appropriate functions for representing the nucleon-nucleus interaction.

In the approximations leading to (8), the strength functions can be expressed as a sum of p-group strength functions

$$s_c(E_\lambda)=\textstyle\sum_p s_{cp}(E_\lambda), \tag{12}$$

and the normalization of the individual s_{cp} is obtained from (10b′) when the sum there is replaced by an integral,

$$\int s_{cp}(E_\lambda)dE_\lambda=\zeta_p^2. \tag{12a}$$

In the following sections we shall attempt to learn something about the dependence of the s_c on E_λ by an investigation of their energy moments with respect to the single-particle level positions.

It has been shown that the p-group strength function for the complex square well representation has the energy dependence[3] (omitting the subscript c)

$$s_p(E_\lambda)=(2\pi)^{-1}\zeta_p^2W/[(E_p-E_\lambda)^2+\tfrac{1}{4}W^2]. \tag{13}$$

When there is no absorption, that is, $\Gamma=0$ in (11), this becomes the reciprocal logarithmic derivative at the radius of the wave function for a complex square well with imaginary part $-\frac{1}{2}W$; when there is absorption, the imaginary part is $-\frac{1}{2}(\Gamma+W)$. It is evident that the second and higher even moments of this strength function are infinite. More generally, it can be shown that the same moments of the group strength functions for any representative complex potential are infinite. Thus, one finds by the methods of the R-function theory that the reciprocal logarithmic derivatives for such complex potentials have expansions of the form

$$\textstyle\sum_p \zeta_p^2/(E_p-E-\tfrac{1}{2}iW_p). \tag{14}$$

The reduced widths $\zeta_p^2=\zeta_{p1}^2+i\zeta_{p2}^2$ are in general complex, and the imaginary parts $-\frac{1}{2}W_p$ of the denominators are equal to the averages of the imaginary part of the complex potential, weighted according to the absolute square of the complex eigenfunctions of the potential. The p-group strength functions corresponding to (14) are $1/\pi$-times the imaginary parts of the contributing terms:

$$s_p(E_\lambda)=\frac{1}{\pi}\cdot\frac{\zeta_{p2}^2(E_p-E_\lambda)+\frac{1}{2}W_p\zeta_{p1}^2}{(E_p-E_\lambda)^2+\frac{1}{4}W_p^2}. \tag{14a}$$

Evidently these functions have in addition the property of becoming negative over part of the range of E_λ values unless the imaginary parts ζ_{p2}^2 should happen to vanish, as in the special case of the complex square well. A test of the validity of representative complex potentials would be a direct evaluation of the second moment of the $s_{0p}(E_\lambda)$ to see whether or not they are infinite.

IV. MOMENTS OF THE C COEFFICIENTS AND OF THE REDUCED WIDTHS

It is possible to obtain directly the formal expressions for the energy moments of the squares of the $C_{\lambda;cp}$ and of the reduced widths with respect to the single-particle level positions E_{cp}. From the wave equation (1) it follows that

$$(H-E_{cp})^{\nu}X_{\lambda}=(E_{\lambda}-E_{cp})^{\nu}X_{\lambda} \tag{15}$$

for positive integers ν. By substituting the expansion (2) into (15) and forming the scalar product of the resulting expression with the $\psi_c u_p$ corresponding to E_{cp}, one obtains

$$\sum_{c'p'} C_{\lambda;c'p'}(\psi_c u_p,\ (H-E_{cp})^{\nu}\psi_{c'}u_{p'}) = (E_{\lambda}-E_{cp})^{\nu}C_{\lambda;cp}. \tag{16}$$

Both sides of this expression are multiplied by $C_{\lambda;cp}$ and then summed over all of the levels λ, whereupon one finds that the moments are

$$M_{\nu}\equiv\sum_{\lambda}(E_{\lambda}-E_{cp})^{\nu}C_{\lambda;cp}^{2}=(\psi_c u_p,\ (H-E_{cp})^{\nu}\psi_c u_p), \tag{17}$$

the reduction of the left side of (16) having been effected by means of (3b).

