Soluble Problem in the Theory of Coulomb Excitation

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(Received January 6, 1955)

The quantum treatment of Coulomb excitation in the limiting case of no energy loss involves radial Coulomb integrals that can be expressed in simple terms. The excitation function, G, and particle parameter for the directional correlation, a_2, are evaluated for this case and compared to the classical limit. The deviation from the classical limit is found to be negligible for the excitation function in the region of experimental interest, but for the particle parameter a_2 the deviation is sizeable.

THE Coulomb excitation of nuclear levels has been customarily treated[1,2] as an interaction involving the electric field of impinging particles traveling in definite Kepler orbits. This use of classical trajectories is valid, according to Bohr,[3] if the parameter $\eta\equiv Z_1Z_2e^2/\hbar v$ is large compared to unity. There have been two types of problem considered: the total cross section for the excitation process[1] as measured, say, by the γ quanta produced, and the directional correlation[2] of these quanta with the incident particle beam. Numerous experiments in the range $\eta\sim 3$ to 10 have shown generally good agreement with the approximate theory for the total cross section, but recent data on the correlation has suggested appreciable deviations.[4] It is therefore of some interest to examine more critically the validity of the classical approximation.[5] An essentially exact quantum mechanical treatment of both problems has been carried out,[6] reducing the problem to integrals over the radial Coulomb wave functions. (Evaluation of these integrals, a rather formidable task, is in progress using electronic computers.) The usual classical approximation results from this exact quantum mechanical treatment by a simultaneous limiting process $\eta\rightarrow\infty$, $1-\rho\rightarrow 0$, $\eta(1-\rho)\equiv\xi\rightarrow$finite, where ρ is the ratio of the emergent to incident wave numbers for the impinging particle. This limit process is, in general, difficult, if not impossible, to carry through explicitly for the case of arbitrary energy losses.[7] For the particular case of no energy loss ($\xi=0$), however, the two limits may be carried out separately, letting first $\rho=1$ and then $\eta\rightarrow\infty$. Fortunately the relevant Coulomb integrals may be integrated exactly in this case,[8] and it is thus possible to treat the approach to the classical limit in detail. It is obvious from Bohr's considerations that the calculations must agree precisely in the limit $\eta\rightarrow\infty$, and we may therefore use the classical limit to normalize our results and simplify the discussion.

The analogy between the classical calculation as given in reference 2, and the quantum mechanical calculations given in reference 6, is remarkably close. Let us restrict attention to the quadrupole transitions in the following. The excitation function then is defined[2] as:

$$g_2(0)=\int_1^\infty \epsilon d\epsilon \sum_{\mu=0,\pm 2}|S_\mu^{(2)}(0)|^2, \tag{1}$$

while the quantum result[6] is, normalized as discussed above,

$$G(\eta,1)=32\eta^2\sum_{L=0}^{\infty}\left[\frac{L(L+1)(2L+1)}{(2L-1)(2L+3)}I_{L,L}^2+\frac{3L(L-1)}{2(2L-1)}\times I_{L,L-2}^2+\frac{3}{2}\frac{(L+1)(L+2)}{(2L+3)}I_{L,L+2}^2\right], \tag{2a}$$

$$I_{L,L'}\equiv\int_0^\infty r^{-3}drF_L(\eta,r)F_{L'}(\eta,r). \tag{2b}$$

(The $F_L(\eta,r)$ are the radial Coulomb wave functions, and the $I_{L,L'}$ is taken to be zero if $L+L'\leqslant 0$.)

Noting that the eccentricity ϵ is related to the angular momentum L by the equation $\epsilon^2=1+L^2/\eta^2$, one sees already the close formal connection between the two calculations. The Coulomb integrals can be evaluated, with the result that:

$$I_{L,L}=[2L(L+1)(2L+1)]^{-1}[2L+1-\pi\eta-i\eta\psi(L+1+i\eta)+i\eta\psi(L+1-i\eta)], \tag{3a}$$

$$I_{L,L+2}=I_{L+2,L}=\tfrac{1}{6}|L+1+i\eta|^{-1}|L+2+i\eta|^{-1}. \tag{3b}$$

The function $\psi(z)$ is the logarithmic derivative of the gamma function.

For large values of L these integrals approach the

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[2] K. Alder and A. Winther, Phys. Rev. **91**, 1518 (1953).

[3] N. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **18**, No. 8 (1948).

[4] P. H. Stelson and F. K. McGowan, Bull. Am. Phys. Soc. **29**, No. 7, 34 (1954).

[5] K. Alder and A. Winther, Phys. Rev. **96**, 237 (1954), have obtained somewhat more accurate results using a WKB approximation. G. Breit and P. B. Daitch, Phys. Rev. **96**, 1447 (1954); Daitch, Lazarus, Hull, Benedict, and Breit, Phys. Rev. **96**, 1449 (1954).

[6] L. C. Biedenharn and M. E. Rose, Oak Ridge National Laboratory Report ORNL-1789, September, 1954.