The expressions for the moments of the reduced widths are somewhat more complicated because of the additional p-sum which appears in the relation (7a). However, if the p-group approximation (8a) or (8b) is valid, then it is of interest to consider the moments $\mathfrak{M}_{\nu}$ of the $\gamma_{\lambda;cp}^{2}$ of (8):

$$\mathfrak{M}_{\nu}\equiv\sum_{\lambda}(E_{\lambda}-E_{cp})^{\nu}\gamma_{\lambda;cp}^{2}=\zeta_p^{2}M_{\nu}. \tag{18}$$

By replacing the sum over levels in (18) by an integral, the moments of the strength functions are obtained,

$$\int(E_{\lambda}-E_{cp})^{\nu}s_{cp}(E_{\lambda})dE_{\lambda}\approx\zeta_p^{2}M_{\nu}. \tag{18a}$$

In view of the choice (5a) of the independent-particle potential, M_1 vanishes, indicating that the center-of-mass of the p-groups coincides with the E_{cp}, as in Fig. 2. In order to simplify the expression for M_2, one uses the Hermitean property of $(H-E_{cp})$ and Eq. (4):

$$\begin{aligned}M_2&=\sum_{\lambda}(E_{\lambda}-E_{cp})^{2}C_{\lambda;cp}^{2}\\&=((H_0-E_{cp}+V)\psi_c u_p,\ (H_0-E_{cp}+V)\psi_c u_p)\\&=(\psi_c u_p,\ V^{2}\psi_c u_p)=(V^{2})_{cp;cp}.\end{aligned} \tag{17a}$$

The second moment, which should provide a measure of the square of the widths W of the giant resonances of Fig. 2, is thus proportional to the mean of the square of the departure potential, as one would expect.

V. CALCULATION OF THE AVERAGE POTENTIAL AND THE SECOND MOMENT OF THE C COEFFICIENTS

In order to determine the E_{cp} and the $\mathfrak{M}_{\nu}$, it is necessary to construct the average potential. Unfortunately, our knowledge of nuclear forces remains inadequate in spite of much excellent work on the subject, and we shall therefore simply adopt as the potential $\mathcal{V}(\mathbf{r}_i,\mathbf{r}_n)$ between two nucleons the expression proposed by Yukawa, with half exchange, half non-exchange character,

$$\mathcal{V}(\mathbf{r}_i,\mathbf{r}_n)=\tfrac{1}{2}(1+P_{in})Ce^{2}|\mathbf{r}_i-\mathbf{r}_n|^{-1}\times\exp(-\kappa|\mathbf{r}_i-\mathbf{r}_n|)+T, \tag{19}$$

the constants being given by Feshbach and Schwinger and by Hall and Powell,[13] namely, $C\approx-40$, $\kappa=2.5\ mc^{2}/e^{2}$. The tensor force T will be neglected because its effect on $\bar{V}$ is small, and the second moment will turn out to be too large anyway. For the same reason the part of (19) containing the Majorana exchange operator P_{in} will also be neglected. These approximations effectively reduce (19) to a central force which is half as strong as that involved at low energies in the neutron-proton system.

The average potential $\bar{V}$ of (5a) is then given by the average density $\rho(\mathbf{r})$ of the nucleons in the residual nucleus,

$$\bar{V}(\mathbf{r}_n)=A\int\rho(\mathbf{r})\mathcal{V}(\mathbf{r},\mathbf{r}_n)d\mathbf{r}, \tag{20}$$

where A is the mass number of the residual nucleus. $\bar{V}(\mathbf{r}_n)$ evidently satisfies the differential equation

$$\nabla^{2}\bar{V}(\mathbf{r}_n)-\kappa^{2}\bar{V}(\mathbf{r}_n)=-2\pi ACe^{2}\rho(\mathbf{r}_n). \tag{21}$$