[7] An exception in the exact Sommerfeld dipole case.

[8] This result is a special case taken from a paper on radial Coulomb matrix elements, in preparation.

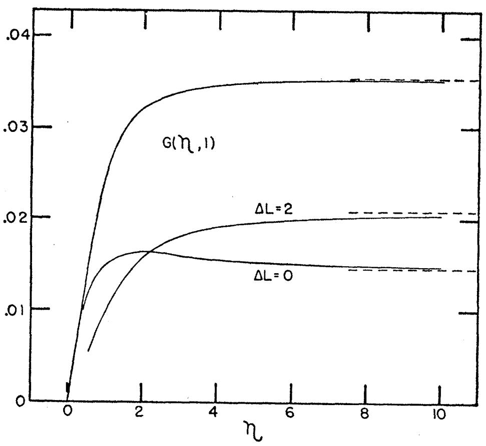

FIG. 1. The excitation function G for no energy loss ($\xi=0$) *versus* η. The lower curves are the $\Delta L=0,2$ components. The dotted lines are the classical limits.

functions $S_\mu^{(2)}$ given by Alder and Winther, and the correspondence between Eqs. (1) and (2a) is complete.

Figure 1 shows the behavior of the exact $G(\eta,1)$ function *versus* η. The classical limit is approached very quickly, the difference being only 9 percent at $\eta=2$. On the other hand, this behavior is to some extent fortuitous, as shown by the curves of the $\Delta L=0,2$ components, also plotted in Fig. 1. These depart in opposite directions from their classical limits by amounts which nearly cancel.

If one measures only the angle of emission of the γ-rays with respect to the incident particles, one obtains an angular distribution which is very similar to the angular correlation between two γ's in cascade. It can be shown that the distribution function is

$$W(\theta)=1+B_2a_2(\eta,\rho)P_2(\cos\theta)+B_4a_4(\eta,\rho)P_4(\cos\theta), \quad (4)$$

which is to be compared with the angular correlation

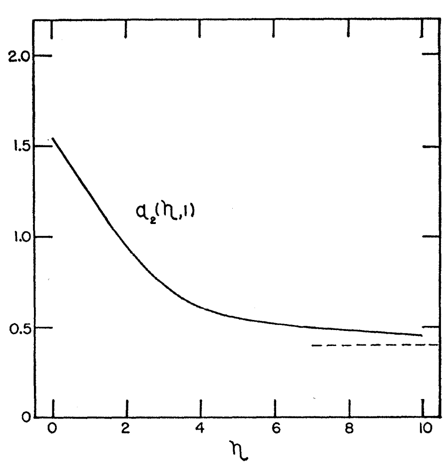

FIG. 2. The particle parameter a_2 *versus* η for no energy loss ($\xi=0$). The classical limit (dotted line) is 0.40567 while the intercept at $\eta=0$ is 1.5467

in the $\gamma-\gamma$ cascade,

$$J_i \xrightarrow{E2} J_{ce} \xrightarrow{\gamma} J_f,$$

given by

$$W'(\theta)=1+B_2P_2(\cos\theta)+B_4P_4(\cos\theta). \quad (5)$$

The J's are the spins of the initial, Coulomb-excited, and the final state after the γ emission, respectively. The first γ transition, being an electric quadrupole radiation, corresponds to the electric quadrupole excitation process. The B_k are tabulated in reference 9, among others.

Restricting attention to the coefficient of $P_2(\cos\theta)$, one finds in the quantum calculation that:

$$\begin{aligned}a_2(\eta,1)=[G(\eta,1)]^{-1}(32\eta^2)\sum_{L=0}^{\infty}\Bigg[&\frac{3(L-2)(L-1)L}{(2L-1)^2}I_{L,L-2}{}^2\\&-\frac{L(L+1)(2L+1)(2L-3)(2L+5)}{(2L-1)^2(2L+3)^2}I_{L,L}{}^2\\&+\frac{3(L+1)(L+2)(L+3)}{(2L+3)^2}I_{L,L+2}{}^2\\&+6\cos(\sigma_L-\sigma_{L-2})\cdot\frac{(L-1)(L)(L+1)}{(2L-1)^2}I_{L,L-2}I_{L,L}\\&+6\cos(\sigma_L-\sigma_{L+2})\cdot\frac{L(L+1)(L+2)}{(2L+3)^2}I_{L,L+2}I_{L,L}\Bigg]. \quad (6)\end{aligned}$$

(The σ_L are the Coulomb phase shifts for angular momentum L.)

In the limit of large L this is seen to correspond precisely term by term to the classical results given by Alder and Winther.

The behavior of the exact a_2 as a function of η is shown in Fig. 2. It is immediately apparent that the deviations from the classical limit are more significant than for the total cross section. This is not unexpected since the particle parameter a_2 is "phase-sensitive," and a classical approximation is generally inadequate. For example, the results differ by a factor of 2 at $\eta=2$, whereas the total cross section here differed by only a few percent.

The calculations at $\xi=0$ show great simplifications, and the origin of this may be found in the fact that this parameter measures the ratio of the collision time to the period of the emitted radiation. For $\xi=0$, therefore, the process is insensitive to all but the grossest details of the motion. The Coulomb integrals, in particular, show that the turning point radius, $r_t\sim(L^2+\eta^2)^{\frac{1}{2}}$, is effectively the only significant feature of the motion.

[9] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).

Since the major contribution to the sums comes from $L \sim \eta$, one sees that large η implies large L. Here the effects of quantization are small, and in this way the calculation becomes classical.

The particular case $\xi = 0$ is not of too much intrinsic importance, but it does serve to illustrate several typical features of the problem and the importance of more accurate calculations for the general case. Such calculations are in progress.

We would like to express our appreciation to the Oak Ridge National Laboratory where this work was begun while one of the authors was a summer visitor. In particular, we would like to thank Dr. M. E. Rose and Dr. R. A. Charpie for their help and encouragement.

PHYSICAL REVIEW VOLUME 98, NUMBER 3 MAY 1, 1955

Giant Resonance Interpretation of the Nucleon-Nucleus Interaction

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(Received January 24, 1955)

In the consideration of the independent-particle model, a distinction can be made between the spacing D of the levels of the whole nucleus and the spacing d of the levels of individual nucleons. Except in the immediate neighborhood of the normal state of closed-shell nuclei, $d \gg D$. In the "giant-resonance" interpretation considered here, the deviations from the independent-particle model are strong enough to mix many states of the whole nucleus, but the mixing is restricted to an energy range which is less than the order of d. According to this interpretation, the reduced particle widths of the levels of the compound nucleus are, on the average, anomalously large close to the energy values of those states of the independent-particle model which correspond to an unexcited target nucleus and a virtual level of the incident particle. As a consequence, the nuclear cross sections have a gross structure which is similar to a giant resonance, such as is implied by the complex square well representation of the nucleon-nucleus interaction. The position, width, and height of these maxima in the average cross sections are expressed in terms of the parameters of the independent-particle model and the departure of the actual nuclear potential which are responsible for the inaccuracy of this model. It is shown, however, that the conventional nuclear potential gives far too large values for the widths of the giant resonances (that is, for the imaginary part of the presentative complex square well potential).

I. INTRODUCTION

FESHBACH, Porter, and Weisskopf[1] have shown that a complex square well potential gives an accurate representation of some of the features of the neutron-nuclei interaction data at low and intermediate energies ($\lesssim 3$ Mev), such as the total cross section measurements by Barschall, Nereson, and collaborators and the angular distribution data of Walt and Barschall.[2] It has also been shown by them and by one of us[3] that one implication of such a representation is that the sum of the reduced neutron widths $\gamma_{\lambda n}^2$ per unit energy interval of the levels λ of the compound nucleus has a giant resonance-like dependence on the real energies E_λ of these levels. This sum plays a decisive part in the theoretical development and is referred to there as the strength function $s_n(E_\lambda) = \langle \gamma_{\lambda n}^2 \rangle_{\mathrm{Av}}/D$, where D is the mean spacing of the E_λ. The maxima of the giant resonances are associated with the positions E_p of the levels p of the real part of the representative potential, and their widths W_p are related to twice the imaginary part. It is presumed that the real part of the potential is essentially that potential which determines the configuration assignments in the shell-model theory, while the imaginary part is considered as representing the departures from this theory which are expected to be important at the higher excitation energies involved in scattering and reaction phenomena. Although it is beyond the scope of the complex potential representation to specify the properties of the individual resonance

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] Feshbach, Porter, and Weisskopf, Phys. Rev. **90**, 166 (1953); **96**, 448 (1954); R. K. Adair, Phys. Rev. **94**, 737 (1954). The first attempt to interpret the long-range fluctuations of the neutron cross sections by means of a simple potential is due to K. W. Ford and D. Bohm, Phys. Rev. **79**, 745 (1950). A similar model was used for the explanation of the high-energy cross sections by Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949). In fact, the early explanations of the large neutron cross sections by Amaldi, D'Agostino, Fermi, Pontecorvo, Rasetti, and Sègre [Proc. Roy. Soc. (London) **A149**, 522 (1935)], by H. A. Bethe [Phys. Rev. **47**, 747 (1935)], by G. Beck and L. H. Horsley [Phys. Rev. **47**, 510 (1935)] by F. Perrin and W. M. Elsasser [J. phys. radium **6**, 194 (1935)], were all based on a similar model. However, Feshbach, Porter, and Weisskopf were the first ones to recognize that the cross section obtained from the simple potential is not the actual cross section but only its average over many resonance levels, and they were the first ones who thoroughly explored the consequences of their model.

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[3] R. G. Thomas, Phys. Rev. **97**, 224 (1955).