By assuming a constant nuclear density $\rho=[(4/3)\pi a^{3}]^{-1}$ inside a spherical nucleus of radius $a=0.45A^{1/3}e^{2}/mc^{2}$ and zero density outside, one can readily integrate (21), obtaining

$$\begin{aligned}\bar{V}(\mathbf{r}_n)/B&=1-(\kappa r)^{-1}(1+\kappa a)\sinh\kappa r\exp(-\kappa a), && r<a,\\&=(\kappa a\cosh\kappa a-\sinh\kappa a)(\kappa r)^{-1}\exp(-\kappa r), && r>a,\end{aligned} \tag{22}$$

where the constant $B=3ACe^{2}/2\kappa^{2}a^{3}=54$ Mev. This potential is plotted in Fig. 3 as a function of (r/a) for $A=30$ ($a=3.9\times10^{-13}$ cm) and for $A=150$ ($a=6.7$).

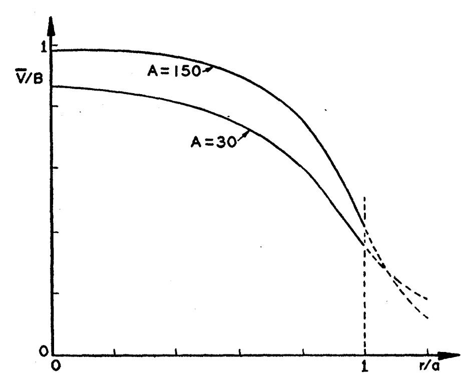

FIG. 3. The average potential $\bar{V}$ of (5a) for the extra nucleon is plotted as a function of the distance r from the center of the nucleus in units of $B=3ACe^{2}/2\kappa^{2}a^{3}\approx54$ Mev, where $a=0.45A^{1/3}e^{2}/mc^{2}$ is the nuclear radius, $\kappa^{-1}=0.4e^{2}/mc^{2}$ is the range of the inter-nucleon force [Eq. (19)], $C\approx-40$ is the corresponding strength, and A is the mass number.

[13] H. Feshbach and J. Schwinger, Phys. Rev. **84**, 194 (1951); H. H. Hall and J. L. Powell, Phys. Rev. **90**, 912 (1953).

The averages of this potential are $\frac{2}{3}B$ and $\frac{3}{4}B$ for $A=30$ and 150, respectively, which are somewhat less than the equivalent 55 Mev necessary to fit the neutron interaction data and the 46 Mev necessary to explain the observed stable single-particle levels, using in each case the same nuclear radius as assumed above. The disagreements are perhaps not serious, even considering that a rather substantial amount of electrostatic potential must be subtracted, because there is considerable uncertainty in the potential (19), particularly its exchange character. The rather strong tail of the potential (22) extending beyond the nuclear radius may be noted; this tail may be helpful for explaining the anomalously small cross section for backward scattering of high-energy protons[14] and the rather large absorption of high-energy neutrons.[15]

The second moment for the normal state is

$$M_2=\int|\psi_0|^2|u_p|^2 \times\{\sum_{ij}\mathcal{V}(\mathbf{r}_i,\mathbf{r}_n)\mathcal{V}(\mathbf{r}_j,\mathbf{r}_n)-[\bar{V}(\mathbf{r}_n)]^2\}d\tau d\mathbf{r}_n. \quad (23)$$

The A terms in the double sum with $i=j$ may be written as

$$A\int\rho(\mathbf{r})|u_p(\mathbf{r}_n)|^2[\mathcal{V}(\mathbf{r},\mathbf{r}_n)]^2d\mathbf{r}d\mathbf{r}_n. \quad (24a)$$

When the integration is carried out over all coordinates of the residual nucleus with the exception of the ith and jth, the remaining $A(A-1)$ terms become

$$A(A-1)\int\rho(\mathbf{r},\mathbf{r}')|u_p(\mathbf{r}_n)|^2\mathcal{V}(\mathbf{r},\mathbf{r}_n) \times\mathcal{V}(\mathbf{r}',\mathbf{r}_n)d\mathbf{r}d\mathbf{r}'d\mathbf{r}_n, \quad (24b)$$

where $\rho(\mathbf{r},\mathbf{r}')$ is the probability for the simultaneous location of two nucleons at $\mathbf{r}$ and $\mathbf{r}'$, respectively. This probability may be approximated by the expression which is valid for an assembly of free fermions,

$$A(A-1)\rho(\mathbf{r},\mathbf{r}')=A^2[\rho(\mathbf{r})\rho(\mathbf{r}') -(\rho/A)J(|\mathbf{r}-\mathbf{r}'|)], \quad (25)$$

where the quantity J represents the correlations caused by the antisymmetric nature of ψ_0. The integration over $\mathbf{r}'$ involving J is unity, corresponding to a "hole" of unit volume in the distribution of the other nucleons around a given nucleon. Therefore, the $\rho(\mathbf{r},\mathbf{r}')$ of (25) is properly normalized, apart from surface corrections. The explicit form of J, which is calculated in the theory of free electrons,[16] is not needed here.

By inserting (25) into (24b) and then (24b) and (24a) into (23), one finds that the first term of (25), just cancels the $[\bar{V}(\mathbf{r}_n)]^2$ of (23). Assuming again a constant $\rho=[(4/3)\pi a^3]^{-1}$ inside, zero outside, of the nuclear sphere, one thereby obtains

$$M_2=A\int\rho|u_p(\mathbf{r}_n)|^2d\mathbf{r}_n\int d\mathbf{r}\mathcal{V}(\mathbf{r},\mathbf{r}_n) \times\int d\mathbf{r}'[\delta(\mathbf{r},\mathbf{r}')-J(|\mathbf{r}-\mathbf{r}'|)]\mathcal{V}(\mathbf{r}',\mathbf{r}_n). \quad (26)$$

The integral in (26) with respect to $\mathbf{r}$ and $\mathbf{r}'$ is the product of the potentials of two coincident nucleons ($\delta(\mathbf{r},\mathbf{r}')$) diminished by the product of the potentials of two nucleons which are correlated by the function J. The integral over $\mathbf{r}'$ is very small if the range of J is very small compared with the range of $\mathcal{V}$; in this event, J could be approximated in (26) by a δ function and would cancel the first term. On the other hand, should the range of J be large compared with the range of $\mathcal{V}$, the second term in (26) would be much smaller than the first.

Although the integral (26) could be evaluated directly, the following indirect but simpler method is used. Instead of restricting the integration over $\mathbf{r}'$ to the inside of the residual nucleus, it is extended over all space, thus neglecting surface corrections. The integration with respect to $\mathbf{r}'$ then has the form of a convolution, and its Fourier transform is therefore the product of the Fourier transforms of $\delta(\mathbf{r},\mathbf{r}')-J(|\mathbf{r}-\mathbf{r}'|)$ and of $\mathcal{V}$. The transform of the former is $1-j(k)$, where[16,17]

$$j(k)=\begin{cases}1-(3k/4K)+(k^3/16K^3), & k<2K\\ 0, & k>2K\end{cases} \quad (27a)$$

is the ratio of the common volume of two spheres of radius K centered a distance k apart, to the volume of one of the spheres; K is the maximum momentum of the free particles which were substituted for ψ_0. The transform of the latter ($\mathcal{V}$) is

$$\int\tfrac{1}{2}Ce^2r^{-1}\exp(-\kappa r)\exp(-i\mathbf{k}\cdot\mathbf{r})d\mathbf{r} =2\pi Ce^2/(\kappa^2+k^2). \quad (27b)$$

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[14] R. D. Woods and D. S. Saxon, Phys. Rev. **95**, 577 (1954).

[15] Measurements and interpretations by M. Walt and J. R. Beyster [Bull. Am. Phys. Soc. **29**, No. 8, 31 (1954)] of the interaction of 4.1-Mev neutrons with nuclei show that the complex square-well potential cannot give enough absorption, no matter what value is used for the imaginary part of the potential. The same difficulty was noticed by H. E. DeWitt in the interpretation of the 14-Mev interaction data of J. H. Coon, Phys. Rev. **94**, 785(A) (1954) and of Phillips, Davis, and Graves, Phys. Rev. **88**, 600 (1952). These difficulties indicate that the abrupt discontinuity at the surface of the square-well potential reflects too much and thus does not allow the incident wave to penetrate into the interior where it can be absorbed.

[16] E. P. Wigner and F. Seitz, Phys. Rev. **43**, 804 (1933), Eq. (6).

[17] $J(\mathbf{r})$ is a sum $\sum_{\mathbf{k}',\mathbf{k}''}\exp[i(\mathbf{k}'-\mathbf{k}'')\cdot\mathbf{r}]$, where both $\mathbf{k}'$ and $\mathbf{k}''$ are vectors inside a sphere of radius K, the maximum momentum of the free fermions. Hence, the Fourier transform of J, that is the coefficient of a definite $\exp(i\mathbf{k}\cdot\mathbf{r})$ in the sum, is the number of ways $\mathbf{k}$ can be written as $\mathbf{k}=\mathbf{k}'-\mathbf{k}''$ with both $|\mathbf{k}'|<K$, $|\mathbf{k}''|<K$. By approximating the double sum by integrations, this number is found to be proportional to the common volume of two spheres of radius K centered a distance k apart. The corresponding expression for J is $J(r)=(3K^3/2\pi^2)[(z\cos z-\sin z)/z^3]^2$, where $z=Kr$.

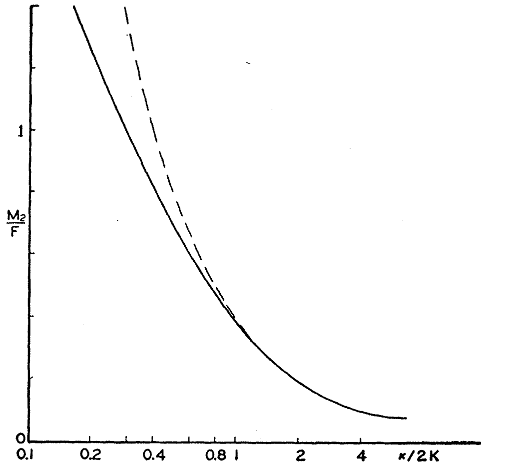

FIG. 4. The solid line is the calculated value of the second moment of the giant resonance, M_2 of Eq. (28), plotted in units of $F=2A\rho C^2e^4/K$ as a function of $x=\kappa/2K$, where κ^{-1} is the range of the internucleon interaction, and K is the maximum momentum of the free particles which were substituted for the normal state of the residual nucleus. The values assumed for the quantities in F are those given in the caption to Fig. 3, and $\rho=[(4/3)\pi a^3]^{-1}$ is the average density of a nucleon in the nucleus. The dashed line is the limiting form $\pi/8x$ of M_2/F for large x, which would apply for all x if the residual nucleus were idealized as an ordinary rather than as a degenerate Fermi gas.

The Fourier transform $f(\mathbf{k})$ of the $\mathbf{r}'$ integral of (26) is therefore, as a function of $\mathbf{r}-\mathbf{r}_n$, given by

$$f(k)=\frac{2\pi Ce^2}{\kappa^2+k^2}\times\begin{cases}(3k/4K)-(k^3/16K^3), & k<2K\\ 1, & k>2K.\end{cases}\tag{27c}$$

Neglecting again the surface effects, the $\mathbf{r}_n$ integration is extended over all space rather than just over the nucleus. As a result, one can integrate the product of the Fourier transforms of the two factors over $\mathbf{k}$ and divide by $(2\pi)^3$ instead of integrating over $\mathbf{r}_n$. The transform of the second factor is (27c) while that of the first factor is $2\pi\rho Ce^2/(\kappa^2+k^2)$ if $|u_p(\mathbf{r}_n)|^2$ is replaced by ρ. The resulting integral no longer depends upon $\mathbf{r}$, and therefore the $\mathbf{r}$ integration simply cancels a ρ, with the result that

$$M_2=A(2\pi)^{-3}\rho\int d\mathbf{k}f(k)2\pi Ce^2/(\kappa^2+k^2)$$

$$=F[\tfrac{1}{4}x^{-1}\tan^{-1}x-\tfrac{1}{4}+(\tfrac{3}{8}+\tfrac{1}{4}x^2)\ln(1+x^{-2})],\tag{28}$$

where $F=2A\rho C^2e^4/K$ and $x=\kappa/2K$. With the constants

$$K=(9\pi A/8a^3)^{\frac{1}{3}}=3.4\,mc^2/e^2,\quad \rho A=2.6\,(mc^2/e^2)^3,$$

the factor $F=(24.4\text{ Mev})^2$. The quantity M_2/F is plotted in Fig. 4 as a function of x. For the assumed value $x=2.5/(2\times3.38)=0.37$, the numerical value of $M_2^{\frac{1}{2}}$ is 22.5 Mev, which is at least an order of magnitude larger than the width of the giant resonance inferred from the low-energy neutron data, that is, the imaginary part of the complex potential of Feshbach *et al.* If the residual nucleus had been idealized as an ordinary rather than as a degenerate Fermi gas, the function M_2/F of (28) would have become equal for all x to $\pi/8x$, its limiting form for large x. Although the application of the exclusion principle to the residual nucleus decreases M_2 and hence the deviation from the independent-particle model,[18] the decrease is far from enough (only 23 percent in the above evaluation) to explain the approximate validity of this model.[19] The above calculation is surely inaccurate, and the calculation of $\bar{V}$ shows that the surface terms may indeed reduce the result by a factor two. On the other hand, the result would be increased by taking into account the Majorana and tensor parts of the interaction. It may also be noted that with the simple potential (19), the third and higher moments are infinite.

As explained in Sec. III, the fact that a finite second moment is calculated is not consistent with any complex potential representation. However, it is of course possible that there are interaction terms more singular than (19) which would make M_2 infinite. Barring this possibility, the finiteness of M_2 and the nonfiniteness of M_4 would be consistent with a strength function of the form

$$s_p(E_\lambda)dE_\lambda=\zeta_p^2(2^{\frac{1}{2}}W^3/8\pi)dE_\lambda/[(E_\lambda-E_p)^4+(\tfrac{1}{2}W)^4],$$

among others. However, the half-width $\frac{1}{2}W$ of this function is equal to the square root of the second moment, indicating that for such a function our calculated value of $M_2^{\frac{1}{2}}$ (about 23 Mev) disagrees by an order of magnitude with the observed value (about $1\frac{1}{2}$ Mev, the half-width of the giant resonance of the complex square well).

Agreement with the observed width $W\sim1\frac{1}{2}$ Mev could be obtained in two ways. One may either arbitrarily increase the range κ^{-1} of the forces by about a factor 3 and decrease their strength C [see Eq. (19)] by a factor 9 so that the space integral of the potential remains the same. This would naturally reduce the fluctuations of the departure potential V, and (28) shows that it would reduce M_2 to the magnitude of the observed value. The possibility that the meson cloud is smeared out in heavier nuclei to such an extent that the potential is fairly smooth has often been considered.

Another way to bring calculation and observations into agreement is based on the fact that decompositions

[18] V. Weisskopf, Science **113**, 101 (1951). See also Morrison, Muirhead, and Rosser, Phil. Mag. **44**, 1326 (1953); M. L. Goldberger, Phys. Rev. **74**, 1269 (1948).

[19] The situation is similar to, but more extreme than in the normal state of nuclei. See E. P. Wigner, *On the Shell Model for Nuclei* (L. Farkas Memorial Volume, Jerusalem, 1952), p. 45. This article calculates the deviations from the independent-particle model wave functions for the normal states of nuclei, assuming a potential similar to (19). Although the exclusion principle is fully taken into account in the calculation, [rather than only partially as in the present calculation, see the remark after (3)] the conclusion is arrived at that the wave function of the independent-particle model (i.e., a single Slater determinant) represents only a rather small part of the actual wave function. However, this fraction becomes even smaller, in fact very much smaller, if one disregards the exclusion principle.

of the X_λ which are more general than (2) may be used. In the first place, although it is probably most convenient to use (5a) for the potential energy of u_p, it is not necessary to do so, and one may consider the use of an independent-particle potential energy which minimizes the spread W of the p-group strength functions. Secondly, it is apparent that, with the exception of (4) and (5), the formulas of II remain valid even if the ψ_c are allowed to depend parametrically on the coordinate $\mathbf{r}_n$ of the incident particle. Such a parametric dependence of ψ_c on $\mathbf{r}_n$ corresponds to a polarization of the residual nucleus[20] by the incident nucleon, and one may hope that the corresponding $\psi_c(\mathbf{r}_1,\mathbf{r}_2,\cdots\mathbf{r}_A;\mathbf{r}_n)u_p(\mathbf{r}_n)$ takes the interaction of the last nucleon with the residual nucleus more fully into account than the $\psi_c u_p$ of (2). As a result, the square integral of $(V-E_{cp})\psi_c u_p$ will become smaller and the calculated width of the giant resonance less than as given above. This second possibility would seem to contradict the calculations carried out above and are, in fact, compatible with them only if (8a) and (8b) are not valid. The explanation is that the terms of (7a) can interfere destructively between giant resonances and constructively at these resonances so that the results based on the absence of such interferences, in particular the calculation of Sec. V, becomes invalid. We have as yet not found convincing evidence for the viewpoints just put forward.

Finally, it is noted that our formula (28) for M_2 shows no dependence on the energy of the incident particle, although it is found that to explain the data the imaginary part of the complex potential must be increased from about $1\frac{1}{2}$ to 8 Mev as the energy of the incident particle increases from a few to about 20 Mev.[14,15] About one-half of this increase could be accounted for by the increase of the absorption width Γ which appears in the Stieltjes transform (11) (see Appendix B of reference 3).

[20] The polarization of the nucleus by one of the particles contained therein was discussed by K. M. Watson, Phys. Rev. **89**, 575 (1953); N. C. Francis and K. M. Watson, Phys. Rev. **92**, 291 (1953); Brueckner, Levinson, and Mahmoud, Phys. Rev. **95**, 655 (1954); Gyo Takedo and K. M. Watson, Phys. Rev. **97**, 1336 (1955).

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Gamma Rays from the Inelastic Scattering of Neutrons in Aluminum, Magnesium, and Silver*

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Monoenergetic neutrons from the $H^2(d,n)He^3$ reaction were used to bombard scatterers each in the form of a ring surrounding an unshielded NaI(Tl) gamma-ray spectrometer. The gamma-ray spectrum for each scatterer was obtained by subtracting the background counting rate from the counting rate with the scatterer in place. An analysis of the gamma-ray spectra yields discrete gamma-rays for each scatterer as follows: Al: 0.422*, 0.843, 0.988, 1.69, and 2.10 Mev; Mg: 0.438, 0.555, 0.688, 0.837, 1.00, 1.34, 1.91, 2.08*, and 2.44 Mev; Ag: 0.332*, 0.696, 0.795, 1.10, 1.99, 2.13*, 2.32, and 2.54* Mev. The starred gamma-ray energies denote those that have not been previously reported.

INTRODUCTION

IN the neutron inelastic scattering process the incident neutron energy is reduced, and the target nucleus is left in an excited state. The excited nucleus generally decays to its ground state by the emission of one or more gamma rays. The nuclear energy levels may be obtained directly from a measurement of the energies of the groups of inelastically scattered neutrons. Very little information[1] has been obtained in this way due to the very poor energy resolution of neutron spectrometers. Energy level separations may be determined by a measurement of the energies of the de-excitation gamma rays. The determination of the energies of the de-excitation gamma rays has been the subject of many investigations.[2–11]

EXPERIMENTAL PROCEDURE

Monoenergetic neutrons with an energy of ~2.7 Mev were obtained from the $H^2(d,n)He^3$ reaction. The deuterons were accelerated in the University of Kentucky 120-kv low-voltage accelerator. A neutron flux

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[11] Rayburn, Lafferty, and Hahn, Phys. Rev. **94**, 1641 (1954).